Search for scalar and tensor glueballs

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Quark systematics of meson states

From three quarks u, d and s the following quark-antiquark nonets can be constructed:

$$K^0 = d\bar{s} \qquad K^+ = u\bar{s}$$

$$\pi^- = d\bar{u} \qquad \pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) \qquad \pi^+ = u\bar{d}$$

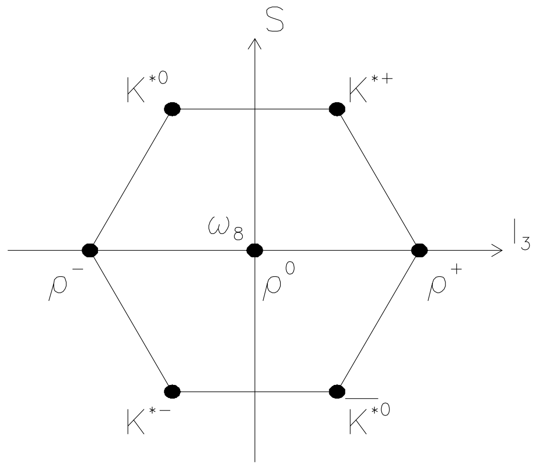
$$K^- = s\bar{u} \qquad \bar{K}^0 = s\bar{d}$$

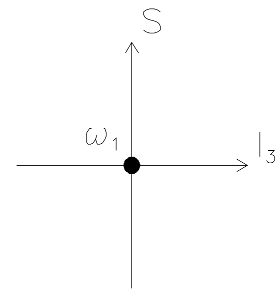
$$\eta_8 = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s}) \qquad \eta_1 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$$

$$\begin{aligned} \eta &= \eta_8 \cos\phi + \eta_1 \sin\phi \\ \eta' &= -\eta_8 \sin\phi + \eta_1 \cos\phi \end{aligned}$$

Vector particles

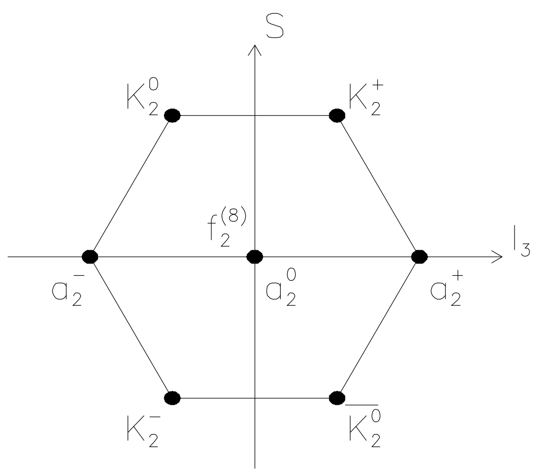
$J^{PC} = 1^{--}$:

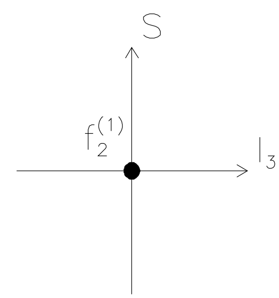




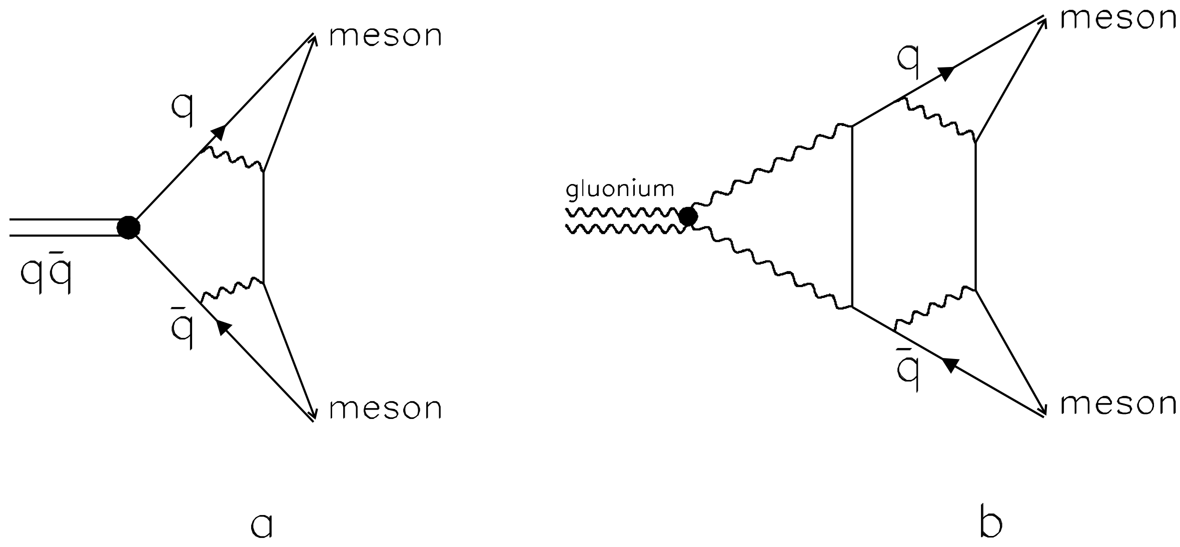
Tensor particles

$J^{PC} = 2^{++}$:





Decay of $\bar{q}q$ and glueball states



The probability to create new $q\bar{q}$-pairs by the gluon field is:

$$u\bar{u} : d\bar{d} : s\bar{s} = 1 : 1 : \lambda \qquad \lambda \simeq 0.5 - 0.8$$

The pure glueball has the quark-antiquark component

$$(q\bar{q})_{glueball} = (u\bar{u} + d\bar{d} + \sqrt{\lambda} s\bar{s})/\sqrt{2+\lambda},$$

$$\varphi_{glueball} \simeq 27^\circ - 33^\circ \ ,$$

For the decay couplings squared for $f_0 \to \pi\pi, K\bar{K}, \eta\eta, \eta\eta'$, the quark-combinatoric rules, in case when the f_0 state is the mixture of the quarkonium ($q\bar{q} = n\bar{n}\ \cos\varphi + s\bar{s}\ \sin\varphi$ where $n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$) and gluonium ($gg$) components, give us:

$$
\begin{aligned}
g^2_{\pi\pi} &= \frac{3}{2}\left(\frac{g}{\sqrt{2}}\cos\varphi + \frac{G}{\sqrt{2+\lambda}}\right)^2 , \\
g^2_{K\bar{K}} &= 2\left(\frac{g}{2}(\sin\varphi + \sqrt{\frac{\lambda}{2}}\cos\varphi) + G\sqrt{\frac{\lambda}{2+\lambda}}\right)^2 , \\
g^2_{\eta\eta} &= \frac{1}{2}\left(g(\frac{\cos^2\Theta}{\sqrt{2}}\cos\varphi + \sqrt{\lambda}\sin\varphi\sin^2\Theta) + \frac{G}{\sqrt{2+\lambda}}(\cos^2\Theta + \lambda\sin^2\Theta)\right)^2 , \\
g^2_{\eta\eta'} &= \sin^2\Theta\cos^2\Theta\left(g(\frac{1}{\sqrt{2}}\cos\varphi - \sqrt{\lambda}\sin\varphi) + G\frac{1-\lambda}{\sqrt{2+\lambda}}\right)^2 .
\end{aligned}
$$

The angle Θ determines contents of η and η' mesons: $\eta = \cos\Theta\, n\bar{n} - \sin\Theta\, s\bar{s}$ and $\eta' = \sin\Theta\, n\bar{n} + \cos\Theta\, s\bar{s}$; we use $\Theta = 38^\circ$.

Nonet classification: the decay properties of all particles are described by one $SU(3)$ coupling, mixing angle and masses of particles.

Scalar sector:

$f_0(980)$	$M = 980 \pm 10$ **MeV**	$\Gamma = 40 - 100$ **MeV**	$\pi\pi$
$a_0(980)$	$M = 984.7 \pm 1.2$ **MeV**	$\Gamma = 50 - 100$ **MeV**	$\pi\eta$
σ	$M = 400 - 1600$ **MeV**	$\Gamma = 600 - 1200$ **MeV**	$\pi\pi$
$K_0(1430)$	$M = 1412 \pm 6$ **MeV**	$\Gamma = 294 \pm 23$ **MeV**	$K\pi$
$f_0(1370)$	$M = 1310 \pm 30$ **MeV**	$\Gamma = 290 \pm 40$ **MeV**	4π
$a_0(1450)$	$M = 1530 \pm 30$ **MeV**	$\Gamma = 180 \pm 30$ **MeV**	$2\pi\omega$
$f_0(1500)$	$M = 1494 \pm 8$ **MeV**	$\Gamma = 112 \pm 8$ **MeV**	$\pi\pi$, 4π
$f_0(1750)$	$M = 1750 \pm 30$ **MeV**	$\Gamma = 210 \pm 60$ **MeV**	4π, $K\bar{K}$, $\eta\eta$
$K_0(1830)$	$M \sim 1830$ **MeV**	$\Gamma \sim 280$ **MeV**	
$f_0(1710)$	$M = 1724 \pm 7$ **MeV**	$\Gamma = 137 \pm 8$ **MeV**	$\pi\pi$, $K\bar{K}$, $\eta\eta$

Two body reactions:

Reaction	Experiment	Reaction	Experiment
$\pi^+\pi^- \to \pi^+\pi^-$ **(all waves)**	**CERN-Münich**		
$\pi\pi \to \pi^0\pi^0$ **(S-wave)**	**GAMS**	$\pi\pi \to \pi^0\pi^0$ **(S-wave)**	**E852**
$\pi\pi \to \eta\eta$ **(S-wave)**	**GAMS**	$\pi\pi \to \eta\eta'$ **(S-wave)**	**GAMS**
$\pi\pi \to K\bar{K}$ **(S-wave)**	**BNL**	$K^-\pi^+ \to K^-\pi^+$ **(S-wave)**	**LASS**

Three body reactions from Crystal Barrel: (L-liquid, G-gaseous targets).

Reaction	Target	Reaction	Target	Reaction	Target
$\bar{p}p \to \pi^0\pi^0\pi^0$	**(L)** H_2	$\bar{p}p \to \pi^+\pi^0\pi^-$	**(L)** H_2	$\bar{p}p \to K_S K_S \pi^0$	**(L)** H_2
$\bar{p}p \to \pi^0\eta\eta$	**(L)** H_2	$\bar{p}n \to \pi^0\pi^0\pi^-$	**(L)** D_2	$\bar{p}p \to K^+K^-\pi^0$	**(L)** H_2
$\bar{p}p \to \pi^0\pi^0\eta$	**(L)** H_2	$\bar{p}n \to \pi^-\pi^-\pi^+$	**(L)** D_2	$\bar{p}p \to K_L K^\pm \pi^\mp$	**(L)** H_2
$\bar{p}p \to \pi^0\pi^0\pi^0$	**(G)** H_2			$\bar{p}n \to K_S K_S \pi^-$	**(L)** D_2
$\bar{p}p \to \pi^0\eta\eta$	**(G)** H_2			$\bar{p}n \to K_S K^- \pi^0$	**(L)** D_2
$\bar{p}p \to \pi^0\pi^0\eta$	**(G)** H_2				

Parametrization of the K-matrix for S-wave:

$$K_{ab}(s) = \left(\sum_{\alpha} \frac{g_a^{(\alpha)} g_b^{(\alpha)}}{M_\alpha^2 - s} + f_{ab} \frac{1\ \text{GeV}^2 + s_0}{s + s_0} \right) \frac{s - s_A}{s + s_{A0}} \quad ,$$

where K_{ab} is a 5×5 matrix (a, b = $\pi\pi$, $K\bar{K}$, $\eta\eta$, $\eta\eta'$, $4\pi + \ldots$)

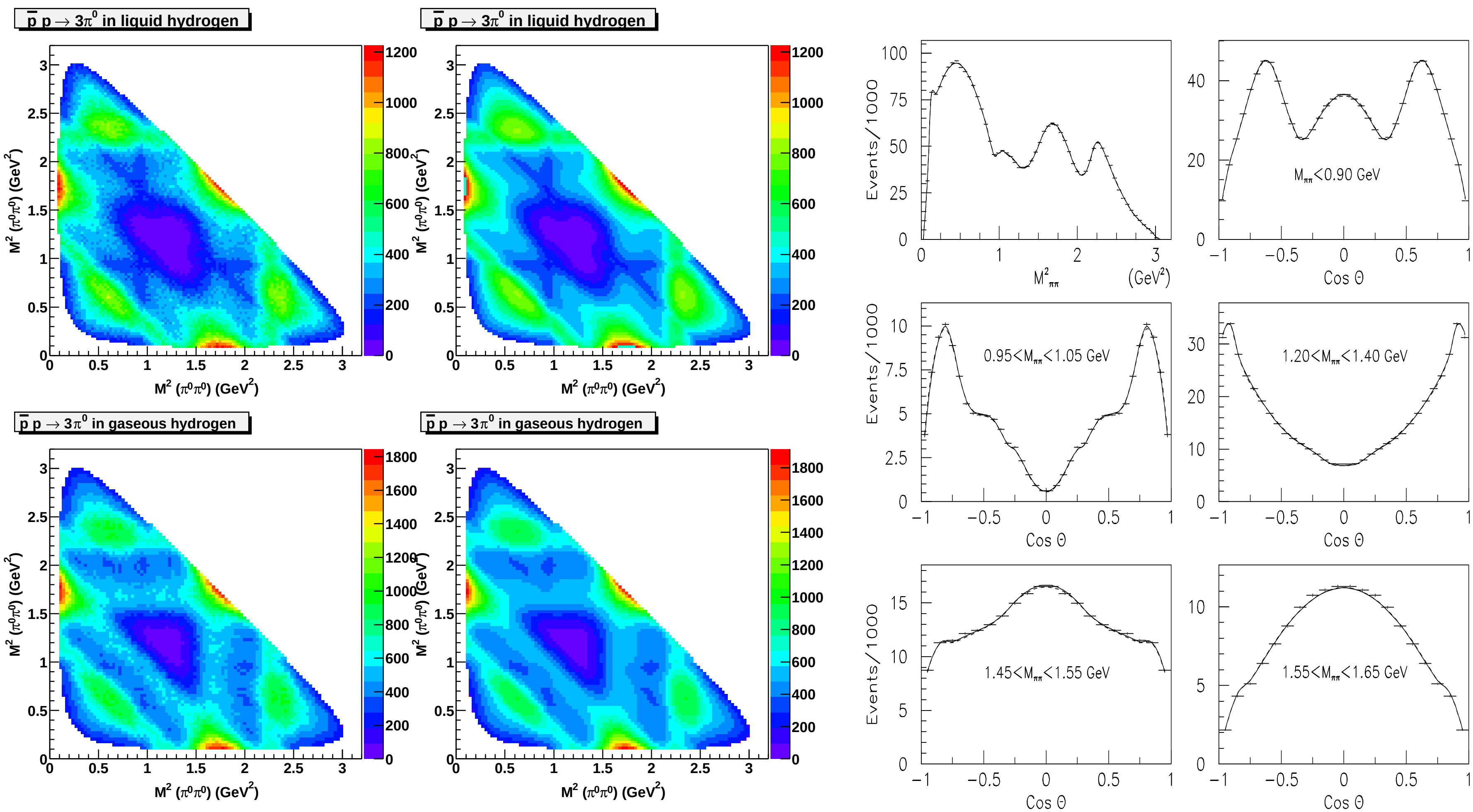
$$\rho_a(s) = \sqrt{\frac{s - (m_{1a} + m_{2a})^2}{s}} \quad , \qquad a = \pi,\ K,\ \eta.$$

The multimeson phase space factor is defined as

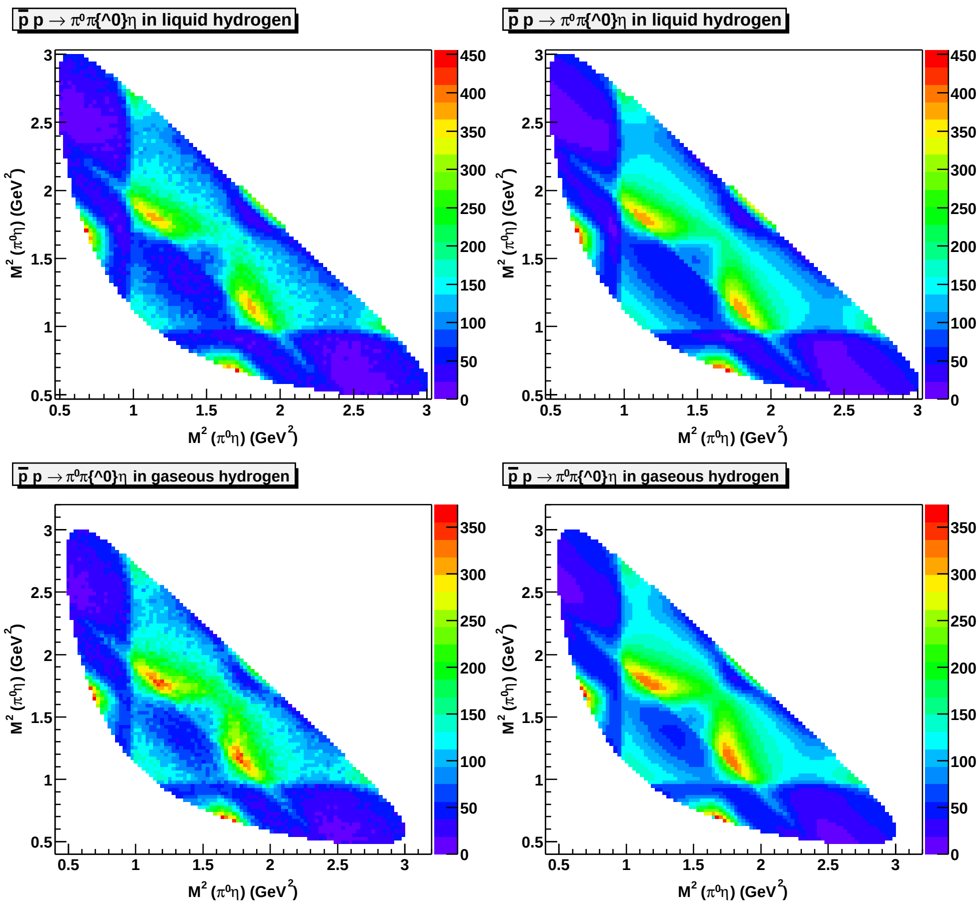
$$\rho_5(s) = \begin{cases} \rho_{51} & \text{at } s < 1\ \text{GeV}^2, \\ \rho_{52} & \text{at } s > 1\ \text{GeV}^2, \end{cases}$$

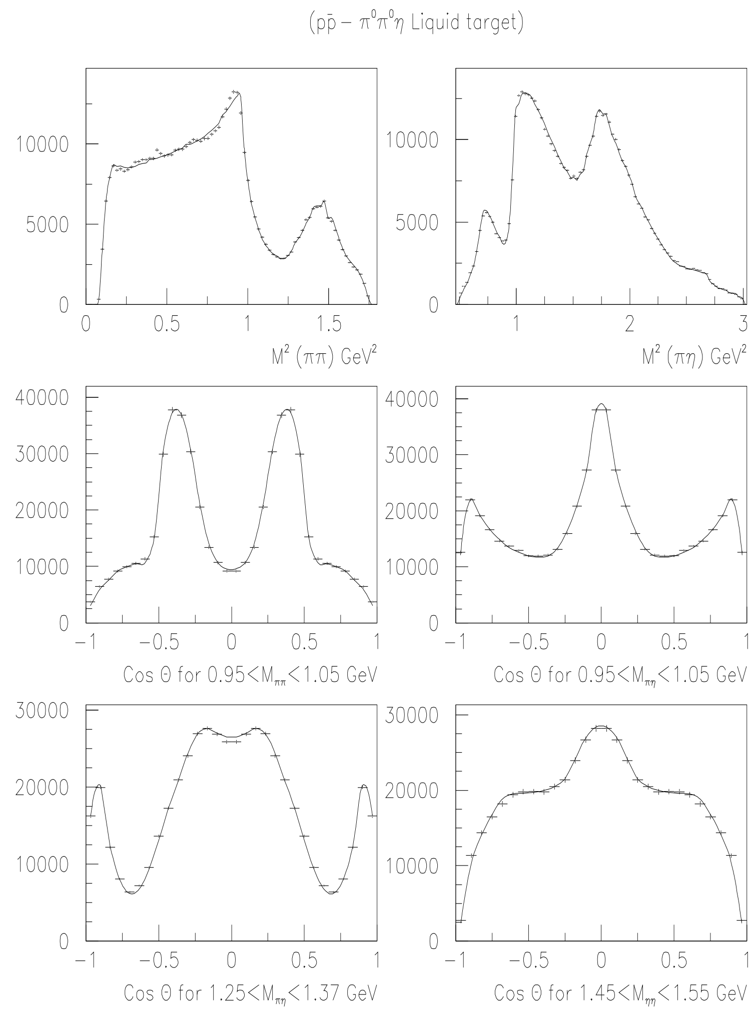
$$\rho_{51} = \rho_0 \int \frac{ds_1}{\pi} \int \frac{ds_2}{\pi} M^2 \Gamma(s_1) \Gamma(s_2) \sqrt{(s + s_1 - s_2)^2 - 4 s s_1} \times$$
$$\times s^{-1} [(M^2 - s_1)^2 + M^2 \Gamma^2(s_1)]^{-1} [(M^2 - s_2)^2 + M^2 \Gamma^2(s_2)]^{-1},$$
$$\rho_{52} = \left(\frac{s - 16 m_\pi^2}{s} \right)^n$$

The description of $p\bar{p} \to 3\pi^0$ CB-LEAR data

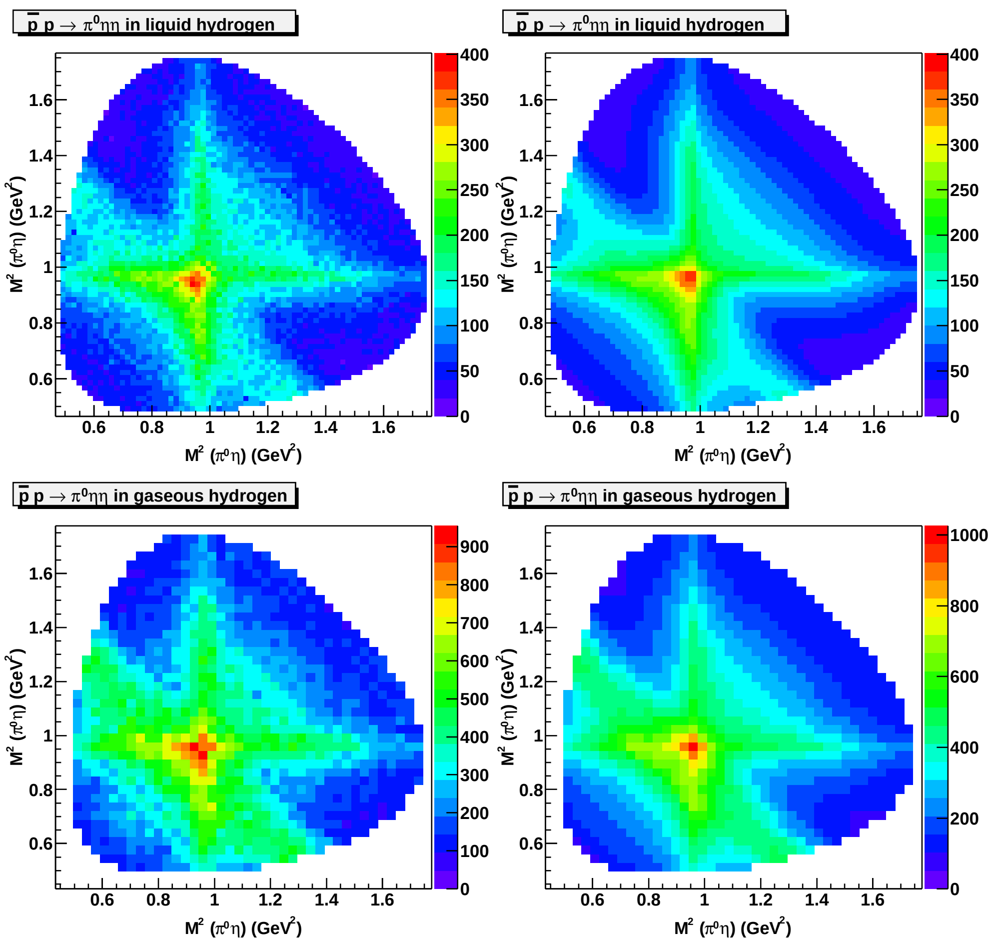


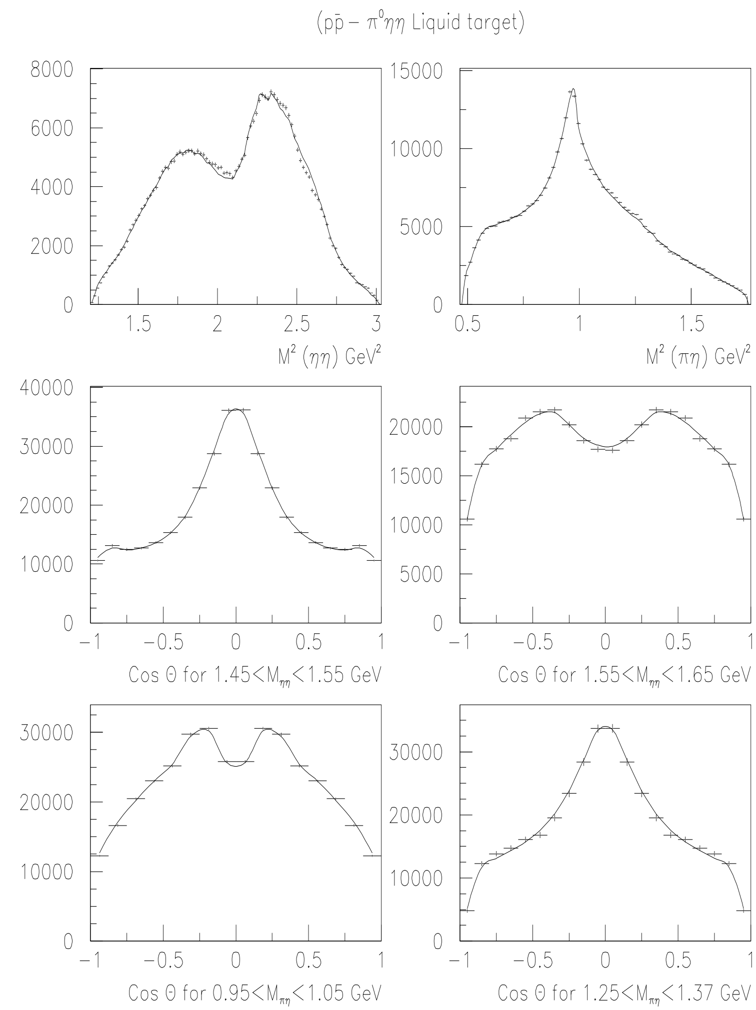
The description of $p\bar{p} \to \pi^0\pi^0\eta$ CB-LEAR data





The description of $p\bar{p} \rightarrow \pi^0\eta\eta$ CB-LEAR data

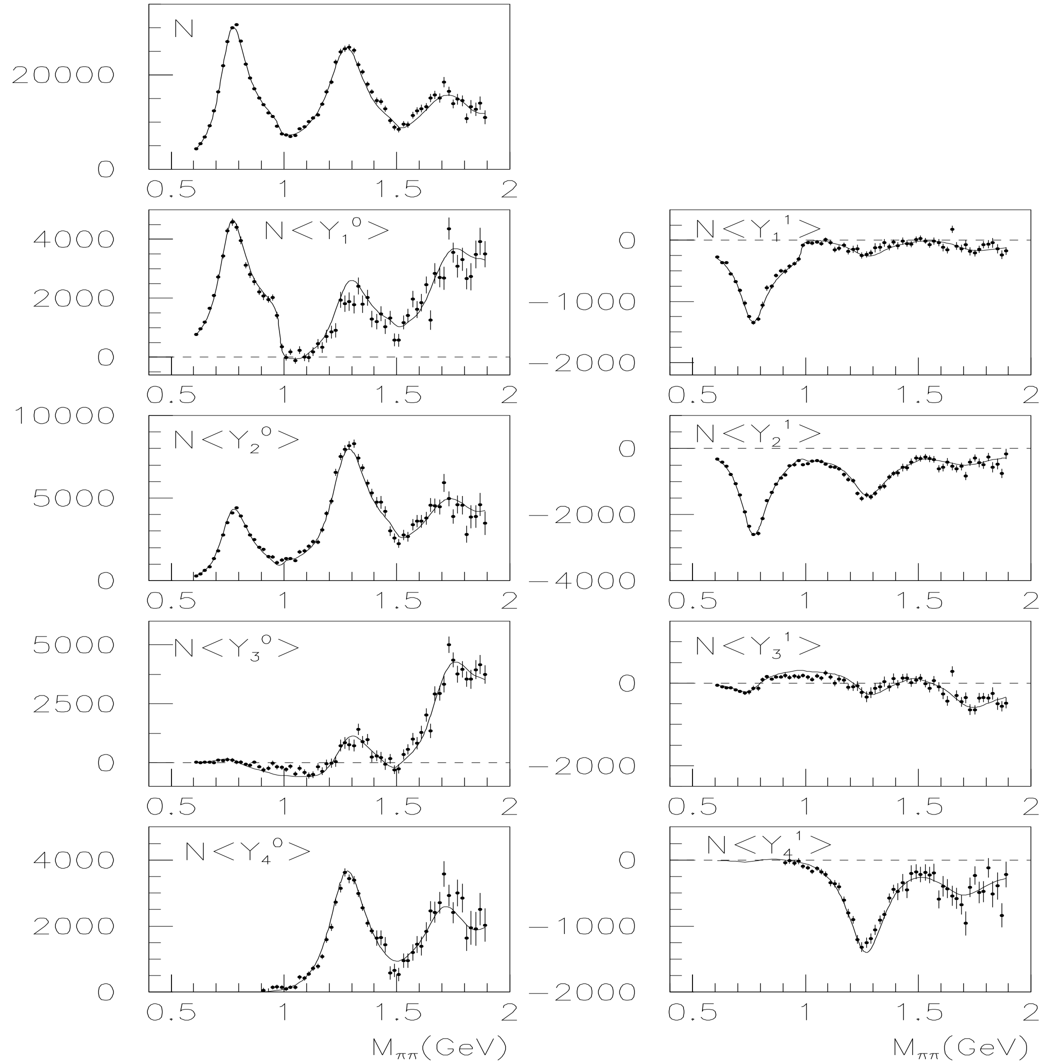




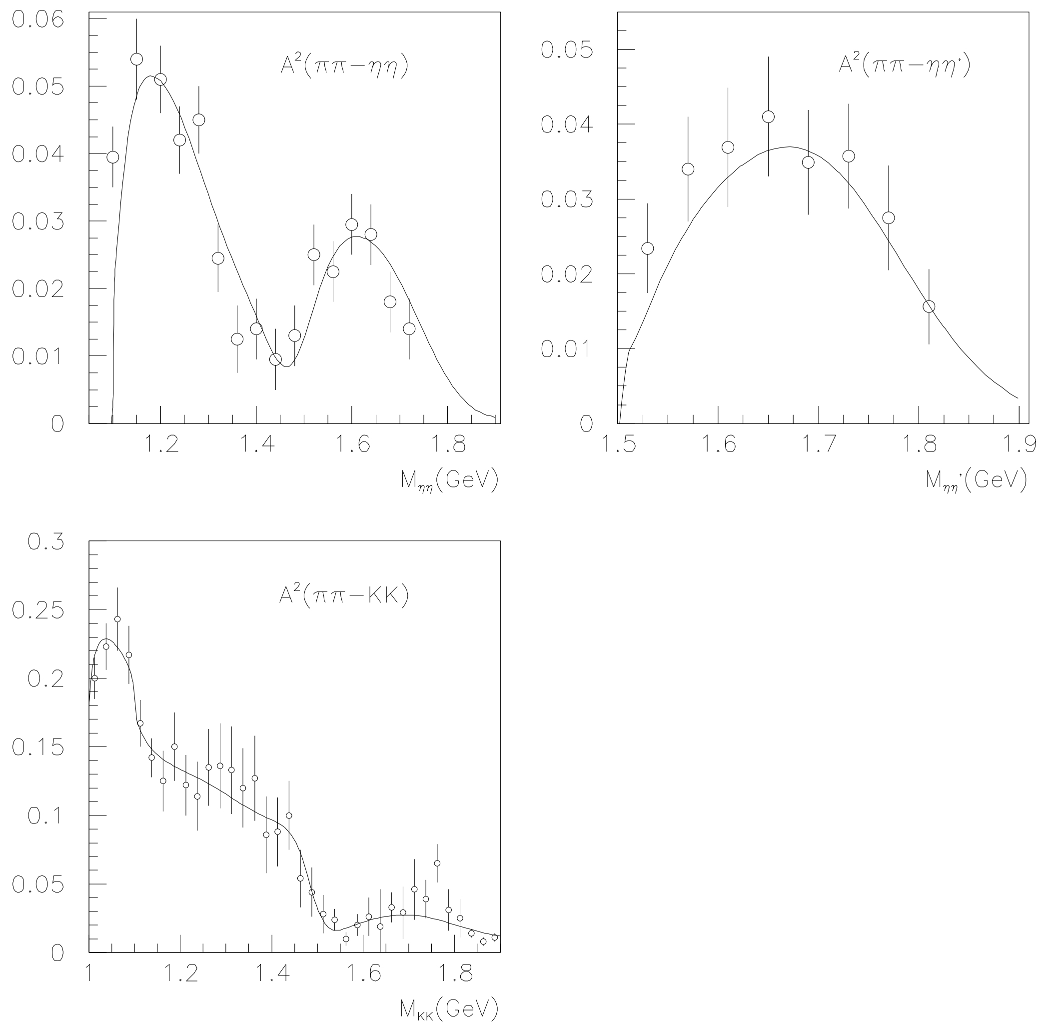
Description of the CERN-Munich data

$$\pi^- p \to \pi^+ \pi^- n$$

$$A_{1j} = K_{1m}(I - i\hat{\rho}(s)\hat{K})^{-1}_{mj}$$



The $\pi\pi \to \eta\eta$, $\pi\pi \to \eta\eta'$ (GAMS) and $\pi\pi \to K^+K^-$ (BNL) data



For the description of the 00^{++} wave in the mass region below 1900 MeV, 5 K-matrix poles are needed:

$$f_0^{\textbf{bare}}(680 \pm 100), \quad \psi = (0.45 \pm 0.1)n\bar{n} - (0.89 \pm 0.05)s\bar{s}\ ,$$
$$f_0^{\textbf{bare}}(1230 \pm 30), \quad \psi = (0.9^{+0.05}_{-0.2})n\bar{n} + (0.45^{+0.3}_{-0.1})s\bar{s}\ ,$$
$$f_0^{\textbf{bare}}(1260 \pm 30), \quad \psi = (0.93^{+0.02}_{-0.1})n\bar{n} + (0.37^{+0.2}_{-0.06})s\bar{s}\ ,$$
$$f_0^{\textbf{bare}}(1600 \pm 50), \quad \psi = (0.95 \pm 0.05)n\bar{n} + (0.3^{+014}_{-0.4})s\bar{s}\ ,$$
$$f_0^{\textbf{bare}}(1810 \pm 50), \quad \psi = \begin{cases} (0.10 \pm 0.05)n\bar{n} + (0.995^{+0.005}_{-0.015})s\bar{s}, \\ (\textbf{Solution } I), \\ (0.67 \pm 0.08)n\bar{n} - (0.74 \pm 0.08)s\bar{s}, \\ (\textbf{Solution } II). \end{cases}$$

Experimental data used in the fit do not fix unambiguously the flavor wave function of $f_0^{\textbf{bare}}(1810 \pm 50)$: two solutions are found for it.

The scattering amplitude has five poles in the energy complex plane, four of them correspond to relatively narrow resonances while the fifth resonance is very broad:

$$f_0(980) \to (1015 \pm 15) - i(43 \pm 8) \quad \textbf{MeV},$$

$$f_0(1300) \to (1310 \pm 20) - i(160 \pm 20) \quad \textbf{MeV},$$

$$f_0(1500) \to (1496 \pm 8) - i(58 \pm 10) \quad \textbf{MeV},$$

$$f_0(1530) \to (1530^{+90}_{-250}) - i(560 \pm 140) \quad \textbf{MeV},$$

$$f_0(1780) \to \begin{cases} (1780 \pm 30) - i(140 \pm 20) \ \textbf{MeV}, \\ (\textbf{Solution } I), \\ (1780 \pm 50) - i(220 \pm 50) \ \textbf{MeV}, \\ (\textbf{Solution } II). \end{cases}$$

Nonet classification:

The lightest scalar $q\bar{q}$ nonet is constructed uniquely as:

$1\,^3P_0$	$2\,^3P_0$ **(1)**	$2\,^3P_0$ **(2)**
$a_0^{\textbf{bare}}(980 \pm 30)$	$a_0^{\textbf{bare}}(1630 \pm 50)$	$a_0^{\textbf{bare}}(1630 \pm 50)$
$K_0^{\textbf{bare}}(1220^{+50}_{-50})$	$K_0^{\textbf{bare}}(1885^{+50}_{-100})$	$K_0^{\textbf{bare}}(1885^{+50}_{-100})$
$f_0^{\textbf{bare}}(680 \pm 100)$	$f_0^{\textbf{bare}}(1600 \pm 50)$	$f_0^{\textbf{bare}}(1230 \pm 30)$
$f_0^{\textbf{bare}}(1260 \pm 30)$	$f_0^{\textbf{bare}}(1810 \pm 50)$	$f_0^{\textbf{bare}}(1810 \pm 50)$
$\Phi(680) = -70^o\ {}^{+5^o}_{-16^o}$	$\Phi(1810) = 84^o \pm 5^o$	$\Phi(1810) = 44^o \pm 10^o$
	$f_0^{\textbf{bare}}(1230 \pm 30)$	$f_0^{\textbf{bare}}(1600 \pm 50)$

However the fit without $f_0(1370)$ is only slightly worse

Data	χ^2	χ^2 without $f_0(1370)$)
$\bar{p}p \longrightarrow \pi^0\pi^0\pi^0$ (Liq)	1.300	1.380
$\bar{p}p \longrightarrow \pi^0\pi^0\pi^0$ (Gas)	1.215	1.390
$\bar{p}p \longrightarrow \eta\pi^0\eta$ (Liq)	1.300	1.400
$\bar{p}p \longrightarrow \eta\pi^0\eta$ (Gas)	1.433	1.405
$\bar{p}p \longrightarrow \pi^0\eta\pi^0$ (Liq)	1.150	1.312
$\bar{p}p \longrightarrow \pi^0\eta\pi^0$(Gas)	1.090	1.200
$\pi\pi \longrightarrow \eta\eta$ (S-wave)	0.86	1.25
$\pi\pi \longrightarrow \eta\eta'$ (S-wave)	0.40	0.42

The description of CERN-Münich 1.20 $\longrightarrow$ 1.65 in combined analysis

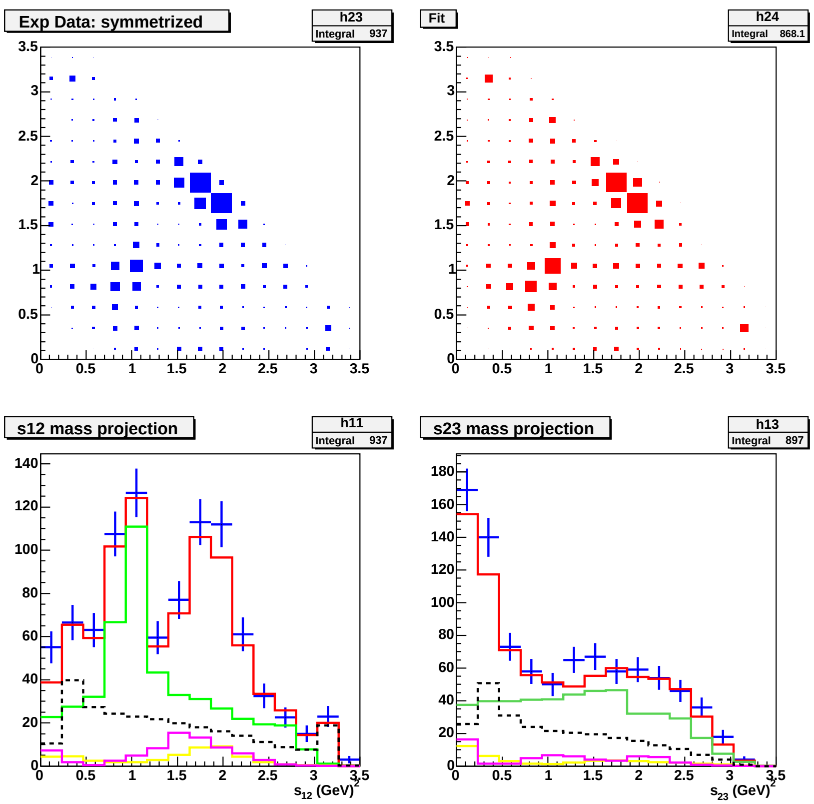
The description of CERN-Münich 1.10 $\longrightarrow$ 1.20 if fitted without $p\bar{p}$ data.

But it is expected that $Br_{\pi\pi}(f_0(1370))$ $<$10%

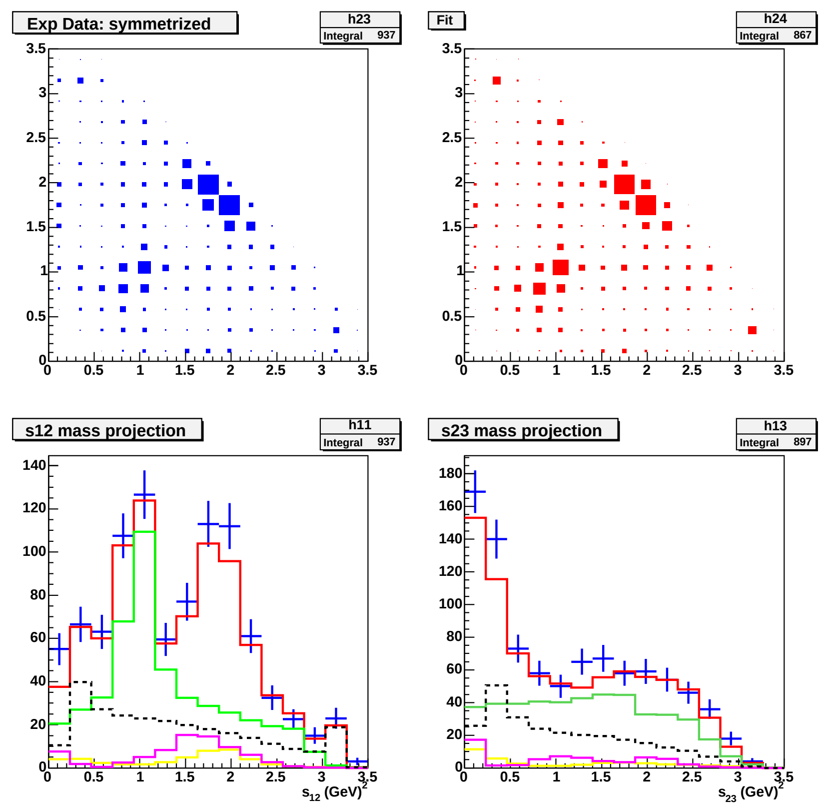
Observation of $f_0(1370)$ in the decay of D-meson

$$D_s^+ \to \pi^+ \pi^- \pi^+$$

The 5-pole K-matrix fit.



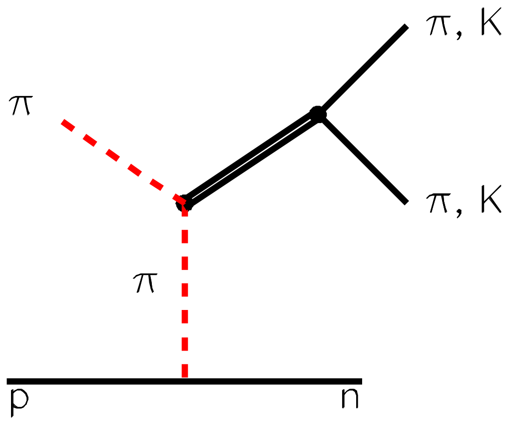
The fit without $f_0(1300)$.



The $f_0(1370)$ provides only marginal improvement in the combined fit.

E. Klempt, M. Matveev, A.V. Sarantsev, Eur.Phys.J.C55:39-50,2008.

The reactions $\pi N \rightarrow \pi\pi N$ at large energy transferred



$$d\sigma = \frac{(2\pi)^4|A|^2}{8\sqrt{s_{\pi N}}|\vec{p_2}|} d\Phi(p_1+p_2,k_1,k_2,k_3)$$

$$d\Phi(p_1+p_2,k_1,k_2,k_3) = (2\pi)^3 d\Phi(P,k_1,k_2)\, d\Phi(p_1+p_2,P,k_3)\, ds\,,$$

Then:

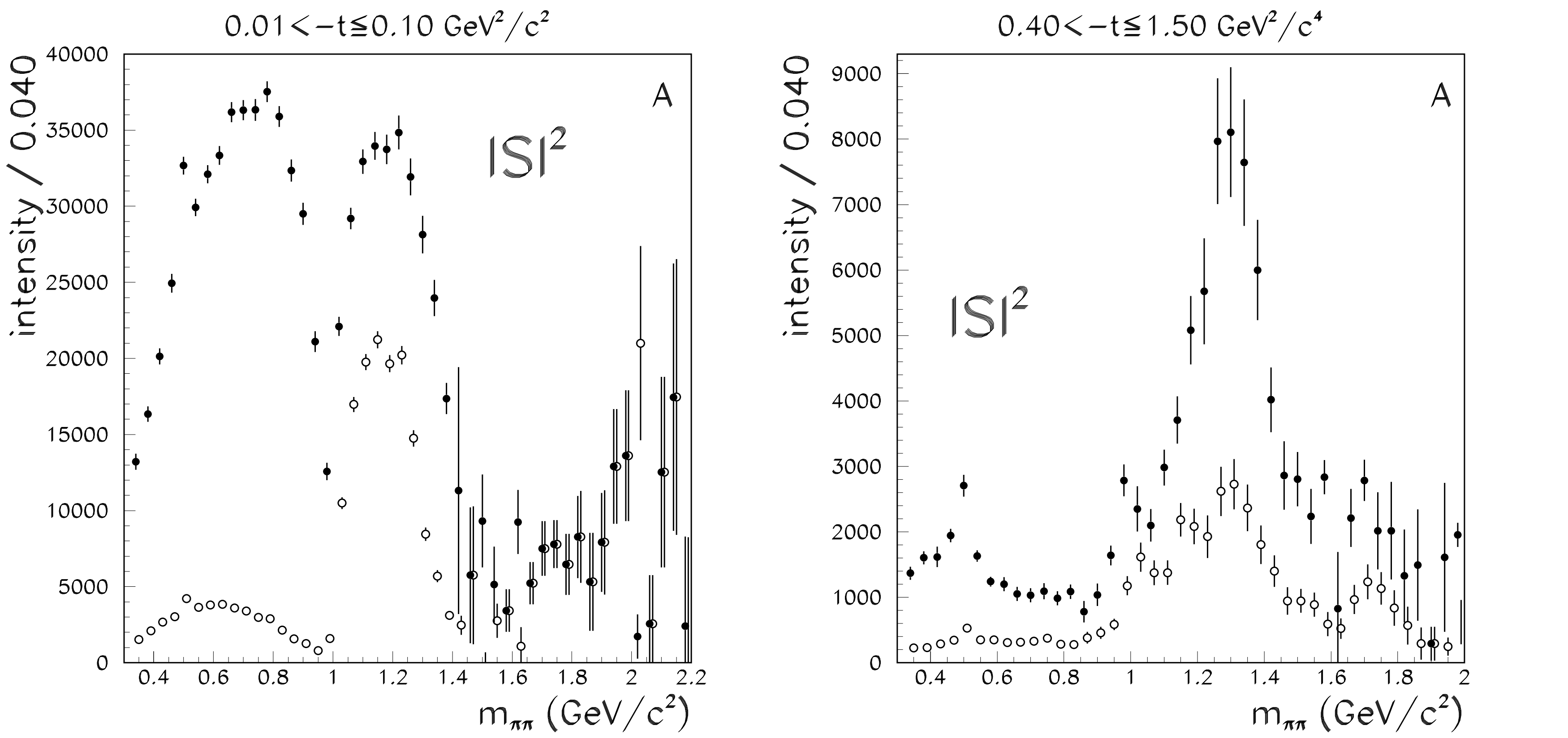
$$d\sigma \quad = \quad \frac{(2\pi)^4|A|^2(2\pi)^3}{8|\vec{p_2}|\sqrt{s_{\pi N}}} \frac{1}{(2\pi)^5} \frac{dt 2M\, dM\, d\Phi(P,k_1,k_2)}{8|\vec{p_2}|\sqrt{s_{\pi N}}} = \frac{(M|A|^2\rho) dt\, dM\, d\Omega}{(2\pi)^3 32|\vec{p_2}|^2\, s_{\pi N}}$$

Unitarity relation:

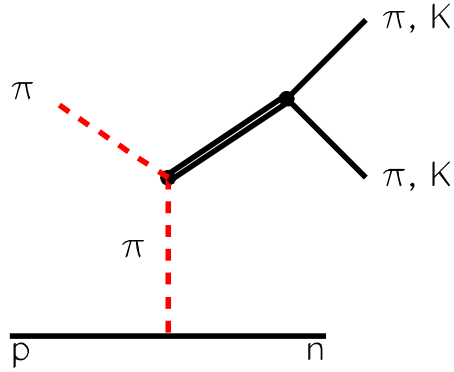
$$Im A = \rho(s)|A|^2$$

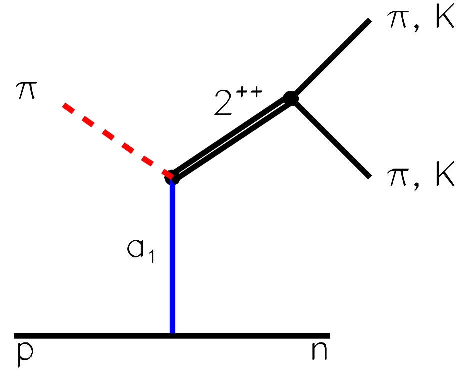
BNL analysis

The S-wave has a very prominent structure at large $|t|$.



Reggezied exchanges (π, a_1, π_2, a_2)





$$A^{(\text{pion trajectories})}_{\pi p\to\pi\pi n} = \sum_{\pi_j} A(\pi\pi_j \to \pi\pi) R_{\pi_j}(s_{\pi N}, q^2)\left(\varphi_n^+(\vec{\sigma}\vec{p}_\perp)\varphi_p\right) g_{pn}^{(\pi_j)}\,.$$

$$A^{(a_1-\text{trajectories})}_{\pi p\to\pi\pi n} = \sum_{a_1^{(j)}} A(\pi a_1^{(j)} \to \pi\pi) R_{a_1^{(j)}}(s_{\pi N}, q^2)\left(\varphi_n^+(\vec{\sigma}\vec{n}_z)\varphi_p\right) g_{pn}^{(a_1 j)}\,.$$

$$R_{\pi_j}(s_{\pi N}, q^2) = \exp\left(-i\frac{\pi}{2}\alpha_\pi^{(j)}(q^2)\right)\frac{(s_{\pi N}/s_{\pi N0})^{\alpha_\pi^{(j)}(q^2)}}{\sin\left(\frac{\pi}{2}\alpha_\pi^{(j)}(q^2)\right)\Gamma\left(\frac{1}{2}\alpha_\pi^{(j)}(q^2)+1\right)}$$

$$R_{a_1^{(j)}}(s_{\pi N}, q^2) = i\exp\left(-i\frac{\pi}{2}\alpha_{a_1}^{(j)}(q^2)\right)\frac{(s_{\pi N}/s_{\pi N0})^{\alpha_{a_1}^{(j)}(q^2)}}{\cos\left(\frac{\pi}{2}\alpha_{a_1}^{(j)}(q^2)\right)\Gamma\left(\frac{1}{2}\alpha_{a_1}^{(j)}(q^2)+\frac{1}{2}\right)}$$

Features of reggezied a_1 exchange:

$$A(\pi a_1^{(j)} \to \pi\pi) = \sum_J \epsilon_\beta^{(-)} \left[A^{(J+)}_{\pi a_1^{(j)} \to \pi\pi} X^{(J+1)}_{\beta\mu_1\ldots\mu_J} + A^{(J-)}_{\pi a_1^{(j)} \to \pi\pi} Z^{\beta}_{\mu_1\ldots\mu_J} \right] X^{(J)}_{\nu_1\ldots\nu_J} \,,$$

$$A(\pi a_1^{(k)} \to \pi\pi) = \sum_J \alpha_J |\vec{p}|^{J-1} |\vec{k}|^J \left(W_0^{(J)} Y_J^0(\Theta,\varphi) + W_1^{(J)} Re Y_J^1(\Theta,\varphi) \right)$$

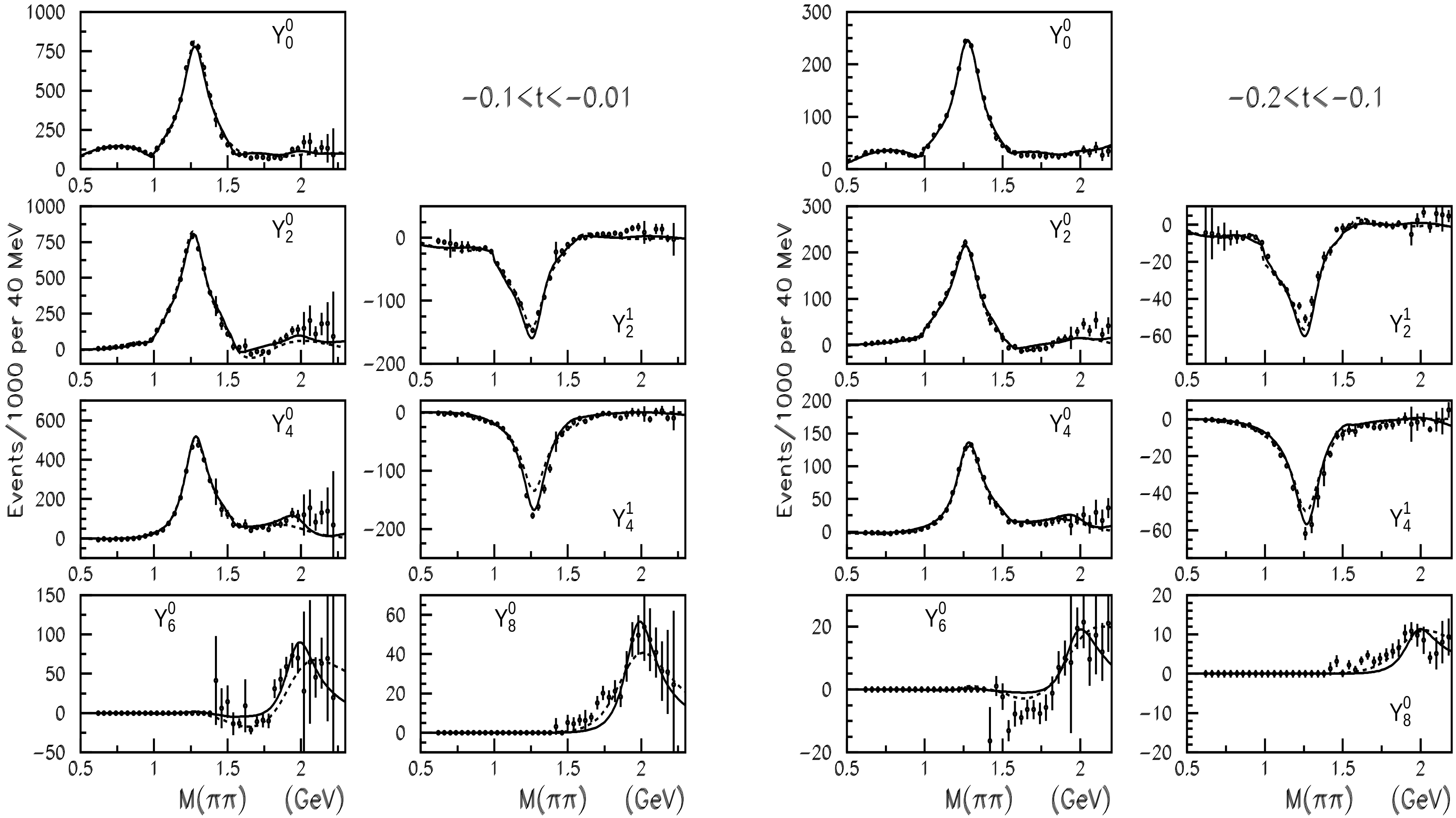
where:

$$\begin{aligned} W_{0k}^{(J)} &= -N_{J0} \left(k_{3z} - \frac{|\vec{p}|}{2} \right) \left(|\vec{p}|^2 A^{(J+)}_{\pi a_1^{(k)} \to \pi\pi} - A^{(J-)}_{\pi a_1^{(k)} \to \pi\pi} \right) \\ W_{1k}^{(J)} &= -\frac{N_{J1}}{J(J+1)} k_{3x} \left(|\vec{p}|^2 J \, A^{(J+)}_{\pi a_1^{(k)} \to \pi\pi} + (J+1) A^{(J-)}_{\pi a_1^{(k)} \to \pi\pi} \right) \end{aligned} \quad (1)$$

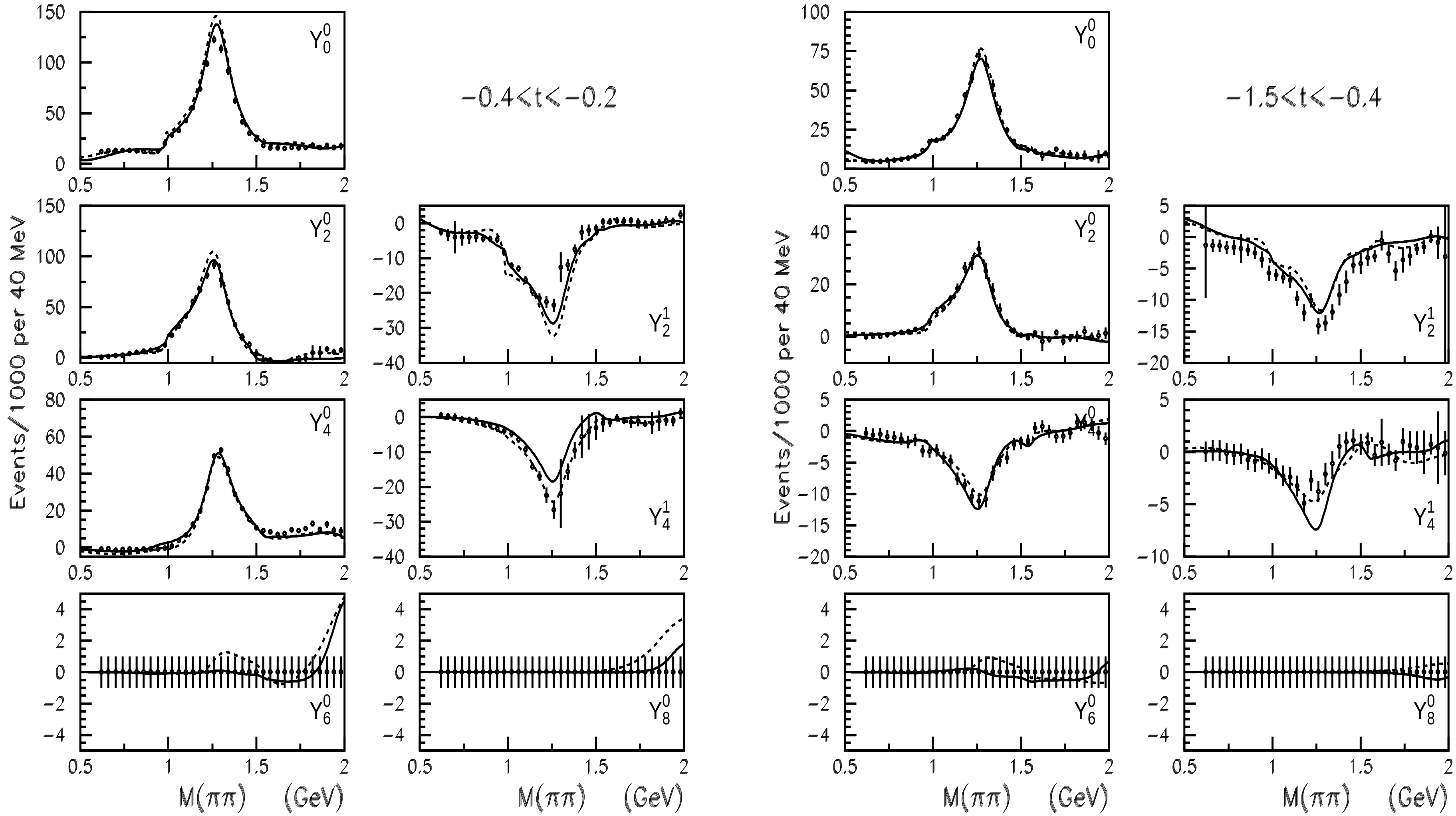
Then $< Y_J^2 >$ moments in the cross section are $(k_{3x}/k_{3z})^2$.

However the contribution to $< Y_J^0 >$ could be rather large already at small t.

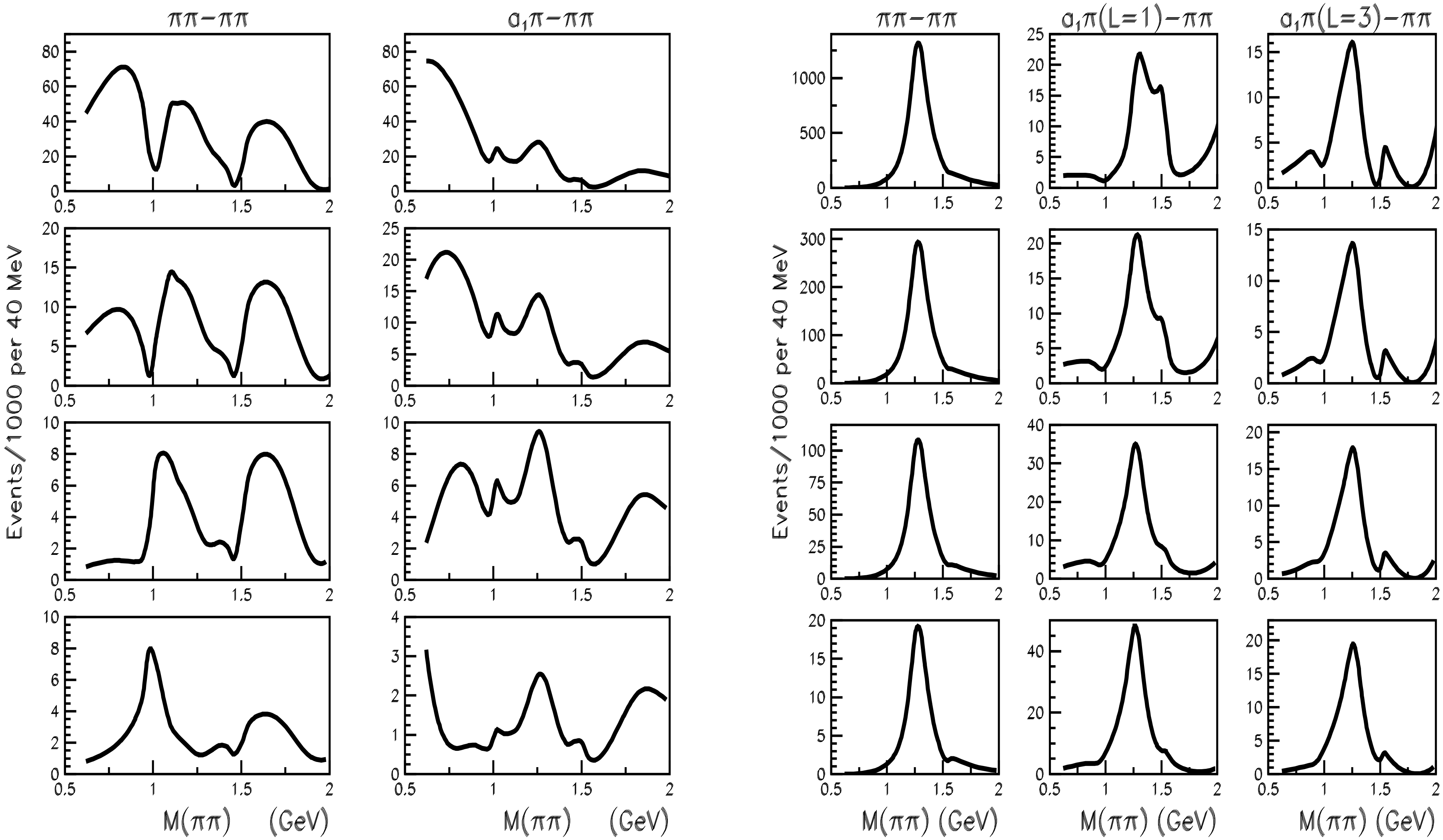
The description of $\pi N \to \pi^0\pi^0 N$ (E852)



The description of $\pi N \to \pi^0\pi^0 N$ (E852)



S and D-waves at different t-intervals



Fit without $f_0(1370)$

Fit of the BNL data deteriorated everywhere. Largest effect at:

$-0.2 < t < -0.1$ $1.84 \to 3.63$

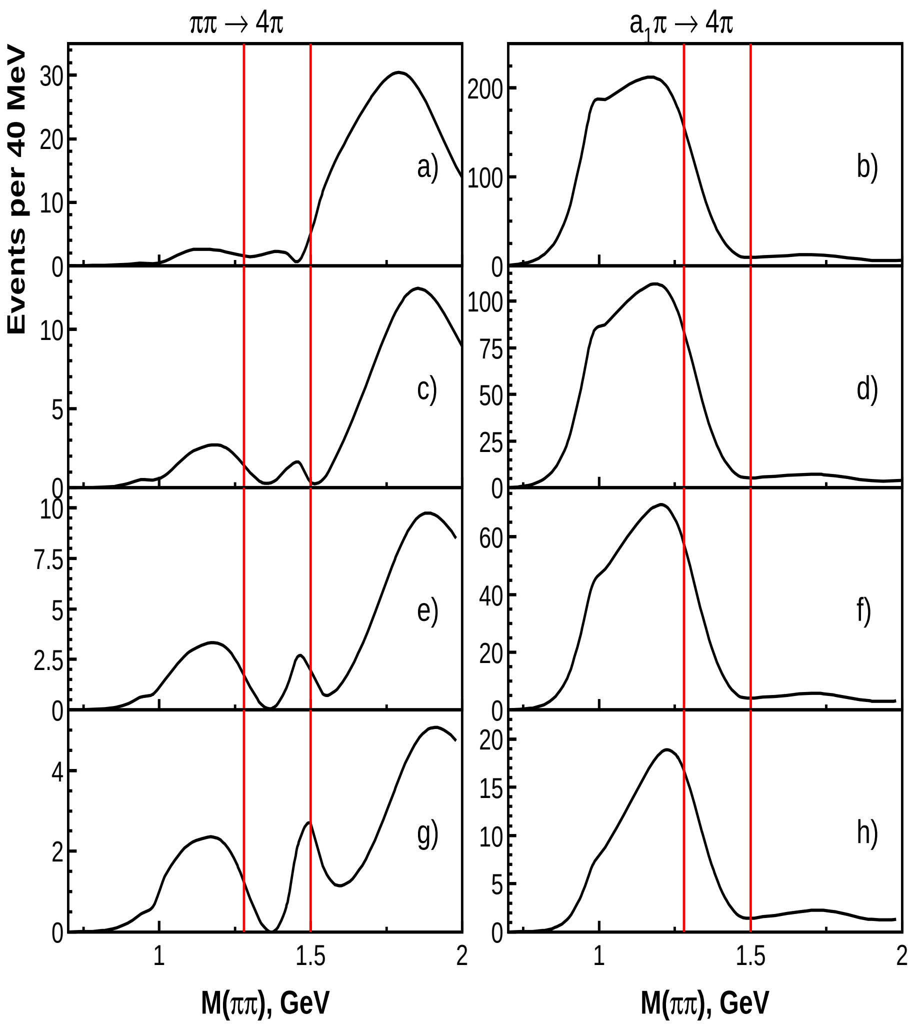
$-0.4 < t < -0.2$ $2.07 \to 4.90$

Fit of other data sets:

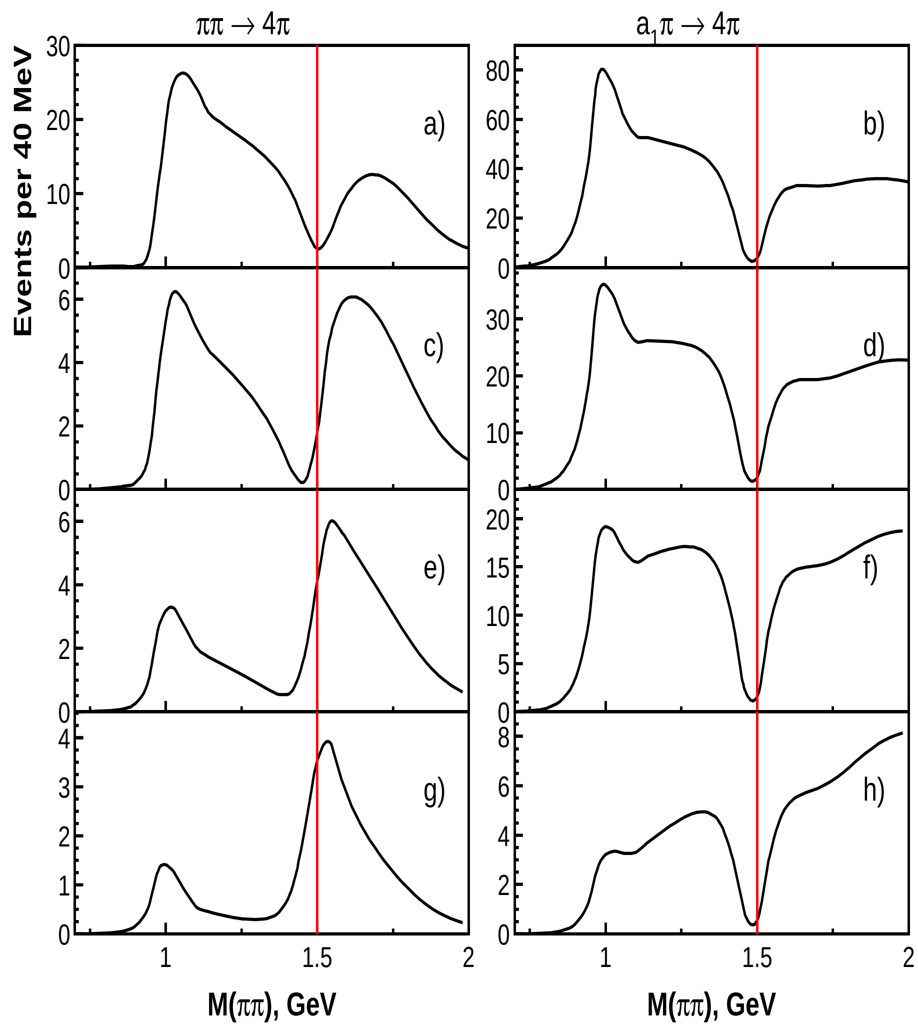
Data	Solution 1	Solution 2	Solution 2(-) (no $f_0(1370)$)
$\bar{p}p \to \pi^0\pi^0\pi^0$ (Liq)	1.360	1.356	1.443
$\bar{p}p \to \pi^0\pi^0\pi^0$ (Gas)	1.238	1.242	1.496
$\bar{p}p \to \eta\pi^0\eta$ (Liq)	1.350	1.442	1.446
$\bar{p}p \to \eta\pi^0\eta$ (Gas)	1.503	1.371	1.315
$\bar{p}p \to \pi^0\eta\pi^0$ (Liq)	1.210	1.236	1.412
$\bar{p}p \to \pi^0\eta\pi^0$(Gas)	1.099	1.119	1.227
$\pi\pi \to \eta\eta$ (S-wave)	1.08	1.19	1.38
$\pi\pi \to \eta\eta'$ (S-wave)	0.26	0.41	0.45

Predictions for S-wave contribution to the $\pi^- p \to 4\pi n$ reaction

The 5-pole K-matrix fit.



The fit without $f_0(1300)$.



Conclusion about scalar glueball

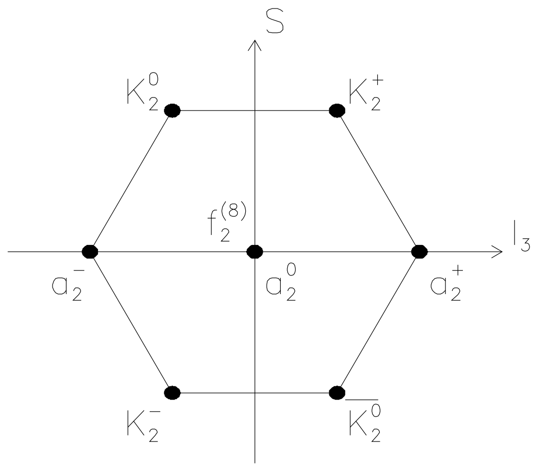
1. **The crucial question is the existence of $f_0(1370)$ state which decays dominantly into 4π channel.**

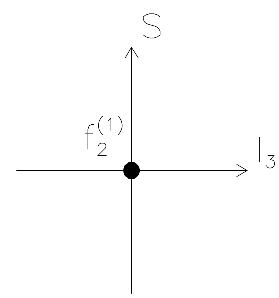
2. **The study of t-dependence in the πN transition into different final states can provide a vital information about this resonance.**

3. **The reggeon exchange approach is a most suitable tool for analysis of the $\pi N \to mesons N$ data, providing a natural connection of the regions of small and large t.**

Systematics of tensor mesons

Tensor particles, ground states $J^{PC} = 2^{++}$:





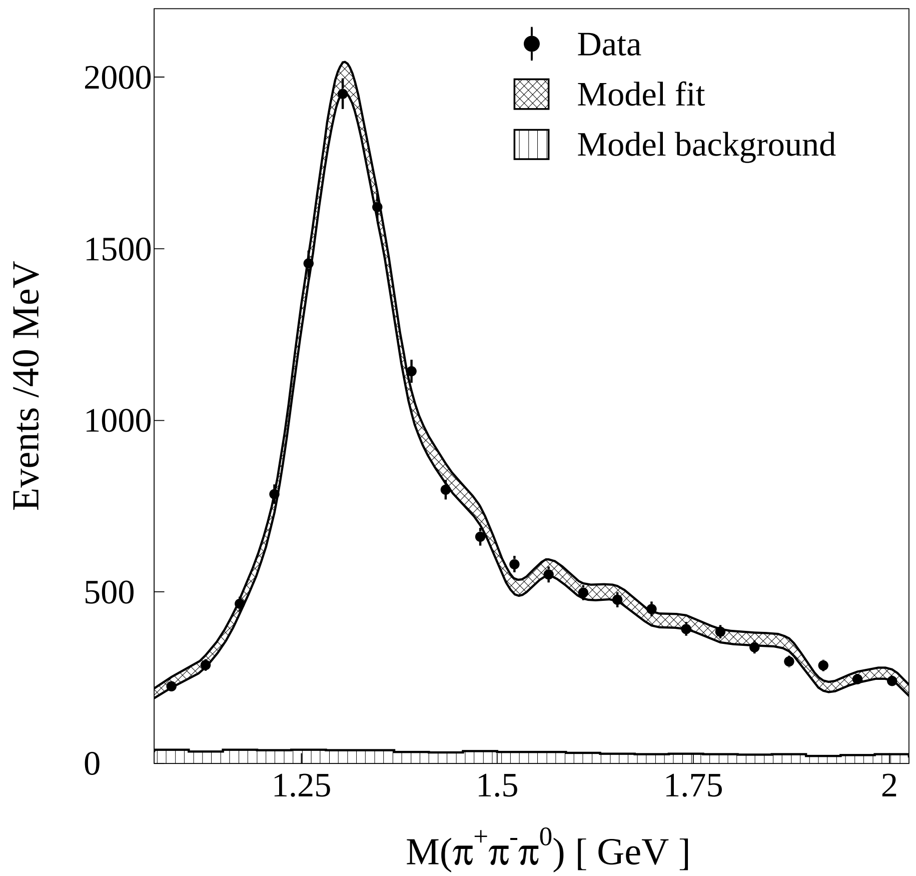
$f_2(1275) \qquad f_2'(1525) \qquad a_2(1320) \qquad K_2(1430)$

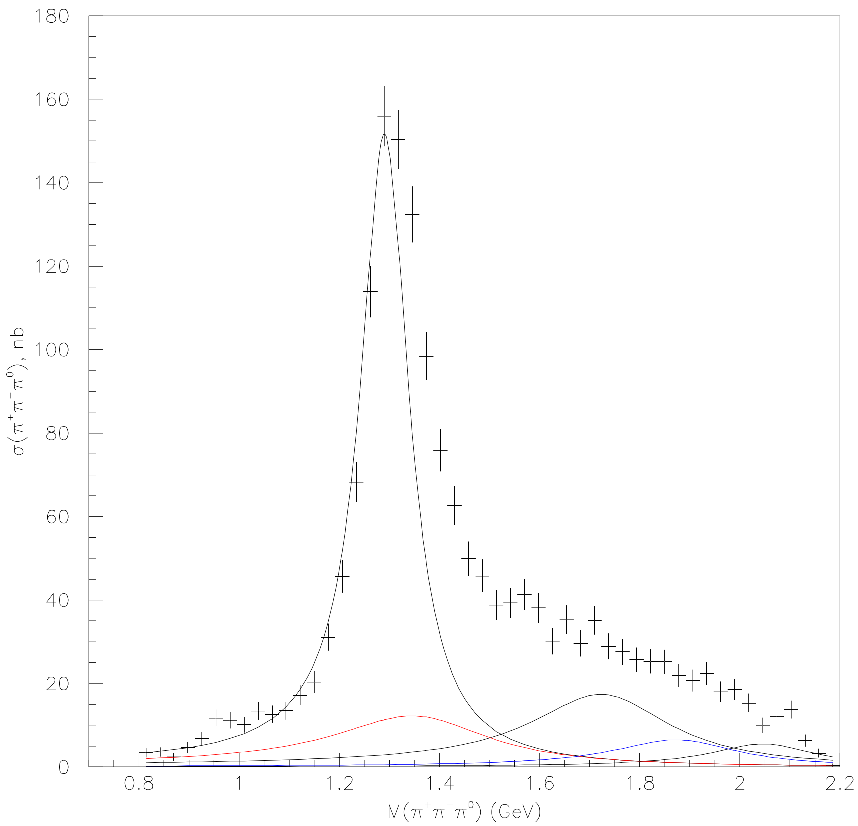
Nonet of first radial excitations of tensor states:

$f_2(1560) \qquad a_2(1700) \qquad f_2(1750)$

Analysis of the L3 data on reaction $\gamma\gamma \to \pi^+\pi^-\pi^0$

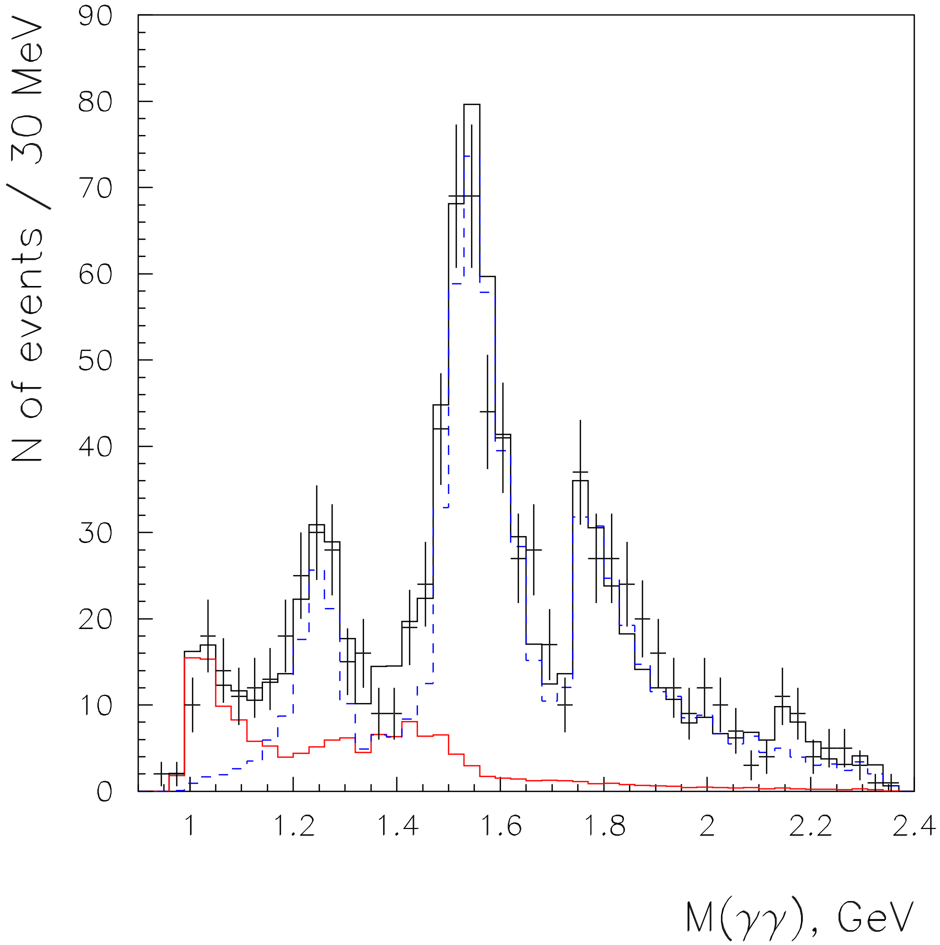
2^{++}, 0^{++} , - 2^{-+} states





V. Schegelsky, A. Sarantsev, A.Anisovich, M.Levchenko, EPJA 27, 199 (2006)

Analysis of the L3 data on the reaction $\gamma\gamma \to K_s K_s$



black histogram - the fit

and contributions:

blue histogram - the tensor states

red histogram - the scalar states

V. Schegelsky, A. Sarantsev, V.Nikonov, A.Anisovich, EPJA 27, 207 (2006)

$$a_2^- = d\bar{u} \qquad a_2^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) \qquad a_2^+ = u\bar{d}$$

$$f_2 = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})\cos\Phi \quad + s\bar{s}\ \sin\Phi$$

$$f_2' = -\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})\sin\Phi \quad + s\bar{s}\ \cos\Phi$$

	First nonet			**Second nonet**		
	$a_2(1320)$	$f_2(1270)$	$f_2'(1525)$	$a_2(1700)$	$f_2(1560)$	$f_2(1750)$
Mass (MeV)	1304 ± 10	1277 ± 6	1523 ± 5	1725 ± 30	1570 ± 20	1755 ± 10
Width (MeV)	120 ± 15	195 ± 15	104 ± 10	340 ± 40	160 ± 20	67 ± 12
g **(GeV)**	0.8 ± 0.1	0.9 ± 0.1	1.05 ± 0.1	0.38 ± 0.05		
Φ **(deg)**	-1 ± 3			-10^{+5}_{-10}		

1 Crystal Barrel data for $p\bar{p}$ annihilation in flight

Very important information was obtained from $p\bar{p}$ annihilation in flight. High statistical data taken at energies of antiproton 600, 900, 1150, 1200, 1350, 1525, 1640, 1800 and 1940 MeV was used to search for meson states in $p\bar{p}$ channel (RAL+PNPI groups)

$\bar{p}p \to \pi^+\pi^-$	$\bar{p}p \to \pi^0\pi^0$	$\bar{p}p \to \pi^0\pi^0\pi^0$
$\bar{p}p \to \eta\eta$	$\bar{p}p \to \eta\eta'$	$\bar{p}p \to \pi^0\pi^0\eta$
$\bar{p}p \to \pi^0\eta$	$\bar{p}p \to \pi^0\eta'$	$\bar{p}p \to \pi^0\eta\eta$
$\bar{p}p \to \pi^0\omega$		$\bar{p}p \to \pi^0\pi^0\omega$
$\bar{p}p \to \eta\omega$		$\bar{p}p \to \pi^0\eta\omega$

The combined analysis was performed together with $\bar{p}p \to \pi^+\pi^-$ data obtained with polarized target (E. Eisenhandler et al., Nucl. Phys. B98 (1975) 109).

The Partial wave analysis of the following data sets:

Crystal Barrel at LEAR data on: $\bar{p}p \to \pi^0\pi^0, \eta\eta, \eta\eta', \pi^0\pi^0\eta$

E. Eisenhandler et al., Nucl. Phys. B98 (1975) 109, on $\bar{p}p(polarized) \to \pi^+\pi^-$

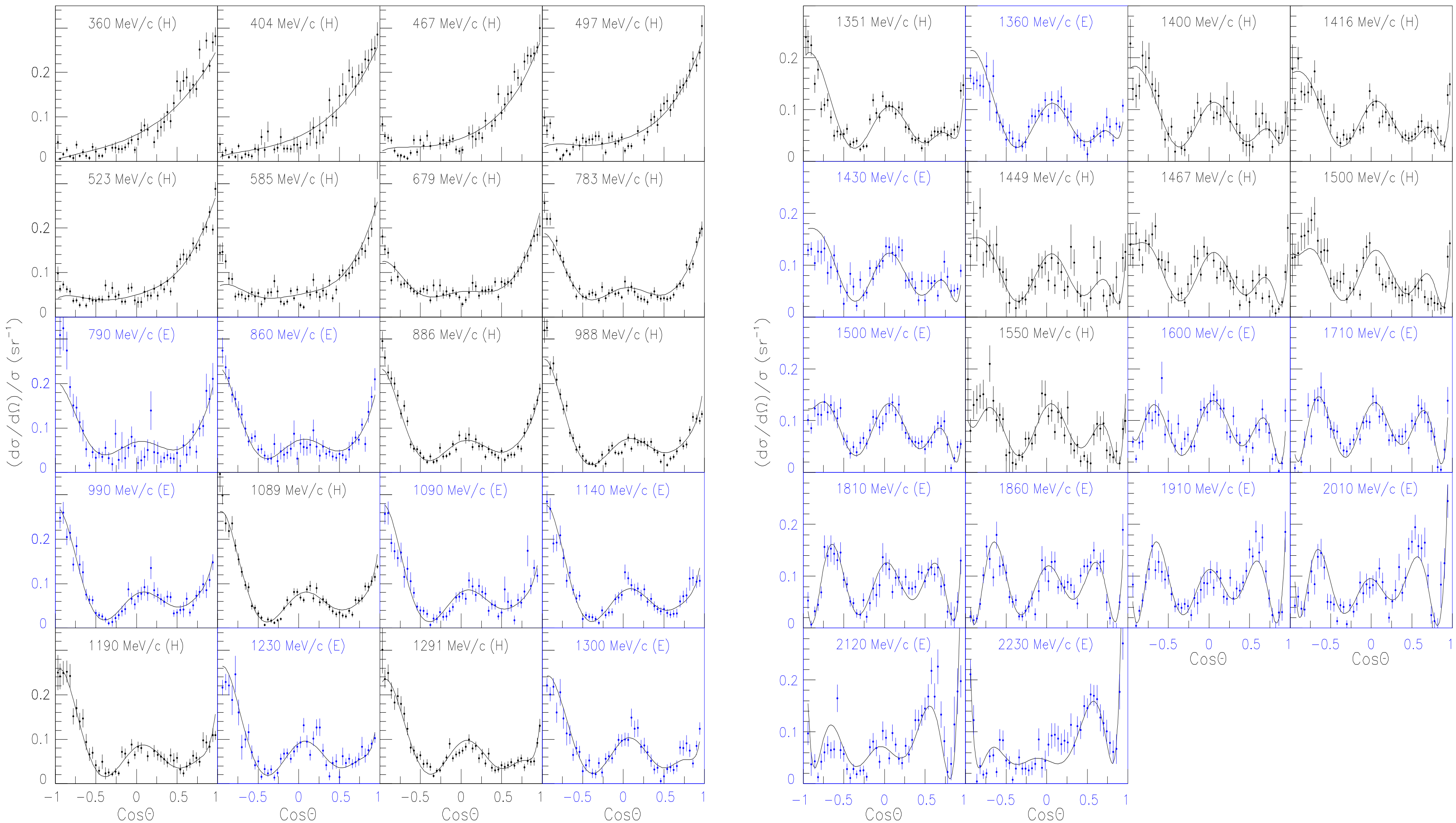
Five tensor states are required to describe the data:

$$f_2(1920), f_2(2000), f_2(2020), f_2(2240), f_2(2300)$$

Resonance	Mass (MeV)	Width (MeV)
$f_2(1920)$	1920 ± 30	230 ± 40
$f_2(2000)$	2010 ± 30	495 ± 35
$f_2(2020)$	2020 ± 30	275 ± 35
$f_2(2200)$	2230 ± 40	245 ± 45
$f_2(2300)$	2300 ± 35	290 ± 50

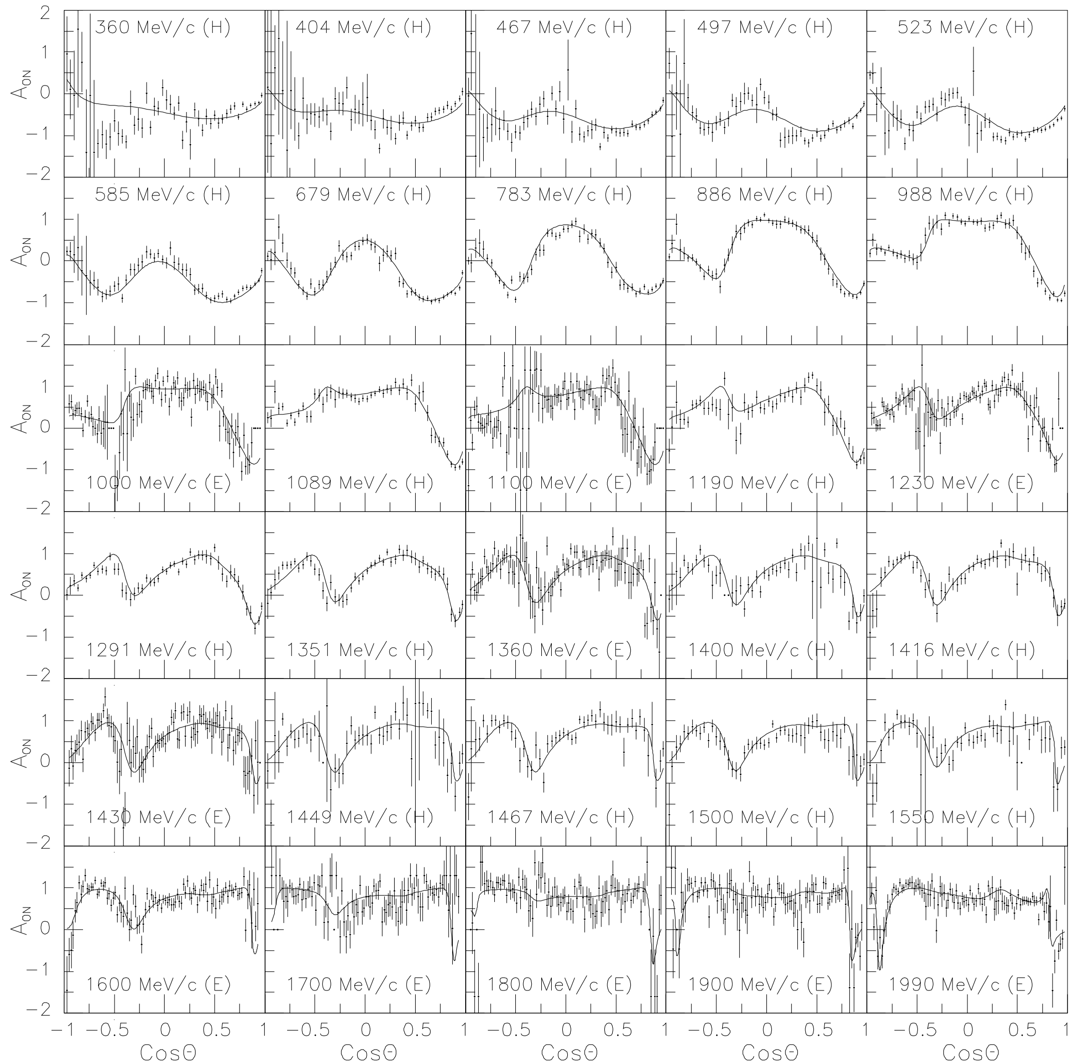
A.V. Anisovich *et al.*, Phys. Lett. B 491, 47 (2000)

Differential cross section $p\bar{p} \rightarrow \pi^+\pi^-$



Polarization

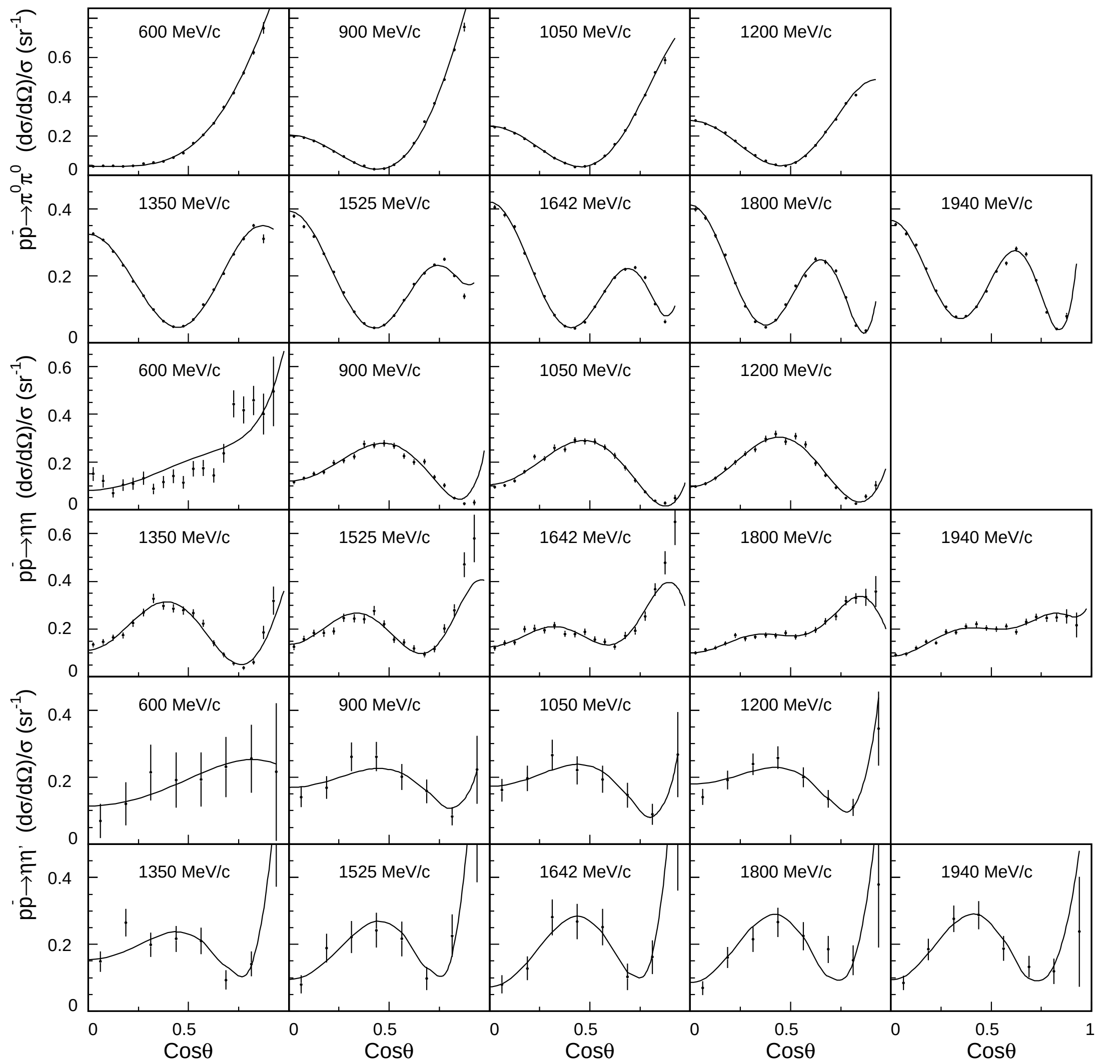
$$p\bar{p} \rightarrow \pi^+\pi^-$$



$\bar{p}p \longrightarrow \pi\pi$

$\bar{p}p \longrightarrow \eta\eta$

$\bar{p}p \longrightarrow \eta\eta'$



The $\bar{p}p \longrightarrow \pi^0\pi^0, \eta\eta, \eta\eta'$ **amplitudes provide the following ratios** $g_{\pi^0\pi^0} : g_{\eta\eta} : g_{\eta\eta'}$**:**

	$g_{\pi\pi}$ $g_{\eta\eta}$ $g_{\eta\eta'}$
$f_2(1920)$	$1 : 0.56 \pm 0.08 : 0.41 \pm 0.07$
$f_2(2000)$	$1 : 0.82 \pm 0.09 : 0.37 \pm 0.22$
$f_2(2020)$	$1 : 0.70 \pm 0.08 : 0.54 \pm 0.18$
$f_2(2240)$	$1 : 0.66 \pm 0.09 : 0.40 \pm 0.14$
$f_2(2300)$	$1 : 0.59 \pm 0.09 : 0.56 \pm 0.17.$

For a pure glueball state it is expected ($\lambda = 0.5 - 0.85$**):**

$$g_{\pi^0\pi^0} : g_{\eta\eta} : g_{\eta\eta'} = 1 : (0.82 - 0.95) : (0.24 - 0.07)$$

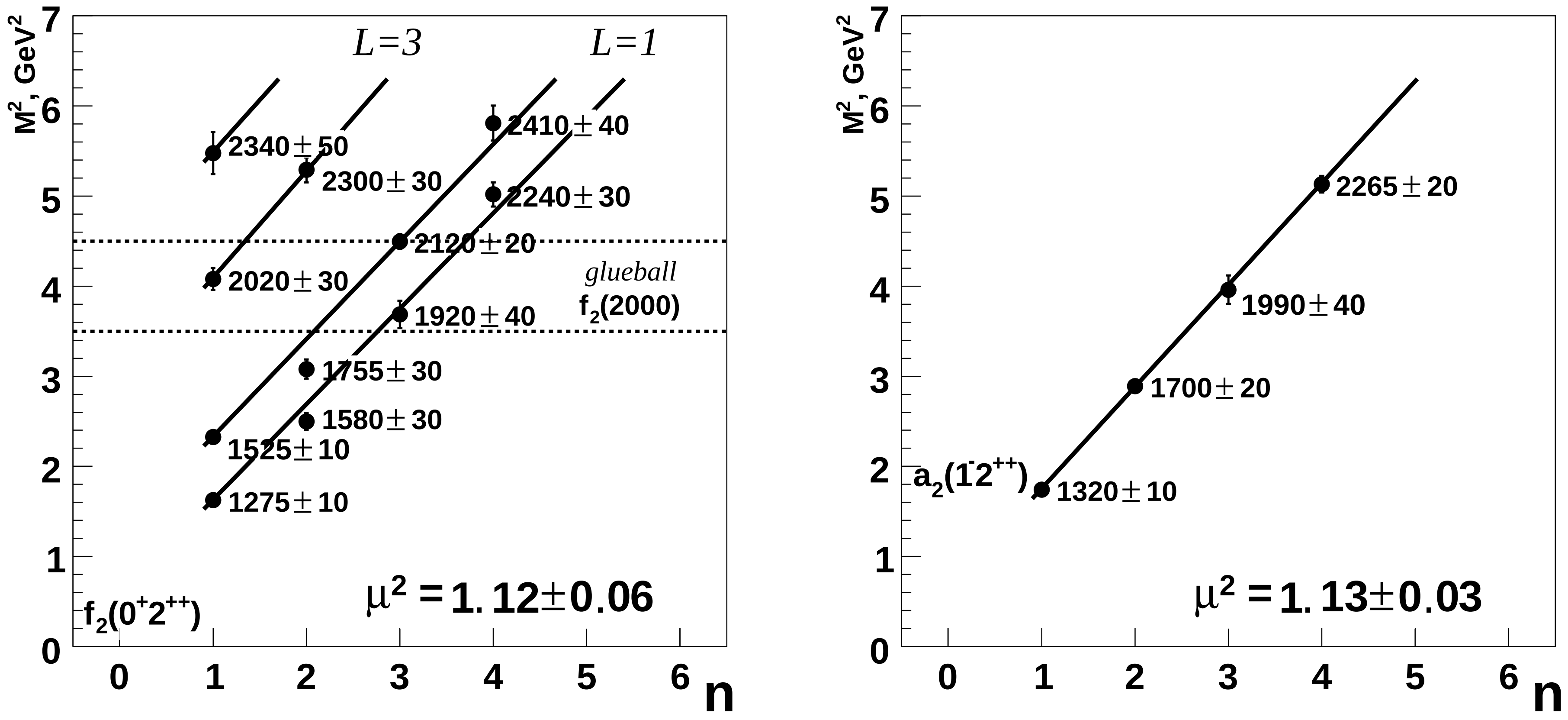
R. S. Longacre, S. J. Lindenbaum, Phys. Rev. D 70 (2004) 094041

"Evidence for a 4th state related to the three $J(PC) = 2^{++}$, $\pi^- p \to \phi\phi n$ states explainable by 2++ glueball production"

Resonance parameters

New Mass(GeV)	New Width(GeV)	Old Mass(GeV)	Old Width(GeV)
$2.049^{+.035}_{-.024}$	$.567^{+.064}_{-.071}$	—	—
$2.123^{+.015}_{-.033}$	$.294^{+.056}_{-.055}$	$2.011^{+.062}_{-.076}$	$.202^{+.067}_{-.062}$
$2.340^{+.013}_{-.013}$	$.148^{+.066}_{-.032}$	$2.297^{+.028}_{-.028}$	$.149^{+.041}_{-.041}$
$2.412^{+.028}_{-.032}$	$.362^{+.100}_{-.053}$	$2.339^{+.055}_{-.055}$	$.319^{+.081}_{-.069}$

The trajectories of the f_2 and a_2 mesons in (J, M^2) plane



Summary

1) The extra tensor state to $q\bar{q}$ pattern was observed in the set of the reactions:

$M = 2010 \pm 25$ **MeV,** $\Gamma = 495 \pm 35$ **MeV in** $p\bar{p} \to \pi^+\pi^-, \pi^0\pi^0, \eta\eta, \eta\eta', \pi^0\pi^0\eta$

A.V. Anisovich *et al.*, Phys. Lett. B 491, 47 (2000)

$M = 1980 \pm 20$ **MeV,** $\Gamma = 520 \pm 50$ **MeV in** $pp \to pp4\pi$

D. Barberis *et al.* (WA 102 Collab.), Phys. Lett.B 471, 440 (2000)

$M = 1940 \pm 50$ **MeV,** $\Gamma = 380^{+120}_{-90}$ **MeV in** $J/\Psi \to \gamma(2\pi^+2\pi^-)$

J.Z. Bai et al. (BES), Phys. Lett.B 472, 207 (2000)

$M = 2050 \pm 30$ **MeV,** $\Gamma = 570 \pm 70$ **MeV in** $\pi^-p \to \phi\phi n$

R.S. Longacre and S.J. Lindenbaum, Phys. Rev. D 70 (2004) 094041

$M = 1930 \pm 25$ **MeV,** $\Gamma = 450 \pm 50$ **MeV in** $\pi^-p \to \eta\eta n$

F. Binon et al. (GAMS) PAN 68,960 (2005)

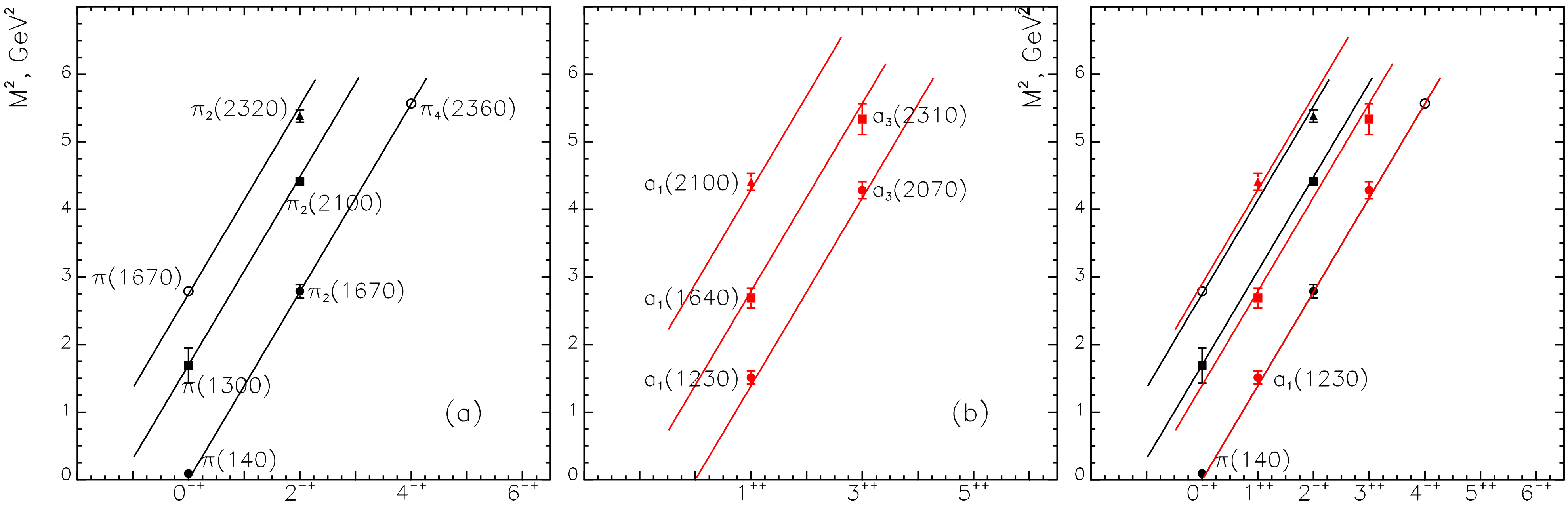
2) The decay ratios to $\pi\pi$, $\eta\eta$ and $\eta\eta'$ channels are compatible with assumption about glueball nature of the state.

Name	$I^G J^{PC}$	**Mass**	**Width**	**Status**	**Units**
π	$1^- 0^{-+}$	2070 ± 35	310 ± 80	*	**MeV**
π	$1^- 0^{-+}$	2360 ± 30	300 ± 80	*	**MeV**
a_1	$1^- 1^{++}$	2270 ± 50	300 ± 70	*	**MeV**
π_2	$1^- 2^{-+}$	2005 ± 20	210 ± 40	*	**MeV**
π_2	$1^- 2^{-+}$	2245 ± 60	320 ± 60	*	**MeV**
a_2	$1^- 2^{++}$	1950 ± 40	180 ± 40	***	**MeV**
a_2	$1^- 2^{++}$	2030 ± 20	205 ± 30	***	**MeV**
a_2	$1^- 2^{++}$	2175^{+80}_{-30}	310 ± 60	*	**MeV**
a_2	$1^- 2^{++}$	2255 ± 20	230 ± 15	***	**MeV**
a_3	$1^- 3^{++}$	2030 ± 20	150 ± 20	**	**MeV**
a_3	$1^- 3^{++}$	2275 ± 40	150 ± 20	*	**MeV**
a_4	$1^- 4^{++}$	2005 ± 30	180 ± 30	***	**MeV**
a_4	$1^- 4^{++}$	2255 ± 40	330 ± 70	**	**MeV**
π_4	$1^- 4^{-+}$	2250 ± 15	215 ± 25	**	**MeV**

Name	$I^G J^{PC}$	Mass	Width	Status	Units
f_0	0^+0^{++}	2105 ± 15	200 ± 25	**	MeV
f_0	0^+0^{++}	2320 ± 30	175 ± 45	*	MeV
f_2	0^+2^{++}	1920 ± 30	230 ± 40	**	MeV
f_2	0^+2^{++}	2020 ± 30	275 ± 35	***	MeV
f_2	0^+2^{++}	2230 ± 40	245 ± 45	***	MeV
f_2	0^+2^{++}	2300 ± 35	290 ± 50	**	MeV
f_2	0^+2^{++}	2010 ± 30	495 ± 35	**	MeV
f_4	0^+2^{++}	2020 ± 25	170 ± 20	***	MeV
f_4	0^+2^{++}	2300 ± 25	280 ± 50	**	MeV
ρ_1	1^+1^{--}	1980 ± 30	165 ± 30	**	MeV
ρ_3	1^+3^{--}	1980 ± 15	175 ± 20	**	MeV
ρ_3	1^+3^{--}	2260 ± 20	200 ± 30	*	MeV

Trajectories on the (J, M^2) plane:

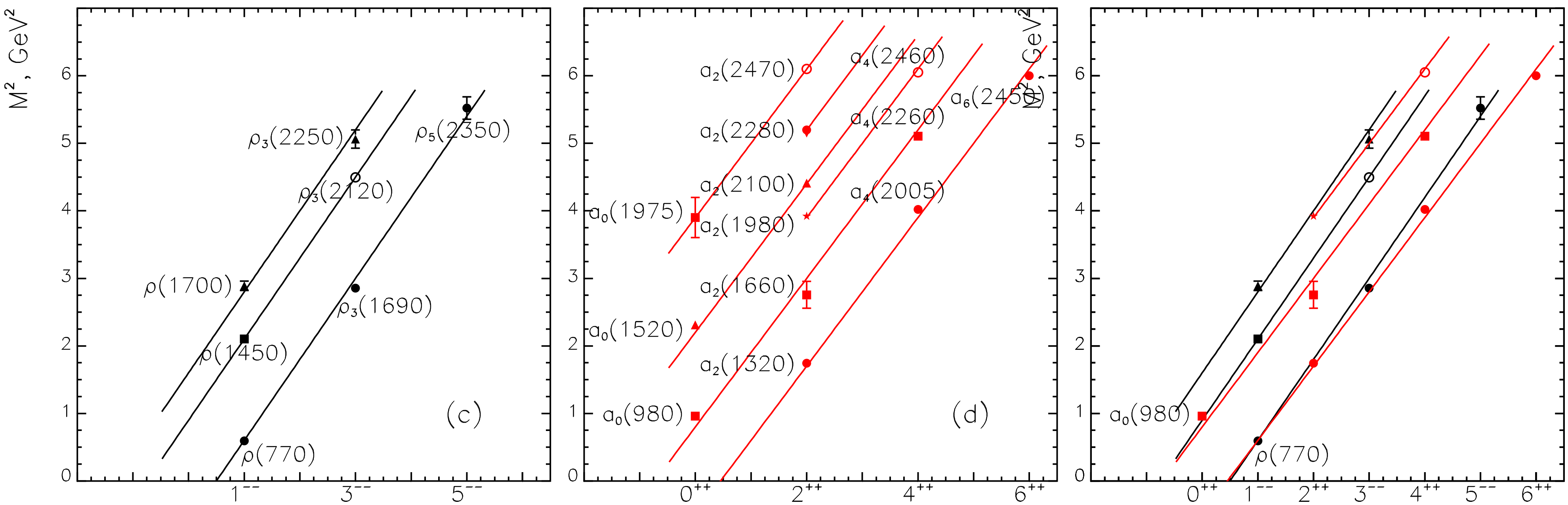
pion trajectory and the daughter ones, a_1-meson and the daughter ones.



Degeneracy of the π and a_1 trajectories, combined presentation.

Trajectories on the (J, M^2) plane:

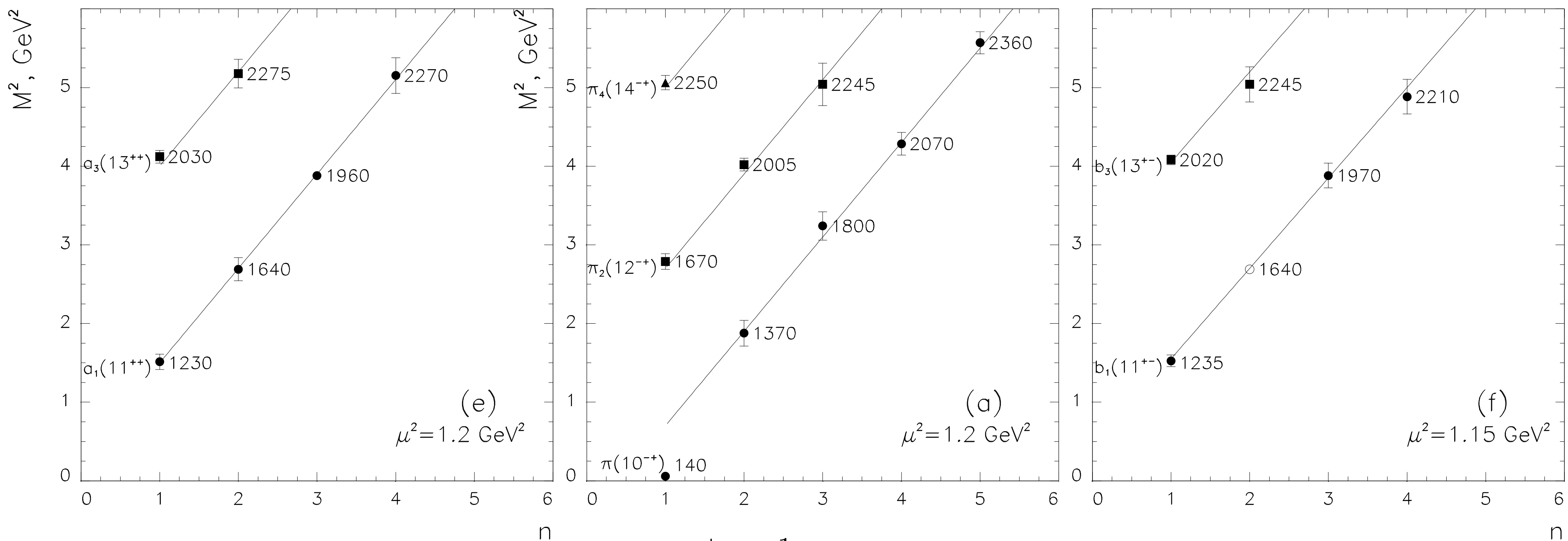
ρ-meson and daughter trajectories, **a_2-meson and daughter trajectories.**



Combined presentation:

ρ and a_2 trajectories are degenerate. $a_0(980)$ is on the daughter trajectory.

Trajectories on the (n, M^2) planes, (I=1)-states



$$\pi_4(14^{-+}) \rightarrow {}^1G_4\ q\bar{q}(L=4)$$

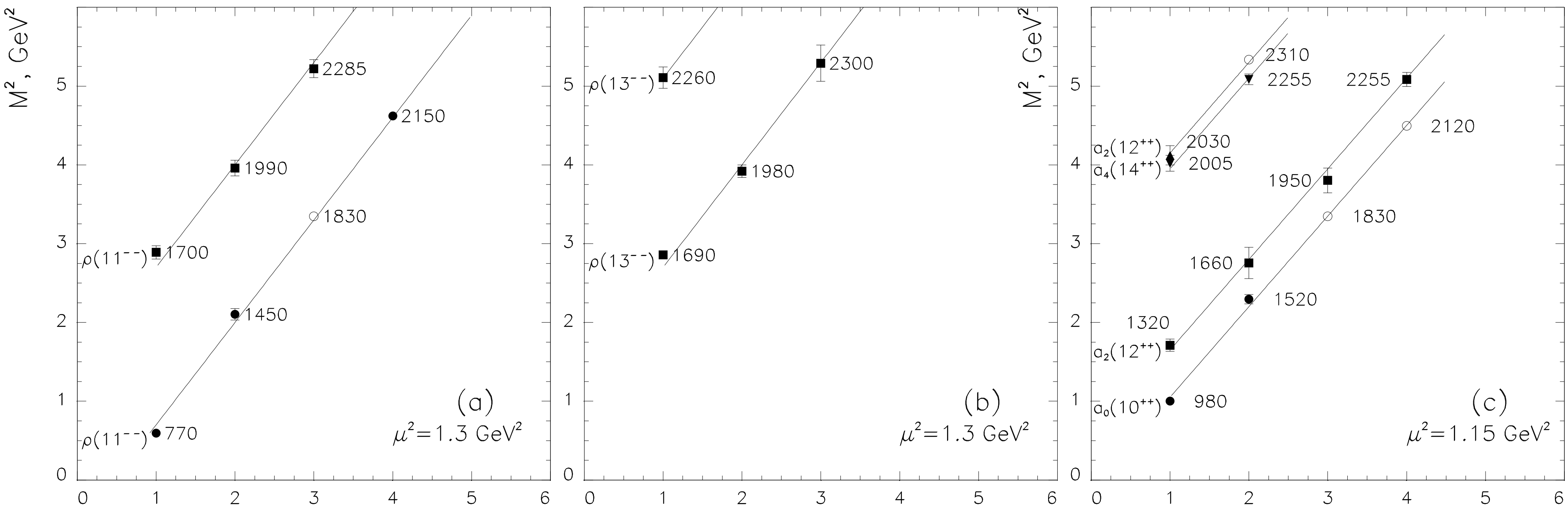
$$a_3(13^{++}) \rightarrow {}^3F_3\ q\bar{q}(L=3) \qquad \pi_2(12^{-+}) \rightarrow {}^1D_2\ q\bar{q}(L=2) \qquad b_3(13^{+-}) \rightarrow {}^1F_3\ q\bar{q}(L=3)$$

$$a_1(11^{++}) \rightarrow {}^3P_1\ q\bar{q}(L=1) \qquad \pi\,(10^{-+}) \rightarrow {}^1S_0\ q\bar{q}(L=0) \qquad b_1(11^{+-}) \rightarrow {}^1P_1\ q\bar{q}(L=1)$$

${}^{2S+1}L_J$, $S \longrightarrow$ **spin of quarks,** $L \longrightarrow$ **angular momentum,** $J \longrightarrow$ **total angular momentum.**

$n \longrightarrow$ **radial quantum number,**

Trajectories on the (n, M^2) planes, (I=1)-states



$^3D_1\ q\bar{q}(L=2)$

$^3S_1\ q\bar{q}(L=0)$

$^3G_3\ q\bar{q}(L=4)$

$^3D_3\ q\bar{q}(L=2)$

$^3F_4\ q\bar{q}(L=3)$

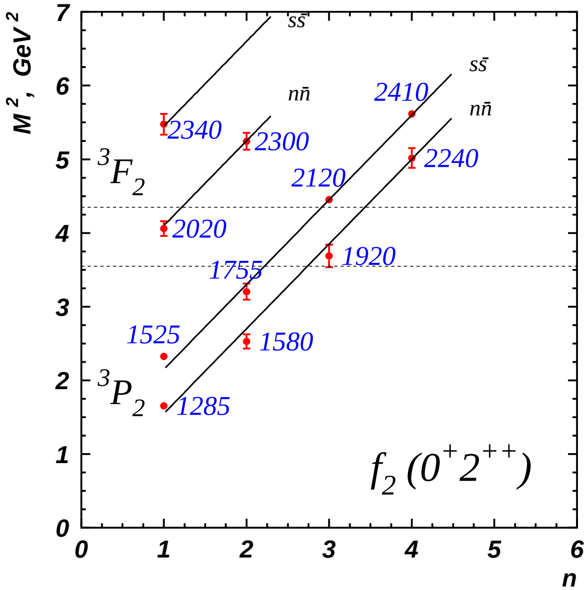
$^3F_2\ q\bar{q}(L=3)$

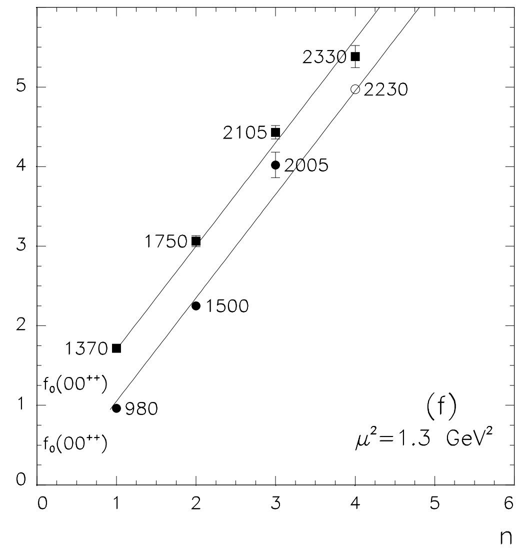
$^3P_2\ q\bar{q}(L=1)$

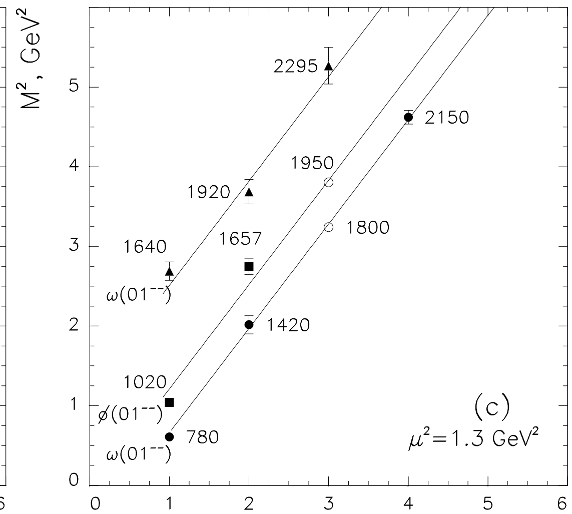
$^3P_0\ q\bar{q}(L=1)$

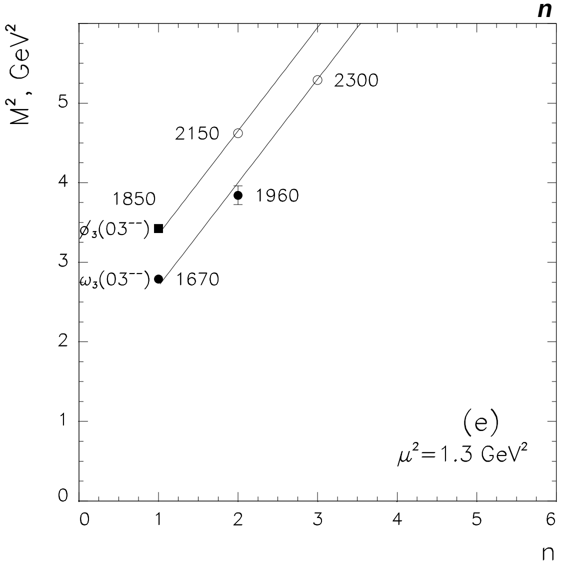
Linear trajectories: $M^2 = M_0^2 + \mu^2(n-1)$, $\mu^2 = (1.15 - 1.30)$ **GeV^2**

$M \rightarrow$ **meson mass,** $M_0 \rightarrow$ **mass of the basic meson** $(n=1)$, μ^2 **is a parameter.**







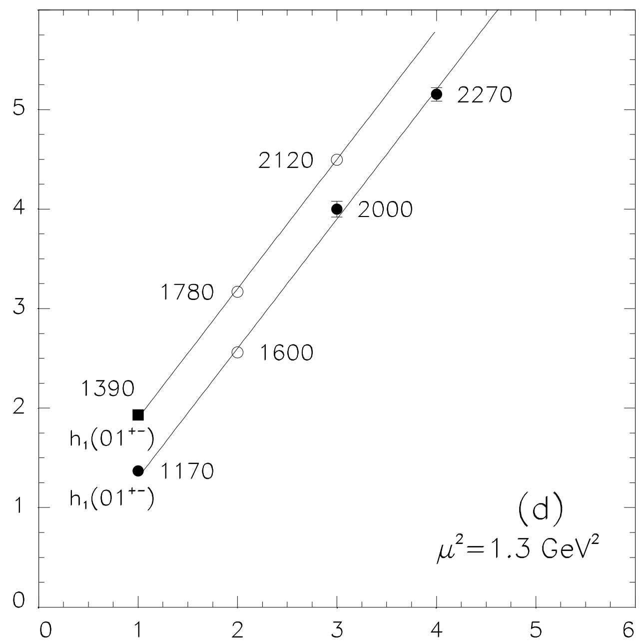


Trajectories on the (n, M^2) planes, (I=0)-states.

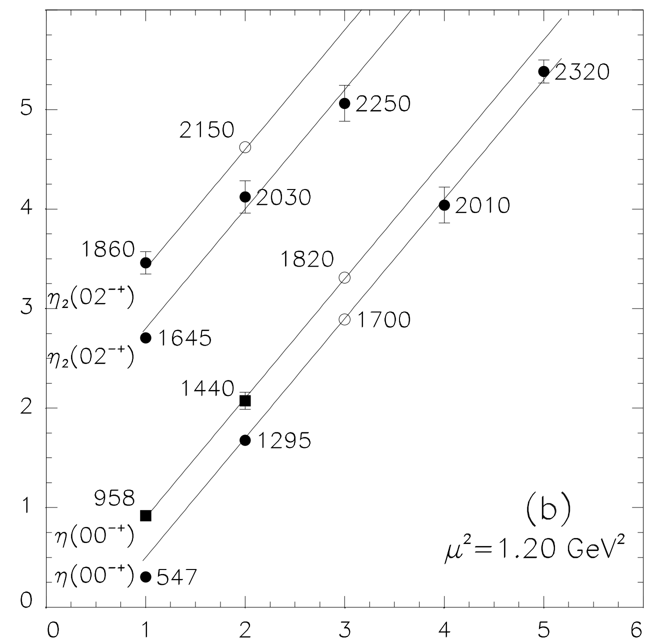
The doubling of trajectories, two flavour states:

$$n\bar{n} = \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}, \quad s\bar{s}.$$

Trajectories on the (n, M^2) planes, (I=0)-states



$^1P_1\ q\bar{q}(L=1)$



$^1D_2\ q\bar{q}(L=2),\ ^1S_0\ q\bar{q}(L=0)$

The doubling of trajectories, two flavor states:

$$n\bar{n} = \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}, \quad s\bar{s}$$

The confirmation of this structure is very important for understanding the strong interactions at low energies

Thank you