Bonn-Gatchina partial wave analysis

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The main objective: the search for new baryon states.

1. **The P-vector based analysis of single and multi-meson photoproduction data. The combined analysis of CB-ELSA data on $\gamma p \to \pi^0 p$, $\gamma p \to \eta p$, $\gamma p \to \pi^0\pi^0 p$, $\gamma p \to \pi^0 \eta p$, $\gamma n \to \eta n$, $\gamma n \to \pi^0 n$ and $\gamma p \to \omega p$ with data from other collaborations: CLAS, GRAAL, LEPS.**

2. **The K-matrix analysis of pion-induced data on the reactions $\pi N \to \pi N$ (on the basis of SAID energy-fixed partial wave analysis), $\pi p \to \pi^0\pi^0 n$ and recently included: $\pi p \to \eta n$, $\pi^- p \to K^0\Lambda$ and $\pi^+ p \to K^+\Sigma$.**

3. **Analysis of NN interaction. The analysis of $pp \to K\Lambda p$ data and analysis of the $pp \to \pi^0 p$ and $np \to pp\pi^-$ data.**

For combined analysis of a larger data set a new approach is needed:

1. **Fully relativistically invariant.**
2. **Convenient for combined analysis of single and multi-meson photoproduction.**
3. **Energy dependent, which allow us to apply directly the unitarity and analyticity conditions.**
4. **Convenient for calculation of the triangle and box diagrams or projection of the t and u-channel exchange amplitudes to the partial waves in s-channel.**

A. Anisovich, E. Klempt, A. Sarantsev and U. Thoma, Eur. Phys. J. A 24, 111 (2005)

A. V. Anisovich and A. V. Sarantsev, Eur. Phys. J. A 30 (2006) 427

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Eur. Phys. J. A 34 (2007) 129.

1 Bosons

1.1 Propagator – Green function

Let us introduce a function which is a solution of the following equation ($t = x_0$):

$$(\frac{\partial^2}{\partial x_\mu \partial x_\mu} + m^2)G(x - x') = \delta^4(x - x')$$

Properties of the Green function:

If the Green function is a solution of the equation:

$$R(x)G(x - x') = \delta^4(x - x')$$

where $R(x)$ is some relativistically invariant operator, then the solution of the equation:

$$R(x)\Psi(x) = V(x)\Psi(x)$$

is:

$$\Psi(x) = \int G(x - x')V(x')d^4x'$$

In the case of equation for a free particle the Green function is called a propagator and is equal to:

$$G(x-x') =< 0|T\Psi(x)\Psi^*(x')|0>$$

If wave function is a spinor or tensor with rank n in Lorentz space then:

$$G^{\mu_1\mu_2\ldots\mu_n}_{\nu_1\nu_2\ldots\nu_n}(x-x') =< 0|T\Psi_{\mu_1\mu_2\ldots\mu_n}(x)\Psi^*_{\nu_1\nu_2\ldots\nu_n}(x')|0>$$

And equation is:

$$R(x)G^{\mu_1\mu_2\ldots\mu_n}_{\nu_1\nu_2\ldots\nu_n}(x-x') = \delta^4(x-x')\prod_{i=1}^{n}\delta_{\mu_i\nu_i}$$

For spinless particle in momentum representation we have:

$$(p^2-m^2)G = 1 \qquad\qquad G = \frac{1}{p^2-m^2}$$

Particles with spin 1

A particle with spin 1 is described by a 4-vector Ψ_μ and obeys the K-G equation:

$$(\frac{\partial^2}{\partial x_\mu \partial x_\mu} + m^2)\Psi_\mu(x) = 0$$

1) In c.m. system such wave function has only three components. It means that in any system only three components are independent.
2)A free propagating particle with fixed spin can not became a particle with another quantum numbers without interaction.

If $\Psi_\mu(x)$ is a wave function of particle with spin 1 and $\Psi(x)$ is a wave function of a particle with spin 0:

$$\int \Psi_\mu(x)\Psi^*(x)d^4x = \alpha p_\mu$$

Where α - is a scalar function of p^2. If such integral is equal to 0 for any wave functions and any p_μ it means that Ψ_μ has no projection on this vector and:

$$p_\mu \Psi_\mu = 0$$

In momentum representation:

$$\Psi_\mu = \frac{1}{\sqrt{2\varepsilon}} u_\mu e^{ipx}$$

and for wave function u_μ we obtain:

$$p_\mu u_\mu = 0$$

A spin 2 particle in c.m. system has 5 independent components and described by symmetrical tensor $\Psi_{\mu\nu}$. For the transition between spin 2 and spin 0 state we have:

$$\int \Psi_{\mu\nu}(x)\Psi^*(x)d^4x = \beta\Big(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\Big)$$

Then an additional condition is:

$$g_{\mu\nu}\Psi_{\mu\nu} = 0 \qquad\qquad g_{\mu\nu}u_{\mu\nu} = 0$$

Such procedure can be repeated for any spin, providing that the tensor have only $2s+1$ independent components. Thus for any integer spin:

$$p^2 u_{\mu_1\mu_2\ldots\mu_n} = m^2 u_{\mu_1\mu_2\ldots\mu_n}$$

$$p_{\mu_i} u_{\mu_1\mu_2\ldots\mu_n} = 0$$

$$g_{\mu_i\mu_j} u_{\mu_1\mu_2\ldots\mu_n} = 0$$

$$u_{\mu_1\ldots\mu_i\ldots\mu_j\ldots\mu_n} = u_{\mu_1\ldots\mu_j\ldots\mu_i\ldots\mu_n}$$

These conditions are the main basis for the construction of the projection operators, which are defined as:

$$G^{\mu_1\mu_2\ldots\mu_n}_{\nu_1\nu_2\ldots\nu_n} = (-1)^n O^{\mu_1\mu_2\ldots\mu_n}_{\nu_1\nu_2\ldots\nu_n} \frac{1}{p^2 - m^2}$$

2 Boson projection operators

In momentum representation:

$$O^{\mu_1\mu_2\ldots\mu_n}_{\nu_1\nu_2\ldots\nu_n} = (-1)^n \sum_{i=1}^{2n+1} u^{(i)}_{\mu_1\mu_2\ldots\mu_n} u^{(i)*}_{\nu_1\nu_2\ldots\nu_n}$$

The projection operator can depends only on the total momentum and the metrical tensor. For spin 0 it is a unit operator. For spin 1 the only possible combination is:

$$O^{\mu}_{\nu} = g^{\perp}_{\mu\nu} = g_{\mu\nu} - \frac{P_\mu P_\nu}{P^2}$$

The propagator for the particle with spin $S > 2$ must be constructed from the tensors $g^{\perp}_{\mu\nu}$: this is the only combination which satisfies:

$$P_\mu g^{\perp}_{\mu\nu} = 0.$$

Then for spin 2 state we obtain:

$$O^{\mu_1\mu_2}_{\nu_1\nu_2} = \frac{1}{2}(g^{\perp}_{\mu_1\nu_1} g^{\perp}_{\mu_2\nu_2} + g^{\perp}_{\mu_1\nu_2} g^{\perp}_{\mu_2\nu_1}) - \frac{1}{3} g^{\perp}_{\mu_1\mu_2} g^{\perp}_{\nu_1\nu_2}$$

There is a recurrent expression for the boson projector operator:

$$O^{\mu_1\ldots\mu_L}_{\nu_1\ldots\nu_L} = \frac{1}{L^2}\left(\sum_{i,j=1}^{L} g^{\perp}_{\mu_i\nu_j} O^{\mu_1\ldots\mu_{i-1}\mu_{i+1}\ldots\mu_L}_{\nu_1\ldots\nu_{j-1}\nu j+1\ldots\nu_L} - \right.$$

$$\left. \frac{4}{(2L-1)(2L-3)} \sum_{i<j,k<m}^{L} g^{\perp}_{\mu_i\mu_j} g^{\perp}_{\nu_k\nu_m} O^{\mu_1\ldots\mu_{i-1}\mu_{i+1}\ldots\mu_{j-1}\mu_{j+1}\ldots\mu_L}_{\nu_1\ldots\nu_{k-1}\nu_{k+1}\ldots\nu_{m-1}\nu_{m+1}\ldots\nu_L}\right)$$

Orbital momentum operator

Let us consider decay of a resonance into two scalar (pseudoscalar) particles. The angular momentum operator can be constructed only from momenta of particles k_1, k_2 and metric tensor $g_{\mu\nu}$.

The wave function of a scalar resonance is a const: $X^0 = 1$

The wave function of vector particle is a vector $X^{(1)}_\mu$, constructed from:
$k_\mu = \frac{1}{2}(k_{1\mu} - k_{2\mu})$ **and** $P_\mu = (k_{1\mu} + k_{2\mu})$.

Orthogonality: no transition between vector and scalar states:

$$\int \frac{d^4k}{4\pi} X^{(1)}_\mu X^{(0)} = 0$$

Then:

$$\int \frac{d^4k}{4\pi} X^{(n)}_{\mu_1\ldots\mu_n} X^{(n-1)}_{\mu_2\ldots\mu_n} = 0$$

The integral over internal momentum can be only proportional to the external momentum. Then:

$$\int \frac{d^4k}{4\pi} X^{(n)}_{\mu_1\ldots\mu_n} X^{(n-1)}_{\mu_2\ldots\mu_n} = \xi P_{\mu_1}$$

$$X^{(1)}_{\mu} P_\mu = 0 \qquad\qquad X^{(n)}_{\mu_1\ldots\mu_n} P_{\mu_j} = 0$$

Then:

$$X^{(1)}_{\mu} = k^{\perp}_{\mu} = k_\nu g^{\perp}_{\nu\mu}; \qquad\qquad g^{\perp}_{\nu\mu} = \left(g_{\nu\mu} - \frac{P_\nu P_\nu}{p^2} \right) ;$$

The production of two operators different by even number of indices is proportional to $g^{\perp}_{\mu\nu}$. To satisfy orthogonality we need the traceless condition.

$$X^{(n)}_{\mu_1\mu_2\ldots\mu_n} g_{\mu_i\mu_j} = 0$$

The orthogonality and symmetry properties can be written as the set of following conditions:

1.	$X^{(n)}_{\mu_1\ldots\mu_i\ldots\mu_j\ldots\mu_n}$	$= X^{(n)}_{\mu_1\ldots\mu_j\ldots\mu_i\ldots\mu_n}$	**(symmetry)**
2.	$P_{\mu_i} X^{(n)}_{\mu_1\ldots\mu_i\ldots\mu_n}$	$= 0$	**(P-orthogonality)**
3.	$g_{\mu_1\mu_2} X^{(n)}_{\mu_1\mu_2\ldots\mu_n}$	$= 0$	**(tracelessness)**

For low orbital momenta:

$$X^0 = 1\,; \quad X^1_\mu = k^\perp_\mu\,; \quad X^2_{\mu\nu} = \frac{3}{2}\left(k^\perp_\mu k^\perp_\nu - \frac{1}{3}\,k^2_\perp g^\perp_{\mu\nu}\right);$$

$$X^3_{\mu\nu\alpha} = \frac{5}{2}\left[k^\perp_\mu k^\perp_\nu k^\perp_\alpha - \frac{k^2_\perp}{5}\left(g^\perp_{\mu\nu}k^\perp_\alpha + g^\perp_{\mu\alpha}k^\perp_\nu + g_{\nu\alpha}k^\perp_\mu\right)\right],$$

The recurrent expression for the operators $X^{(n)}_{\mu_1\ldots\mu_n}$

$$X^{(n)}_{\mu_1\ldots\mu_n} = \frac{2n-1}{n^2}\sum_{i=1}^{n} k^{\perp}_{\mu_i} X^{(n-1)}_{\mu_1\ldots\mu_{i-1}\mu_{i+1}\ldots\mu_n} - \frac{2k^2_\perp}{n^2}\sum_{\substack{i,j=1\\ i<j}}^{n} g_{\mu_i\mu_j} X^{(n-2)}_{\mu_1\ldots\mu_{i-1}\mu_{i+1}\ldots\mu_{j-1}\mu_{j+1}\ldots\mu_n}$$

Taking into account the traceless property of $X^{(n)}$ we have:

$$X^{(n)}_{\mu_1\ldots\mu_n} X^{(n)}_{\mu_1\ldots\mu_n} = \alpha(n)(k^2_\perp)^n \qquad \alpha(n) = \prod_{i=1}^{n}\frac{2i-1}{i} = \frac{(2n-1)!}{n!}.$$

From the recourcive procedure one can get the following expression for the operator $X^{(n)}$:

$$X^{(n)}_{\mu_1\ldots\mu_n} = \left(\prod_{k=1}^{n}\frac{2k-1}{k}\right)\Bigg[k^{\perp}_{\mu_1}k^{\perp}_{\mu_2}\ldots k^{\perp}_{\mu_n} - \frac{k^2_\perp}{2n-1}\left(g^{\perp}_{\mu_1\mu_2}k^{\perp}_{\mu_3}\ldots k^{\perp}_{\mu_n} + \cdots\right) +$$
$$\frac{k^4_\perp}{(2n-1)(2n-3)}\left(g^{\perp}_{\mu_1\mu_2}g^{\perp}_{\mu_3\mu_4}k^{\perp}_{\mu_5}\cdots k_{\mu_4} + \cdots\right) + \cdots\Bigg].$$

2.1 Scattering amplitude of two pseudoscalar (scalar) particles.

If we denote the relative momentum of the system before and after interaction as k and q, then amplitude in the partial wave L ($L = n$) is:

$$A = a(m_1^2, m_2^2, P^2) X^{(n)}_{\mu_1\ldots\mu_n}(k) X^{(n)}_{\mu_1\ldots\mu_n}(q)$$

The $a(k_\perp^2, q_\perp^2, P^2)$ is a kinematical factor which depends only on invariants.

$$X^{(n)}_{\mu_1\ldots\mu_n}(k) X^{(n)}_{\mu_1\ldots\mu_n}(q) = \left(\sqrt{k_\perp^2}\right)^n \left(\sqrt{q_\perp^2}\right)^n P_n(z), \qquad z = \frac{(k^\perp q^\perp)}{\sqrt{k_\perp^2}\sqrt{q_\perp^2}}$$

Then:

$$A = a(m_1^2, m_2^2, P^2) \left(\sqrt{k_\perp^2}\right)^n \left(\sqrt{q_\perp^2}\right)^n P_n(z)$$

Structure of the fermion propagator

The orthogonality condition has a different form in a fermion case:

$$\int \Psi_\mu(x)\Psi^*(x)d^4x = A\,p_\mu + B\,\gamma_\mu$$

where A and B are matrices in spinor space.

It means that we have an additional condition:

$$\gamma_\mu \Psi_\mu = 0 \qquad\qquad \gamma_\mu u_\mu = 0$$

Thus in momentum space we have:

$$(\hat{p} - m)u_{\mu_1\ldots\mu_n} = 0$$

$$p_{\mu_i} u_{\mu_1\ldots\mu_n} = 0$$

$$u_{\mu_1\ldots\mu_i\ldots\mu_j\ldots\mu_n} = u_{\mu_1\ldots\mu_j\ldots\mu_i\ldots\mu_n}$$

$$g_{\mu_i\mu_j} u_{\mu_1\ldots\mu_n} = 0$$

$$\gamma_{\mu_i} \Psi_{\mu_1\ldots\mu_n} = 0$$

These properties define the structure of the fermion projection operator $P^{\mu_1\ldots\mu_n}_{\nu_1\ldots\nu_n}$:

$$G^{\mu_1\ldots\mu_n}_{\nu_1\ldots\nu_n} = \frac{(-1)^n}{2m}\frac{m+\hat{p}}{m^2-p^2}P^{\mu_1\ldots\mu_n}_{\nu_1\ldots\nu_n}$$

The $P^{\mu_1\ldots\mu_L}_{\nu_1\ldots\nu_n}$ is equal to 1 for $J=1/2$ particle. For particle with spin $3/2$ it has form:

$$\frac{1}{2}\Big(g^{\perp}_{\mu\nu} - \gamma^{\perp}_{\mu}\gamma^{\perp}_{\nu}/3\Big)$$

where

$$\gamma^{\perp}_{\mu} = g^{\perp}_{\mu\nu}\gamma_{\nu}.$$

The boson projector operator projects any operator to one which satisfies all boson properties. It means that we can write:

$$P^{\mu_1\ldots\mu_n}_{\nu_1\ldots\nu_n} = O^{\mu_1\ldots\mu_n}_{\alpha_1\ldots\alpha_n}T^{\alpha_1\ldots\alpha_n}_{\beta_1\ldots\beta_n}O^{\beta_1\ldots\beta_n}_{\nu_1\ldots\nu_n}$$

The $T^{\alpha_1\ldots\alpha_n}_{\beta_1\ldots\beta_n}$ operator can have a rather simple form: all symmetry and orthogonality conditions will be imposed by O-operators. First of all T-operator can be constructed only out of the metrical tensor and γ-matrices. Second a construction like $\gamma_{\alpha_i}\gamma_{\alpha_j}$:

$$\gamma_{\alpha_i}\gamma_{\alpha_j}=\frac{1}{2}g_{\alpha_i\alpha_j}+\sigma_{\alpha_i\alpha_j}, \qquad \text{where} \qquad \sigma_{\alpha_i\alpha_j}=\frac{1}{2}(\gamma_{\alpha_i}\gamma_{\alpha_j}-\gamma_{\alpha_j}\gamma_{\alpha_i})$$

gives zero after production with O-operator (first term due to traceless and second due to symmetrical properties). Then the only one structure which can be constructed out of gamma matrices is $g_{\alpha_i\beta_j}$ and $\sigma_{\alpha_i\beta_j}$. Moreover, taking into account symmetrical properties of O-operator the latest can be used as for example $\sigma_{\alpha_1\beta_1}$:

$$T^{\alpha_1\ldots\alpha_L}_{\beta_1\ldots\beta_L}=\frac{L+1}{2L+1}\Big(g_{\alpha_1\beta_1}-\frac{L}{L+1}\sigma_{\alpha_1\beta_1}\Big)\prod_{i=2}^{L}g_{\alpha_i\beta_i}$$

Here the coefficients are calculated to satisfy the orthogonality condition with γ-matrices.

πN interaction

Pion has quantum numbers $J^{PC}=0^{-+}$, proton $1/2^+$. Then in S-wave the only state can be formed is $1/2^-$. P-wave can form two states $1/2^+$ and $3/2^+$.

In PDG states are defined by the quantum numbers in πN decay: $L_{2I,2J}$. For example D_{13} means $3/2^-$ N^* state.

States where $J=L-1/2$ called '-' states ($1/2^+$, $3/2^-$,$5/2^+$,$\ldots$) and states with $J=L+1/2$ called '+' states ($1/2^-$, $3/2^+$,$5/2^-$,$\ldots$).

For '+' states and for '-' states:

$$N^+_{\mu_1\ldots\mu_n}=X^{(n)}_{\mu_1\ldots\mu_n} \qquad N^-_{\mu_1\ldots\mu_n}=i\gamma_\nu\gamma_5 X^{(n+1)}_{\nu\mu_1\ldots\mu_n}$$

$$A=\bar u(k_1)\tilde N^{\pm}_{\mu_1\ldots\mu_n}F^{\mu_1\ldots\mu_n}_{\nu_1\ldots\nu_n}(P)N^{\pm}_{\nu_1\ldots\nu_n}u(q_1)BW^{\pm}_L(s)$$

where

$$F^{\mu_1\ldots\mu_L}_{\nu_1\ldots\nu_L}(p)=(m+\hat p)O^{\mu_1\ldots\mu_L}_{\alpha_1\ldots\alpha_L}\frac{L+1}{2L+1}\big(g^{\perp}_{\alpha_1\beta_1}-\frac{L}{L+1}\sigma_{\alpha_1\beta_1}\big)\prod_{i=2}^{L}g_{\alpha_i\beta_i}O^{\beta_1\ldots\beta_L}_{\nu_1\ldots\nu_L}$$

In c.m.s. of the reaction

$$A_{\pi N} = \omega^* \left[G(s,t) + H(s,t) i(\vec{\sigma}\vec{n})\right] \omega' \ ,$$

$$G(s,t) = \sum_L \left[(L+1)F_L^+(s) - LF_L^-(s)\right] P_L(z) \ ,$$

$$H(s,t) = \sum_L \left[F_L^+(s) + F_L^-(s)\right] P'_L(z) \ .$$

$$F_L^+ \quad = -(|\vec{k}||\vec{q}|)^L \sqrt{\chi_i \chi_f}\ \frac{\alpha(L)}{2L+1} BW_L^+(s) \ ,$$

$$F_L^- \quad = (|\vec{k}||\vec{q}|)^L \sqrt{\chi_i \chi_f}\ \frac{\alpha(L)}{L} BW_L^-(s) \ .$$

$$\chi_i = m_i + k_{i0} \qquad\qquad \alpha(L) = \prod_{l=1}^{L} \frac{2l-1}{l} = \frac{(2L-1)!!}{L!} \ .$$

γN interaction

Photon has quantum numbers $J^{PC}=1^{--}$, proton $1/2^+$. Then in S-wave two states can be formed is $1/2^-$ and $3/2^-$.

Then P-wave $1/2^+, 3/2^+$ and $1/2^+, 3/2^+, 5/2^+$.

In general case: $1/2^-$, $1/2^+$ described by two amplitudes and higher states by three amplitudes.

$$
\begin{aligned}
V^{(1+)\mu}_{\alpha_1\ldots\alpha_n} &= \gamma_\mu i\gamma_5 X^{(n)}_{\alpha_1\ldots\alpha_n}\,, &\qquad V^{(1-)\mu}_{\alpha_1\ldots\alpha_n} &= \gamma_\xi\gamma_\mu X^{(n+1)}_{\xi\alpha_1\ldots\alpha_n}\,,\\
V^{(2+)\mu}_{\alpha_1\ldots\alpha_n} &= \gamma_\nu i\gamma_5 X^{(n+2)}_{\mu\nu\alpha_1\ldots\alpha_n}\,, & V^{(2-)\mu}_{\alpha_1\ldots\alpha_n} &= X^{(n+1)}_{\mu\alpha_1\ldots\alpha_n}\,,\\
V^{(3+)\mu}_{\alpha_1\ldots\alpha_n} &= \gamma_\nu i\gamma_5 X^{(n+1)}_{\nu\alpha_1\ldots\alpha_n} g^\perp_{\mu\alpha_n}\,, & V^{(3-)\mu}_{\alpha_1\ldots\alpha_n} &= X^{(n-1)}_{\alpha_2\ldots\alpha_n} g^\perp_{\alpha_1\mu}\,.
\end{aligned}
$$

Gauge invariance: $\varepsilon_\mu q_{1\mu}=0$ where q_1-photon momentum.

$$\varepsilon_\mu V^{(2\pm)\mu}_{\alpha_1\ldots\alpha_n} = C^\pm \varepsilon_\mu V^{(3\pm)\mu}_{\alpha_1\ldots\alpha_n}$$

where $C^\pm$ do not depend on angles.

$$A=\bar u(k_1) N^\pm_{\mu_1\ldots\mu_n} F^{\mu_1\ldots\mu_n}_{\nu_1\ldots\nu_n}(P) V^{(i\pm)\mu}_{\nu_1\ldots\nu_n} u(q_1) BW^\pm_L(s)\varepsilon_\mu$$

The amplitude for the photoproduction of a single pseudoscalar has structure

$$A = \omega^* J_\mu \varepsilon_\mu \omega' \ ,$$

$$J_\mu = i\mathcal{F}_1 \sigma_\mu + \mathcal{F}_2 (\vec{\sigma}\vec{q}) \frac{\varepsilon_{\mu ij}\sigma_i k_j}{|\vec{k}||\vec{q}|} + i\mathcal{F}_3 \frac{(\vec{\sigma}\vec{k})}{|\vec{k}||\vec{q}|} q_\mu + i\mathcal{F}_4 \frac{(\vec{\sigma}\vec{q})}{\vec{q}^2} q_\mu \ .$$

$$\mathcal{F}_1(z) = \sum_{L=0}^{\infty} [LM_L^+ + E_L^+] P'_{L+1}(z) + [(L+1)M_L^- + E_L^-] P'_{L-1}(z) \ ,$$

$$\mathcal{F}_2(z) = \sum_{L=1}^{\infty} [(L+1)M_L^+ + LM_L^-] P'_L(z) \ ,$$

$$\mathcal{F}_3(z) = \sum_{L=1}^{\infty} [E_L^+ - M_L^+] P''_{L+1}(z) + [E_L^- + M_L^-] P''_{L-1}(z) \ ,$$

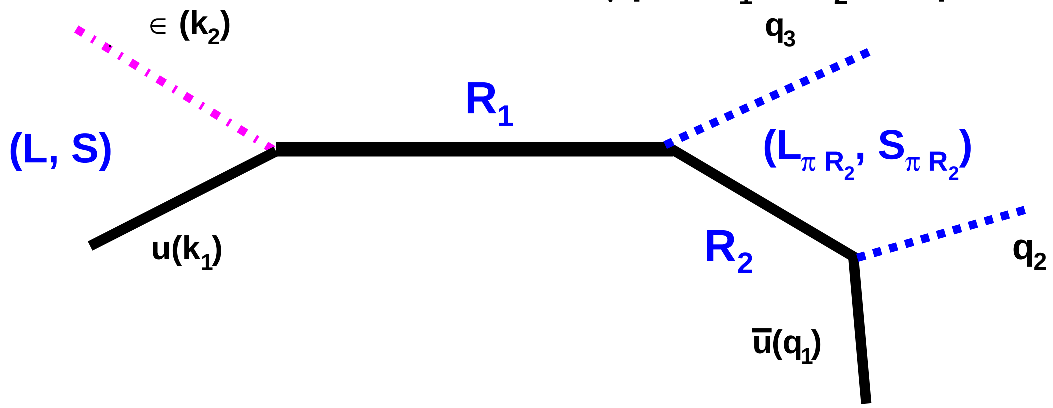
$$\mathcal{F}_4(z) = \sum_{L=2}^{\infty} [M_L^+ - E_L^+ - M_L^- - E_L^-] P''_L(z).$$

Our amplitudes can be algebraically rewritten to multipole representation:

$$E_L^{+(\frac{1}{2})} = \sqrt{\chi_i \chi_f}\,\frac{\alpha(L)}{2L+1}\,\frac{(|\vec{k}||\vec{q}|)^L}{L+1} BW^+(s)\ ,$$
$$M_L^{+(\frac{1}{2})} = E_L^{+(\frac{1}{2})}\ .$$

$$E_L^{+(\frac{3}{2})} = \sqrt{\chi_i \chi_f}\,\frac{\alpha(L)}{2L+1}\,\frac{(|\vec{k}||\vec{q}|)^L}{L+1} BW^+(s)\ ,$$
$$M_L^{+(\frac{3}{2})} = -\frac{E_L^{+(\frac{3}{2})}}{L}\ .$$

The resonance amplitudes for meson photoproduction



The general form of the angular dependent part of the amplitude:

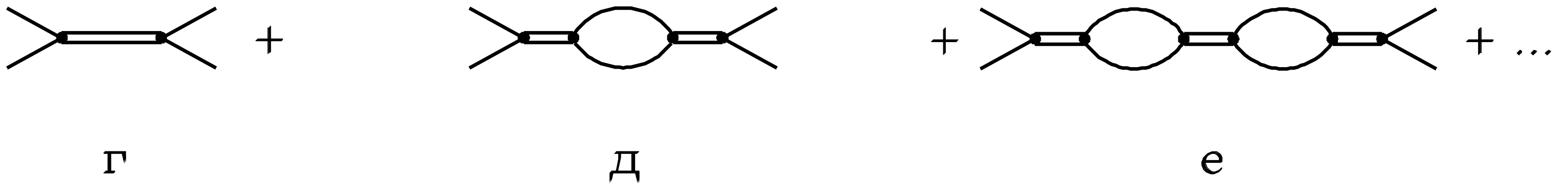
$$\bar{u}(q_1)\tilde{N}_{\alpha_1\ldots\alpha_n}(R_2\to\mu N)F^{\alpha_1\ldots\alpha_n}_{\beta_1\ldots\beta_n}(q_1+q_2)\tilde{N}^{(j)\beta_1\ldots\beta_n}_{\gamma_1\ldots\gamma_m}(R_1\to\mu R_2)$$
$$F^{\gamma_1\ldots\gamma_m}_{\xi_1\ldots\xi_m}(P)V^{(i)\mu}_{\xi_1\ldots\xi_m}(R_1\to\gamma N)u(k_1)\varepsilon_\mu$$

$$F^{\mu_1\ldots\mu_L}_{\nu_1\ldots\nu_L}(p)=(m+\hat{p})O^{\mu_1\ldots\mu_L}_{\alpha_1\ldots\alpha_L}\frac{L+1}{2L+1}\left(g^{\perp}_{\alpha_1\beta_1}-\frac{L}{L+1}\sigma_{\alpha_1\beta_1}\right)\prod_{i=2}^{L}g_{\alpha_i\beta_i}O^{\beta_1\ldots\beta_L}_{\nu_1\ldots\nu_L}$$

$$\sigma_{\alpha_i\alpha_j}=\frac{1}{2}(\gamma_{\alpha_i}\gamma_{\alpha_j}-\gamma_{\alpha_j}\gamma_{\alpha_i})$$

Diagram approach to the calculation of the scattering amplitude

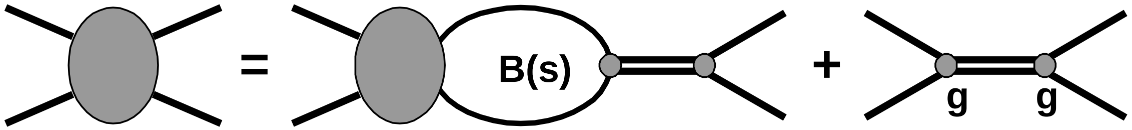
Let us consider a state which is produced from two interacting particles, then propagates and decays into same two particles in the final state. The amplitude for such process can be represented as a sum of the diagrams:



The amplitude can be found as direct sum of the diagrams:

$$A = \frac{1}{M_0^2 - s}\frac{1}{1 - \frac{B(s)}{M_0^2 - s}}$$

This amplitude also can be found by solving the following equation:



$$A = A\frac{B(s)}{M_0^2 - s} + \frac{g^2}{M_0^2 - s}$$

$$A = \frac{g^2}{M_0^2 - s}\frac{1}{1 - \frac{B(s)}{M_0^2 - s}} = \frac{g^2}{M_0^2 - s - B(s)}$$

Here M_0 is a bare mass of the state and $B(s)$ is the two body loop diagram:

$$B^F = \int \frac{d^4k}{i(2\pi)^4} \frac{g^2}{(m^2 - k^2)(m^2 - (P-k)^2)},$$

Let us assume that the vertex (coupling) g has no singularities and is a smooth function in the physical region. The imaginary part of the loop diagram appears at the energy $s > 4m^2$ and can be calculated by:

$$(m^2 - k^2)^{-1}(m^2 - (P-k)^2)^{-1} \to (-2\pi)^2 i\, \delta(m^2 - k^2)\Theta(k_0)$$
$$\delta(m^2 - (P-k)^2)\Theta(P_0 - k_0)$$

Then loop diagram can be rewritten in the dispersion representation:

$$B(s) = \int\limits_{4m^2}^{\infty} \frac{ds'}{\pi} \frac{g(s')\rho(s')g(s')}{s' - s - i0} = Re \int\limits_{4m^2}^{\infty} \frac{ds'}{\pi} \frac{g(s')\rho(s')g(s')}{s' - s} - i\rho(s)g^2(s)$$

The real part of the B-function does not have any singularities in the physical region. It is a smooth function and can be used for renormalization of the bare state mass (or neglected in some cases). Then:

$$A(s) = \frac{g^2}{M^2 - s - i\rho(s)g^2} \qquad\qquad M^2 = M_0^2 - ReB(M^2)$$

And we obtain a Breit-Wigner expression with $M\Gamma = g^2\rho(M^2)$.

Let us calculate analytically the loop diagram assuming that g is a constant.

First we need a renormalization:

$$B(s)=B(M^2)+(s-M^2)\int\limits_{4m^2}^{\infty}\frac{ds'}{\pi}\frac{g^2}{(s'-s)(s'-M^2)}\sqrt{\frac{s'-4m^2}{s'}}$$

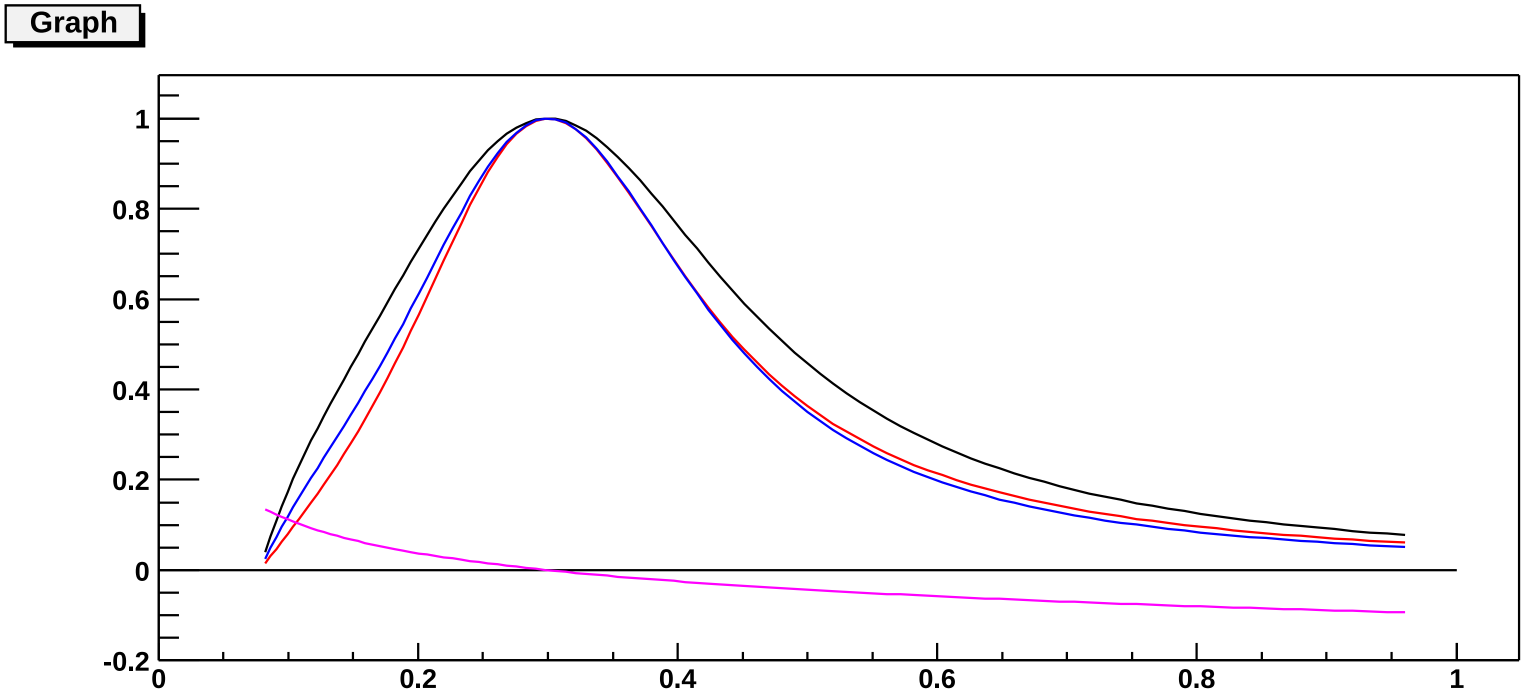
Such integral is equal to:

$$B(s)=ReB(M^2)+\frac{g^2}{\pi}[\rho(s)\ ln\frac{1-\rho(s)}{1+\rho(s)}-\rho(M^2)\ ln\frac{1-\rho(M^2)}{1+\rho(M^2)}]+i\rho(s)g^2$$

At $s\longrightarrow 0$:

$$i\rho(s)=i\sqrt{\frac{s-4m^2}{s}}\rightarrow-\infty$$

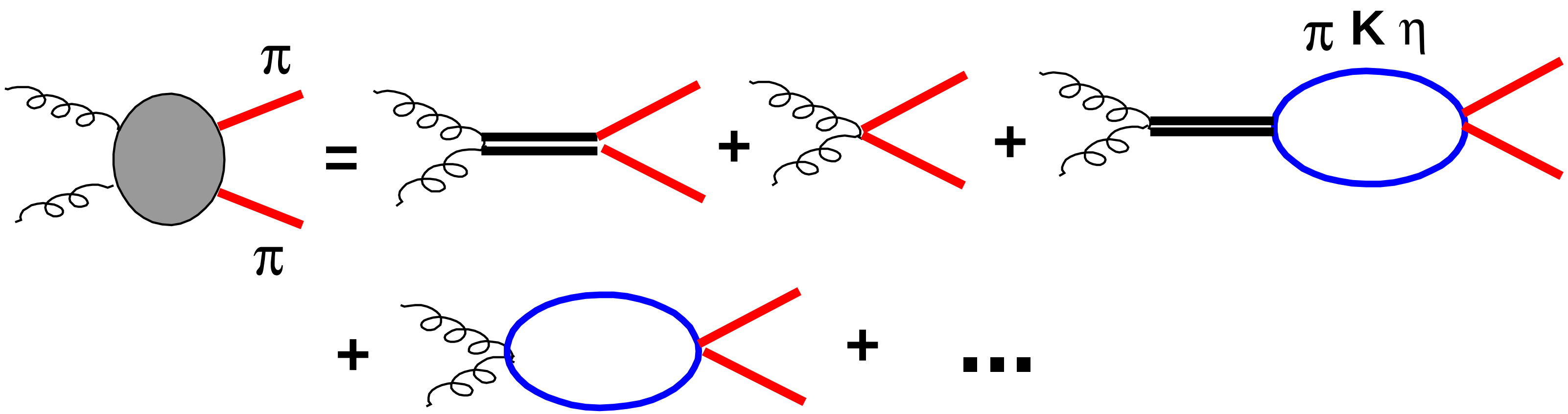
$$i\rho(s)(1.-\frac{i}{\pi}ln\frac{1-\rho(s)}{1+\rho(s)})=i\sqrt{\frac{s-4m^2}{s}}(1-\frac{2}{\pi}arctg\frac{4m^2-s}{s})\rightarrow const$$



Black curve - BW amplitude, red curve - full B(s) calculation, blue curve - BW amplitude with reduced width, magenta - dispersion correction of the real part.

P-vector approach

Let us consider photoproduction of two pions. This case is different from the $\pi\pi$ scattering by the first interaction:



The first interaction can be the direct production of K-matrix poles or nonresonant production: $\gamma\gamma \to \pi\pi$, $\gamma\gamma \to K\bar{K}$ and so on.

Combined analysis of the different reactions:

For pion induced reactions the transition partial wave amplitude can be written as:

$$A_{1i} = K_{1j}(I - i\rho K)^{-1}_{ji}$$

and

$$K_{ij} = \sum_\alpha \frac{g_i^\alpha g_j^\alpha}{M_\alpha^2 - s} + f_{ij}(s) \qquad f_{ij} = \frac{f_{ij}^{(1)} + f_{ij}^{(2)}\sqrt{s}}{s - s_0^{ij}}\,.$$

where f_{ij} is nonresonant transition part.

For the photoproduction:

$$A_k = P_j(I - i\rho K)^{-1}_{jk}$$

The vector of the initial interaction has the form:

$$P_j = \sum_\alpha \frac{\Lambda^\alpha g_j^\alpha}{M_\alpha^2 - s} + F_j(s)$$

Here F_j is nonresonant production of the final state j.

D-vector approach

For πN transition into channel 'a' the amplitude can be written as:

$$A_a = \hat{D}_a + [\hat{K}(\hat{I} - i\hat{\rho}\hat{K})^{-1}\,\hat{\rho}]_{ab}\hat{D}_b\ ,$$

For strong channels:

$$D_a = K_{1a} \qquad A_a = K_{1j}(I - i\rho K)^{-1}_{ji}$$

For weak channels:

$$D_a = \sum_\alpha \frac{g_1^\alpha \Lambda_a^{dec}}{M_\alpha^2 - s} + d_{1a}(s)$$

For photoproduction of 'weak channels'

$$A_{ab} = \hat{G}_{ab} + \hat{P}_a(\hat{I} - i\hat{\rho}\hat{K})^{-1}\,\hat{\rho}\hat{D}_b \qquad G_{ab} = \sum_\alpha \frac{\Lambda_b \Lambda_a^{dec}}{M_\alpha^2 - s} + b_{ab}(s)$$

Phase volumes

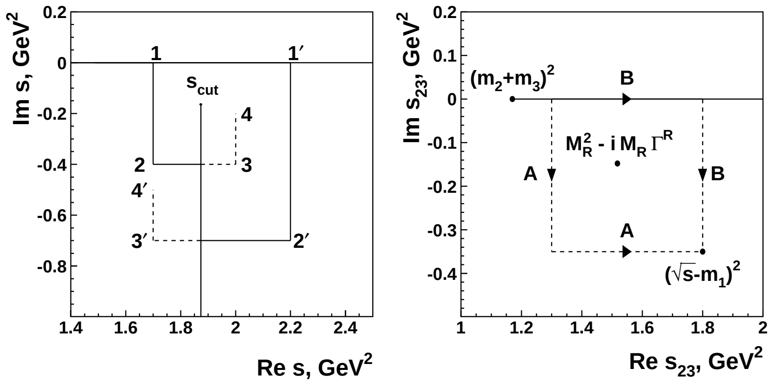
Two body phase volume:

$$\rho(s,m_1,m_2)=\frac{2k}{s}=\frac{\sqrt{(s-(m_1+m_2)^2)(s-(m_1-m_2)^2)}}{s}$$

Three body phase volume:

$$\rho_3(s)=\int\limits_{(m_2+m_3)^2}^{(\sqrt{s}-m_1)^2}\frac{ds_{23}}{\pi}\,\frac{\rho(s,\sqrt{s_{23}},m_1)\,M_R\Gamma^R_{tot}}{(M_R^2-s_{23})^2+(M_R\Gamma^R_{tot})^2}\;,$$

$$M_R\Gamma^R_{tot}=\rho(s_{23},m_2,m_3)g^2(s_{23})\;,$$



Pole position in the complex plane.

Two sheets are defined as:

$$\rho(s,m_1,m_2) = \frac{\sqrt{(s-(m_N+m_\eta)^2)(s-(m_N-m_\eta)^2)}}{s} \qquad I\ sheet$$

$$\rho(s,m_1,m_2) = i\frac{\sqrt{((m_N+m_\eta)^2-s)(s-(m_N-m_\eta)^2)}}{s} \qquad II\ sheet$$

We search for

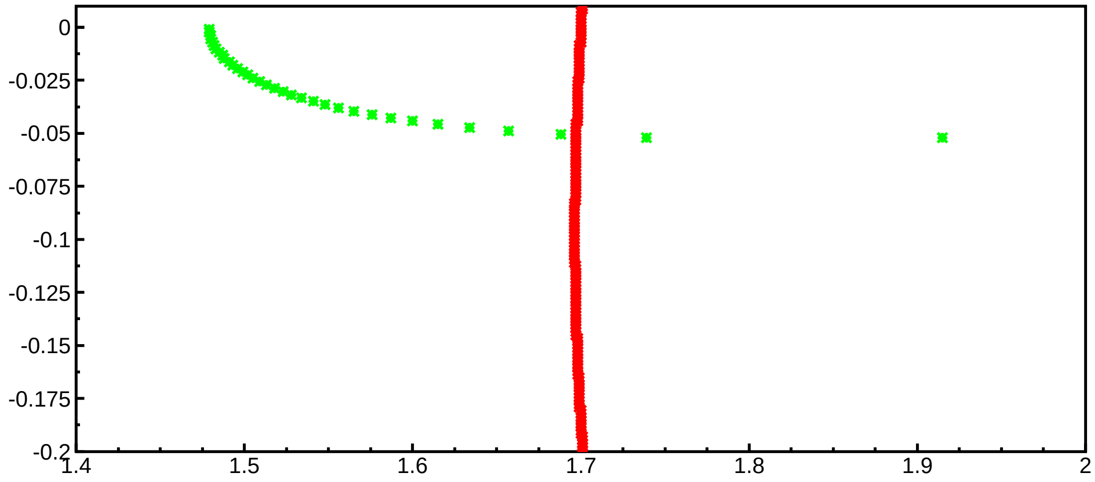
$$Im\ det(I-i\rho K)\prod_\alpha (M_\alpha^2-s) = 0$$

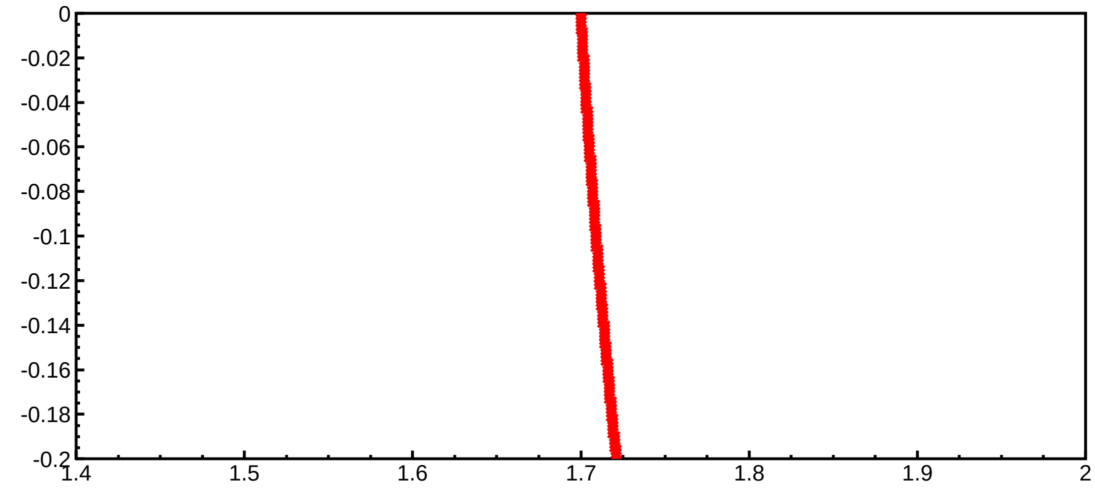
$$Re\ det(I-i\rho K)\prod_\alpha (M_\alpha^2-s) = 0$$

For BW approximation (1pole K-matrix)

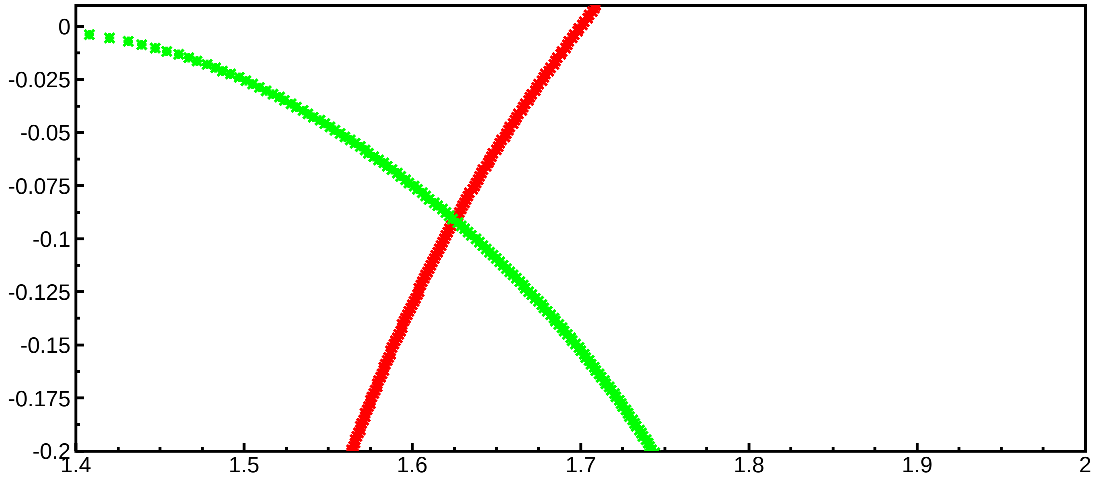
$$det(I-i\rho K)(M_\alpha^2-s) = M_\alpha^2 - s - i\sum_i g_i^2\rho_i$$

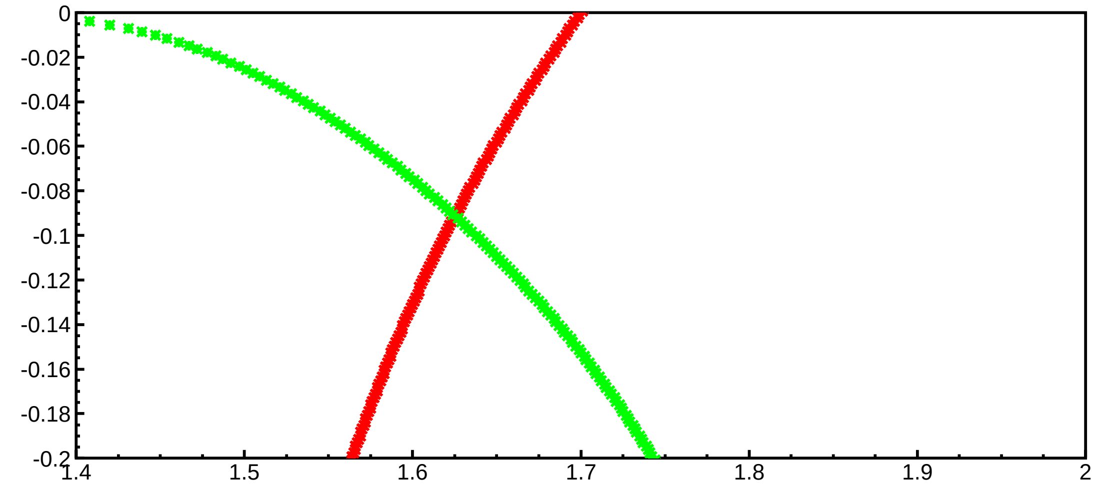
One pole one channel (ηN) K-matrix



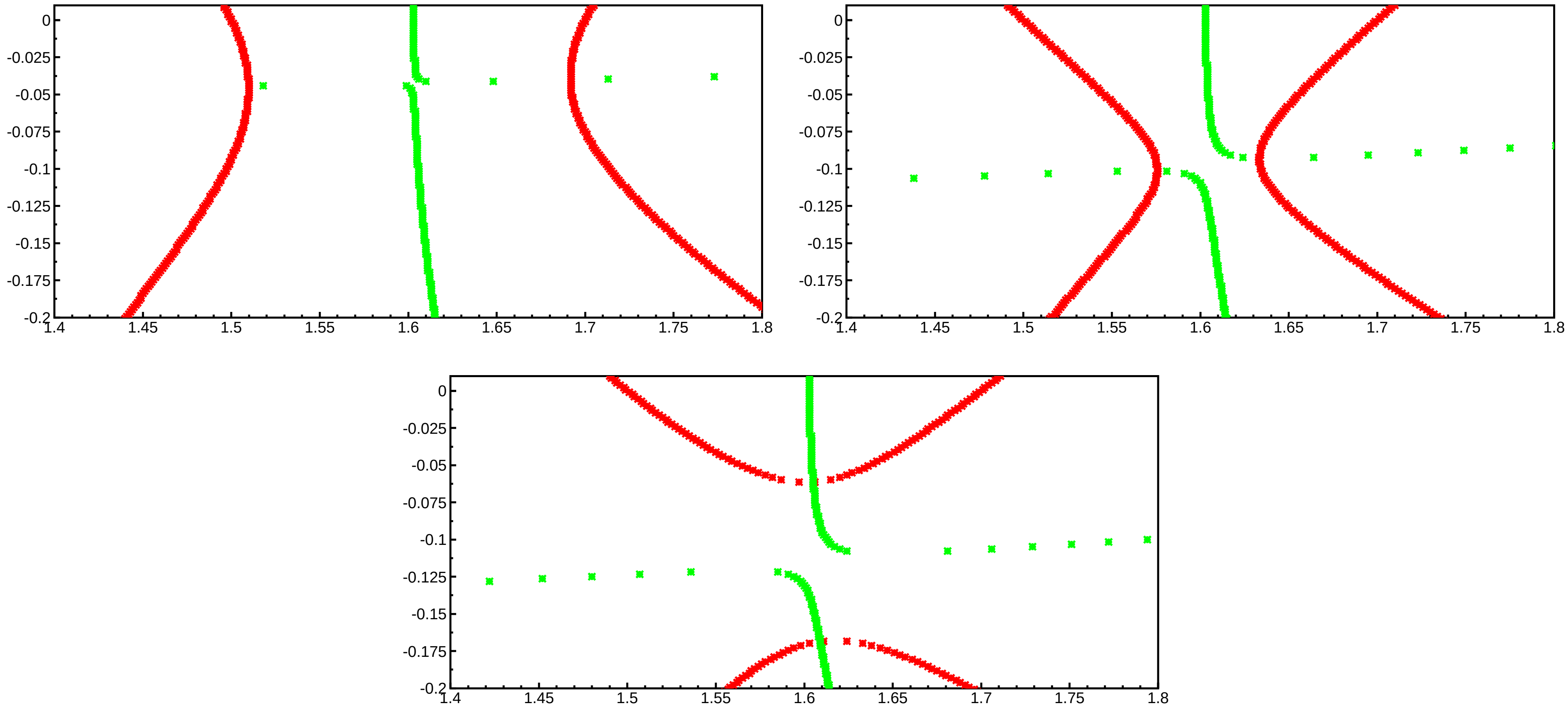


One pole one channel ($\pi\Delta$) K-matrix





Two poles one channel (πN) K-matrix



Calculation of the residues

For any function of complex variables $F(s)$:

$$\oint dsF(s) = \sum_{\alpha} 2\pi i \; Res_{\alpha}$$

This does not depend on the form of counter.

K-matrix amplitude A_{ij}, full factorization:

$$\oint dsA_{ij}(s) = 2\pi i \; g_i g_j$$

In our article:

$$A_{BW} = \frac{X}{M^2_{BW} - s - i\beta \sum_i g_i^2 \rho_i}$$

where M_{BW} and β are fitted to reconstruct pole position.

The fitted reactions. Recently included data sets. New points added

Observable	$N_{\rm data}$	$\frac{\chi^2}{N_{\rm data}}$		Observable	$N_{\rm data}$	$\frac{\chi^2}{N_{\rm data}}$	
$\sigma(\gamma p\to p\pi^0)$	1106	1.27	CB-ELSA	$\sigma(\gamma p\to p\pi^0)$	861	1.74	GRAAL
$\sigma(\frac{3}{2}-\frac{1}{2})(p\pi^0)$	140	1.41	A2GDH	$\Sigma(\gamma p\to p\pi^0)$	1492	3.38	SAID
$P(\gamma p\to p\pi^0)$	607	3.16	SAID	$T(\gamma p\to p\pi^0)$	389	4.01	SAID
$H(\gamma p\to p\pi^0)$	71	1.92	SAID	$G(\gamma p\to p\pi^0)$	75	2.58	SAID
$Ox(\gamma p\to p\pi^0)$	7	1.01	SAID	$Oz(\gamma p\to p\pi^0)$	7	0.38	SAID
$\sigma(\gamma p\to n\pi^+)$	1583	1.87	SAID	$\sigma(\gamma p\to n\pi^+)$	408	2.09	A2GDH
$\Sigma(\gamma p\to n\pi^+)$	899	4.23	SAID	$\sigma(\frac{3}{2}-\frac{1}{2})(n\pi^+)$	231	2.49	A2GDH
$P(\gamma p\to n\pi^+)$	252	3.90	SAID	$T(\gamma p\to n\pi^+)$	661	3.66	SAID
$H(\gamma p\to p\pi^0)$	71	1.92	SAID	$G(\gamma p\to p\pi^0)$	75	2.58	SAID
$S_{11}(\pi N\to\pi N)$	126	1.40	SAID	$P_{11}(\pi N\to\pi N)$	110	2.24	SAID
$P_{13}(\pi N\to\pi N)$	108	2.57	SAID	$P_{33}(\pi N\to\pi N)$	130	4.56	SAID
$D_{33}(\pi N\to\pi N)$	136	4.51	SAID	$D_{13}(\pi N\to\pi N)$	106	5.06	SAID
$\sigma(\gamma p\to p\eta)$	667	0.92	CB-ELSA	$\sigma(\gamma p\to p\eta)$	100	2.72	TAPS
$\Sigma(\gamma p\to p\eta)$	51	2.06	GRAAL 98	$\Sigma(\gamma p\to p\eta)$	100	2.01	GRAAL 04
$T(\gamma p\to p\eta)$	50	1.52	Phoenics	$\sigma(\pi^- p\to n\eta)$	288	2.76	CBALL+Richards

The fitted reactions. Recently included data sets.

Observable	$N_{\rm data}$	$\frac{\chi^2}{N_{\rm data}}$		Observable	$N_{\rm data}$	$\frac{\chi^2}{N_{\rm data}}$	
$C_x(\gamma p\to\Lambda K^+)$	160	1.22	CLAS	$C_x(\gamma p\to\Sigma^0K^+)$	94	2.29	CLAS
$C_z(\gamma p\to\Lambda K^+)$	160	1.53	CLAS	$C_z(\gamma p\to\Sigma^0K^+)$	94	2.19	CLAS
$\sigma(\gamma p\to\Lambda K^+)$	1377	1.80	CLAS	$\sigma(\gamma p\to\Sigma^0K^+)$	1280	2.68	CLAS
$P(\gamma p\to\Lambda K^+)$	202	2.31	CLAS	$P(\gamma p\to\Sigma^0K^+)$	95	1.56	CLAS
$\Sigma(\gamma p\to\Lambda K^+)$	66	2.70	GRAAL	$\Sigma(\gamma p\to\Sigma^0K^+)$	42	0.67	GRAAL
$\Sigma(\gamma p\to\Lambda K^+)$	45	1.75	LEP	$\Sigma(\gamma p\to\Sigma^0K^+)$	45	1.03	LEP
$T(\gamma p\to\Lambda K^+)$	66	2.11	GRAAL	$\sigma(\gamma p\to\Sigma^+K^0)$	48	3.36	CLAS
$Ox(\gamma p\to\Lambda K^+)$	66	1.40	GRAAL	$\sigma(\gamma p\to\Sigma^+K^0)$	160	0.95	CB-ELSA
$Oz(\gamma p\to\Lambda K^+)$	66	1.86	GRAAL	$P(\gamma p\to\Sigma^+K^0)$	72	0.72	CB-ELSA
$\sigma(\gamma p\to p\pi^0\pi^0)$	CB-ELSA (1.4 GeV)			$E(\gamma p\to p\pi^0\pi^0)$	16	2.08	MAMI
$\sigma(\gamma p\to p\pi^0\eta)$	CB-ELSA (3.2 GeV)			$\Sigma(\gamma p\to p\pi^0\eta)$	180	2.68	GRAAL
$\sigma(\gamma p\to p\pi^0\pi^0)$	CB-ELSA (3.2 GeV)			$\Sigma(\gamma p\to p\pi^0\pi^0)$	128	0.85	GRAAL
$\sigma(\pi^-p\to K\Lambda)$	479	1.55	RAL	$P(\pi^-p\to K\Lambda)$	261	1.76	RAL+ANL
$\sigma(\pi^+p\to K^+\Sigma)$	609	1.91	RAL	$P(\pi^+p\to K^+\Sigma)$	420	2.74	RAL

$\gamma p \to \pi^0 p$ from Crystal Barrel at ELSA ($E_\gamma \leq 3.2$ GeV)

$\Delta(1232)P_{33}$

$N(1520)D_{13}$ S_{11}

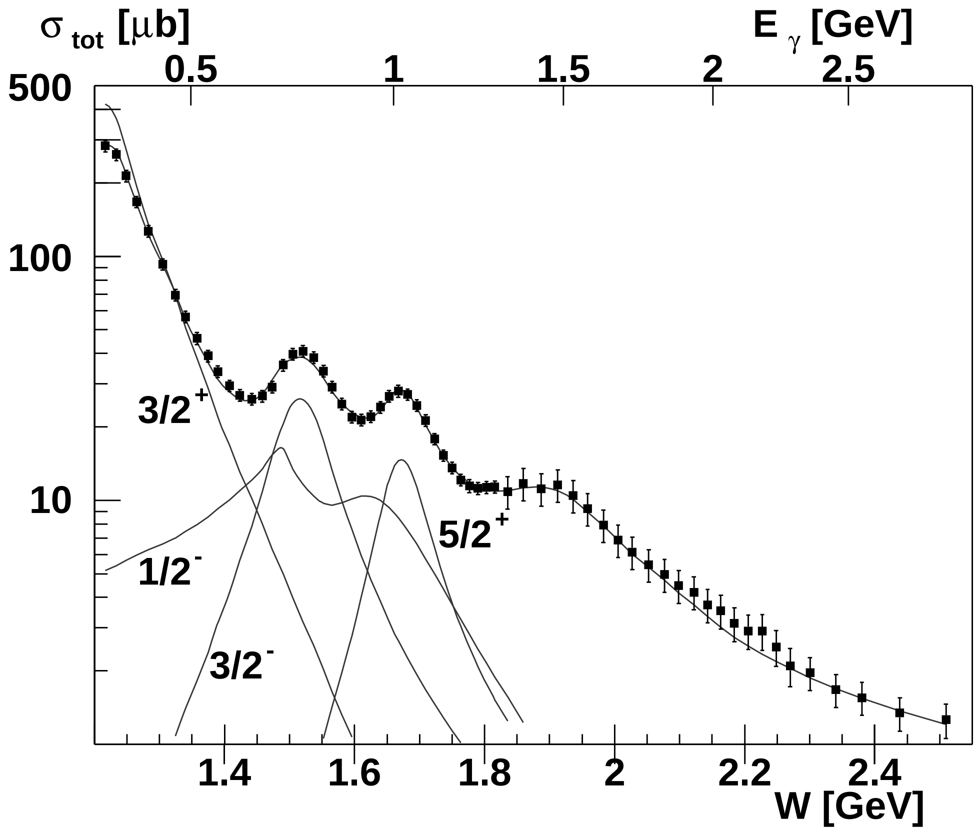
$N(1680)F_{15}$

$\Delta(1700)D_{33}$

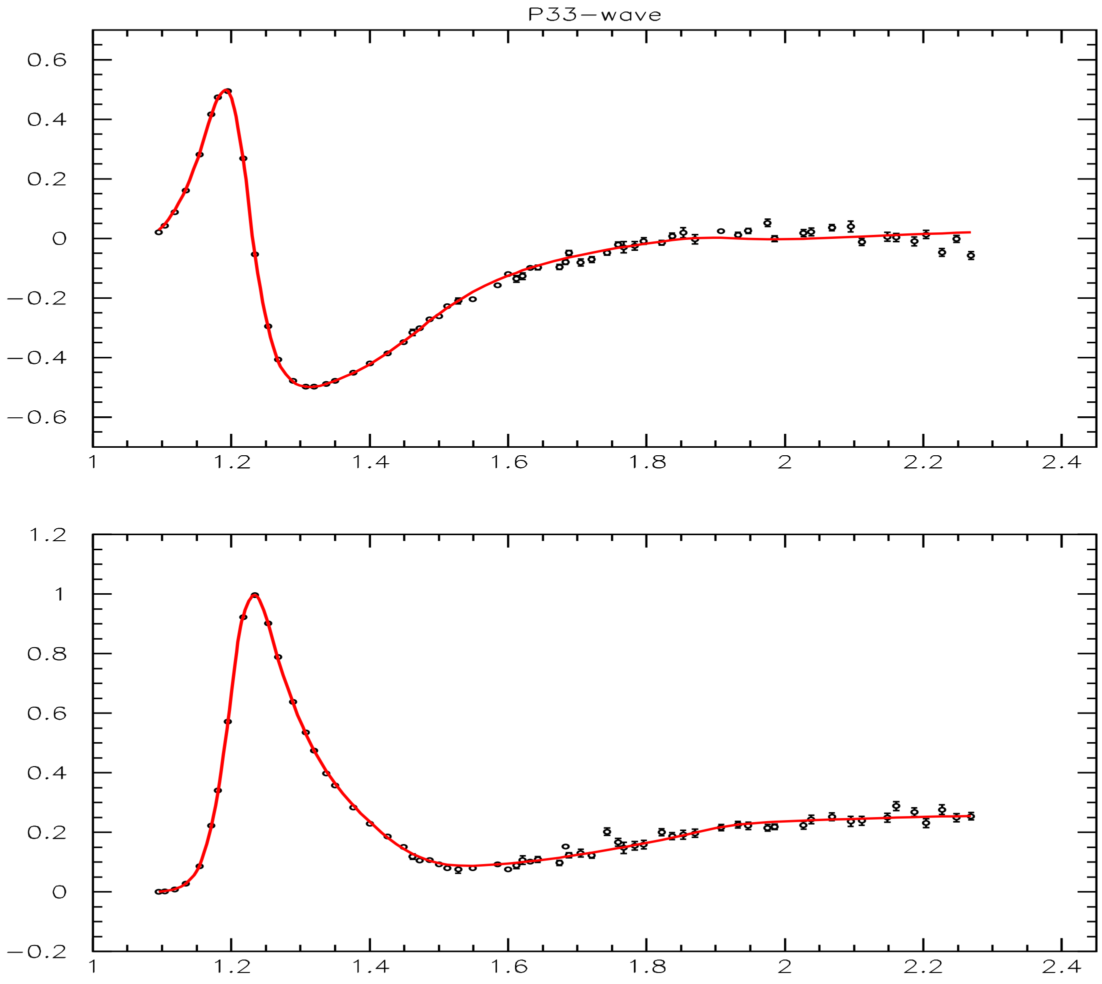
$\Delta(1920)P_{33}$

Non-resonance contribution:

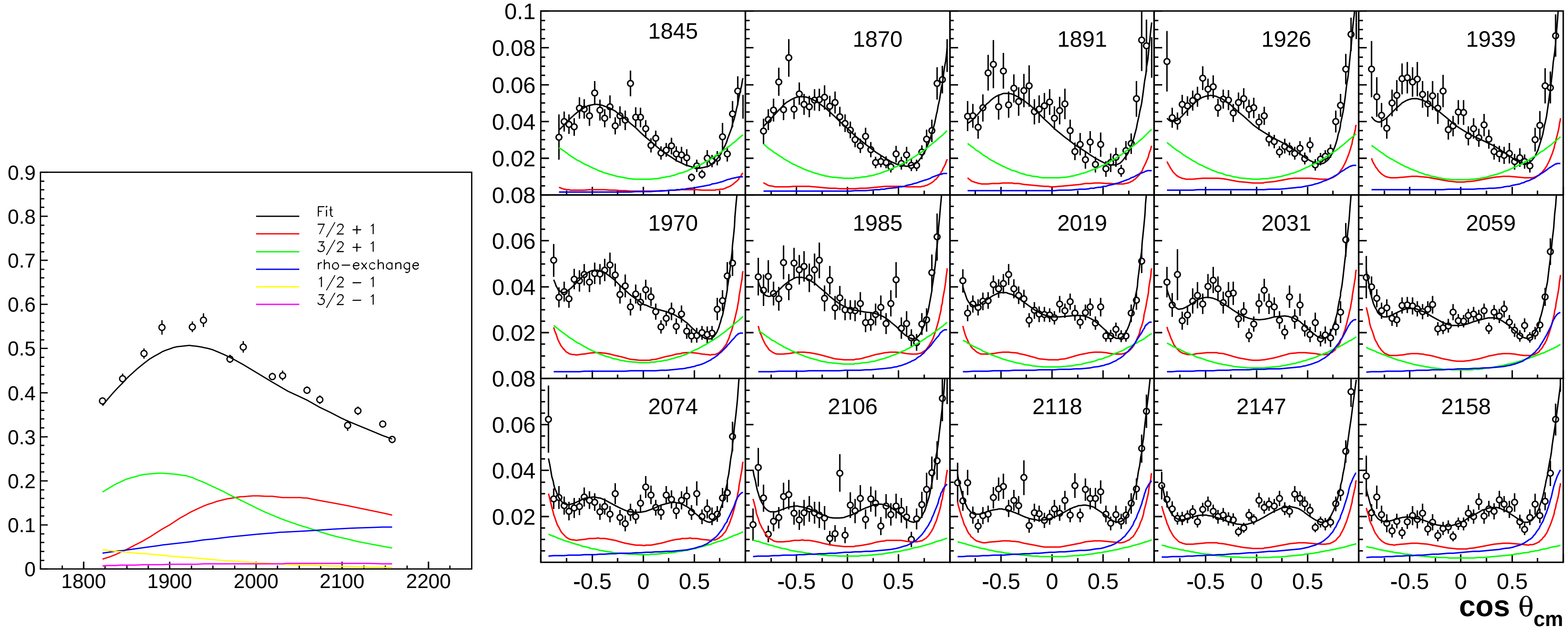
t-channel $\rho-\omega$ exchange, u-exchange and non-resonance production in $J^P = 3/2^+$ wave



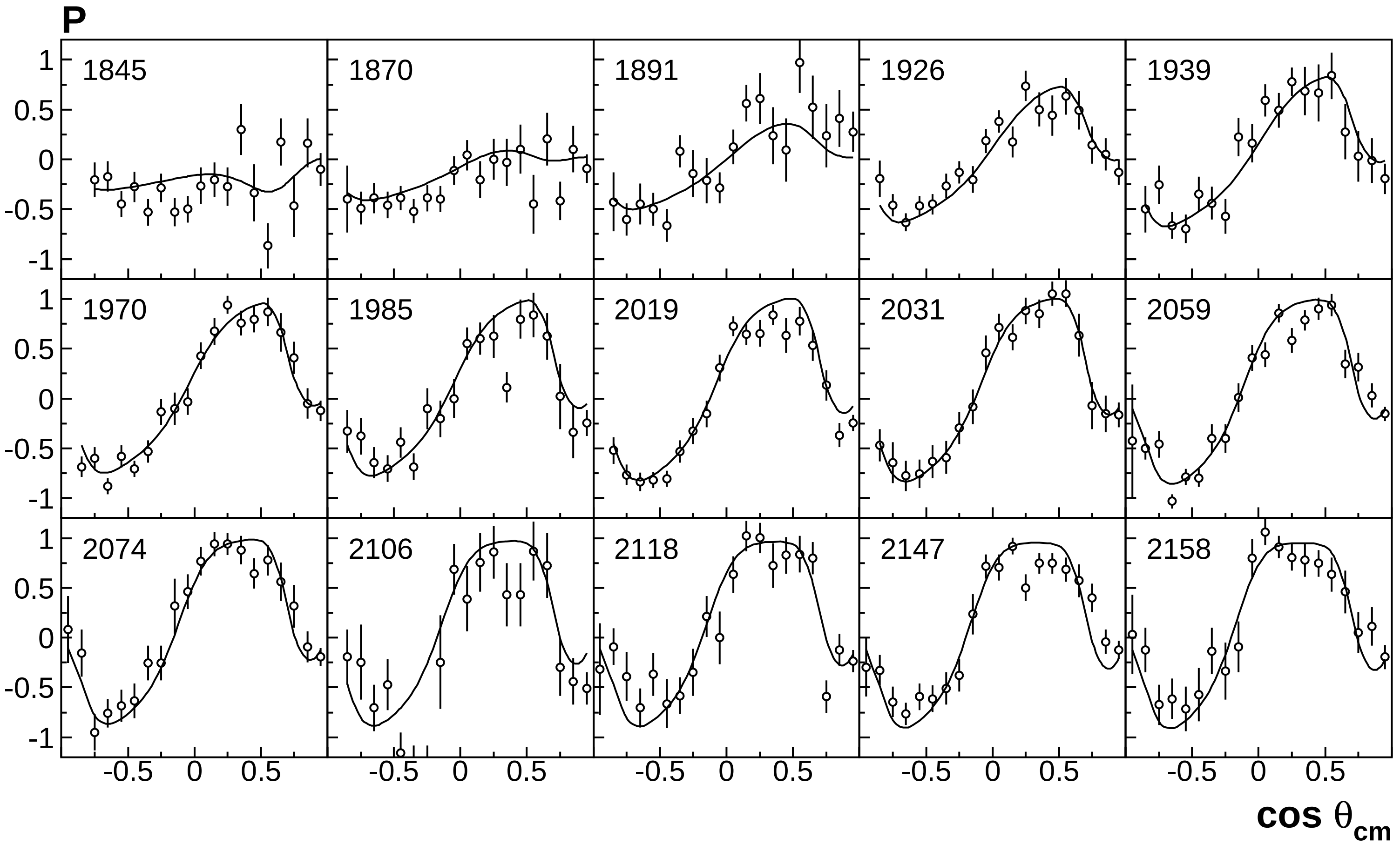
$\mathrm{N}\pi \to \mathrm{N}\pi$, P_{33} wave (3 pole 4 channel K-matrix)



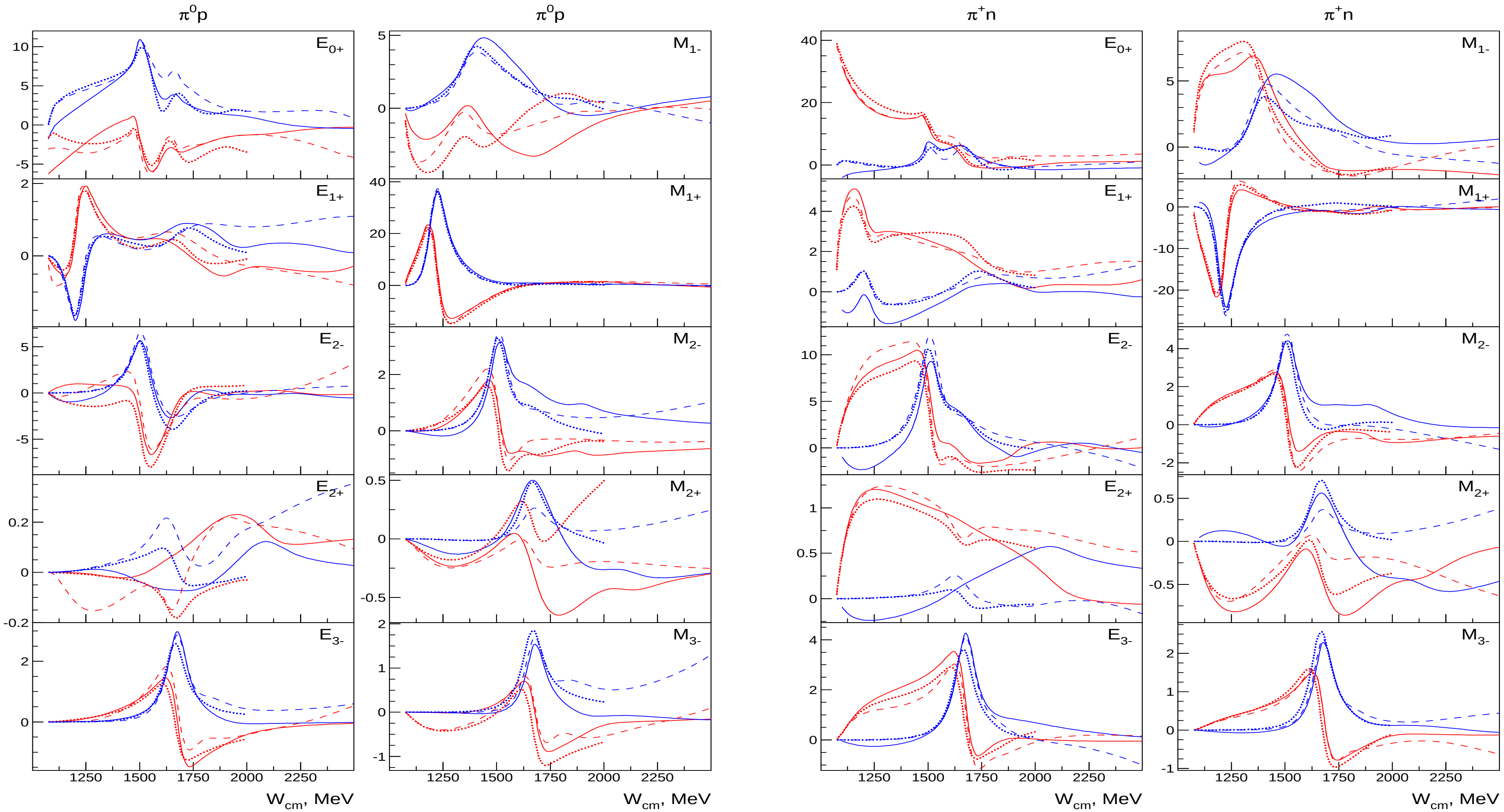
The Δ-states decaying into $K\Sigma$ can be fixed from the $\pi^+ \to K^+\Sigma$ data. The main contribution comes from $P_{33}(1920)$ -red curves and $F_{37}(1900)$ - blue curves.



The recoil asymmetry for the $\pi^+ p \to K\Sigma$ reaction. also shows a clear contribution from this state:



The multipoles for single pion production. Red - real part, Blue - imaginary part. Solid curves BoGa -solution, dashed curves - SAID solution, dotted - MAID 2009.



$\gamma p \to \eta p$ from Crystal Barrel at ELSA ($E_\gamma \leq 3.2$ GeV)

Main resonance contributions:

$N(1535)S_{11}$

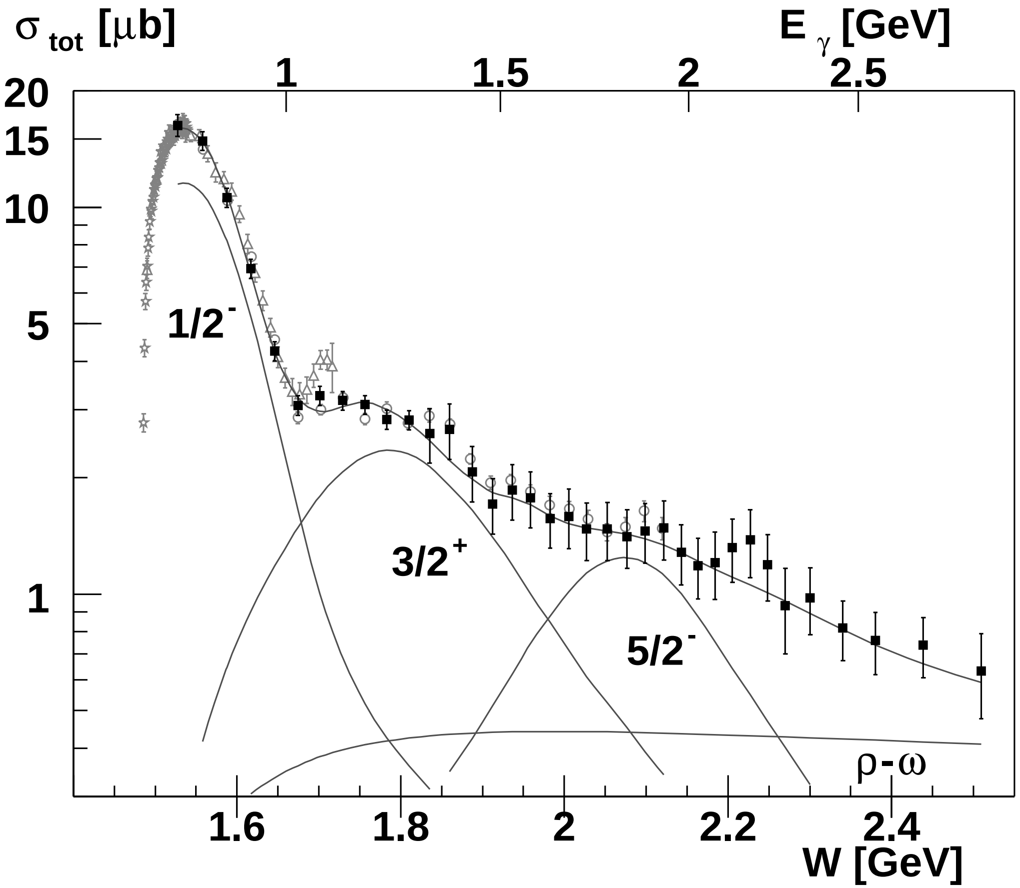
$N(1650)S_{11}$

$N(1720)P_{13}$

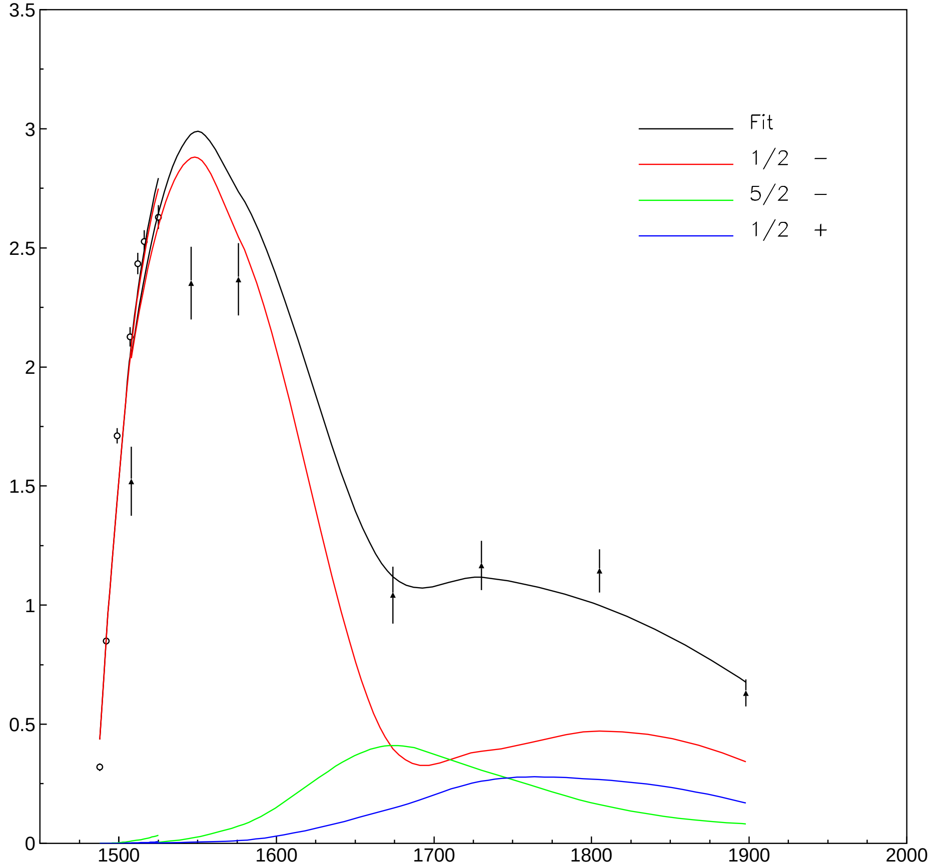
new $N(2070)D_{15}$

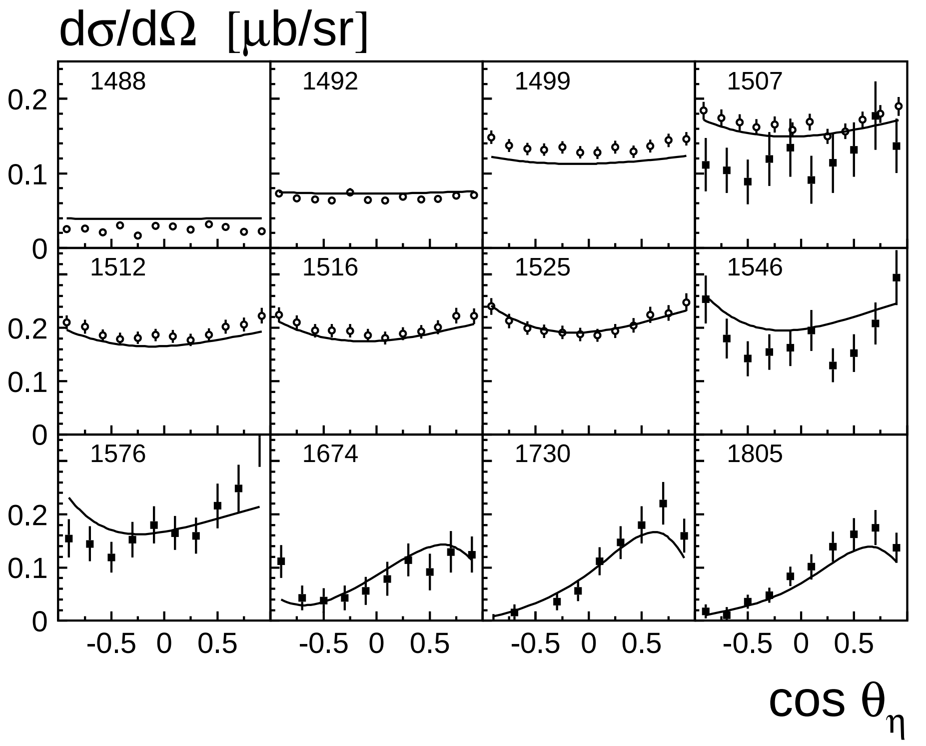
Non-resonance contribution: reggezied t-channel $\rho - \omega$ **exchange.**

No evidence for third $N(1800)S_{11}$



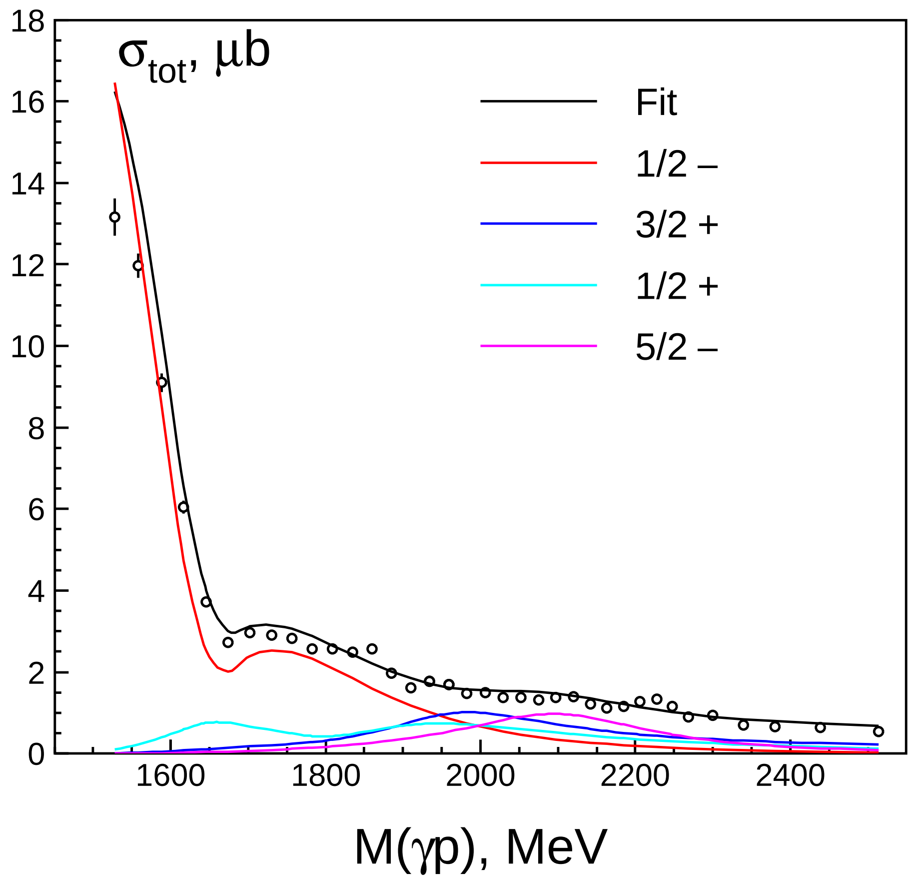
The data on $\pi^- p \to \eta n$ and the target asymmetry $\gamma p \to \eta p$ fix the position and couplings of $P_{11}(1710)$ state and reduce ηN coupling of the $P_{13}(1720)$ state.

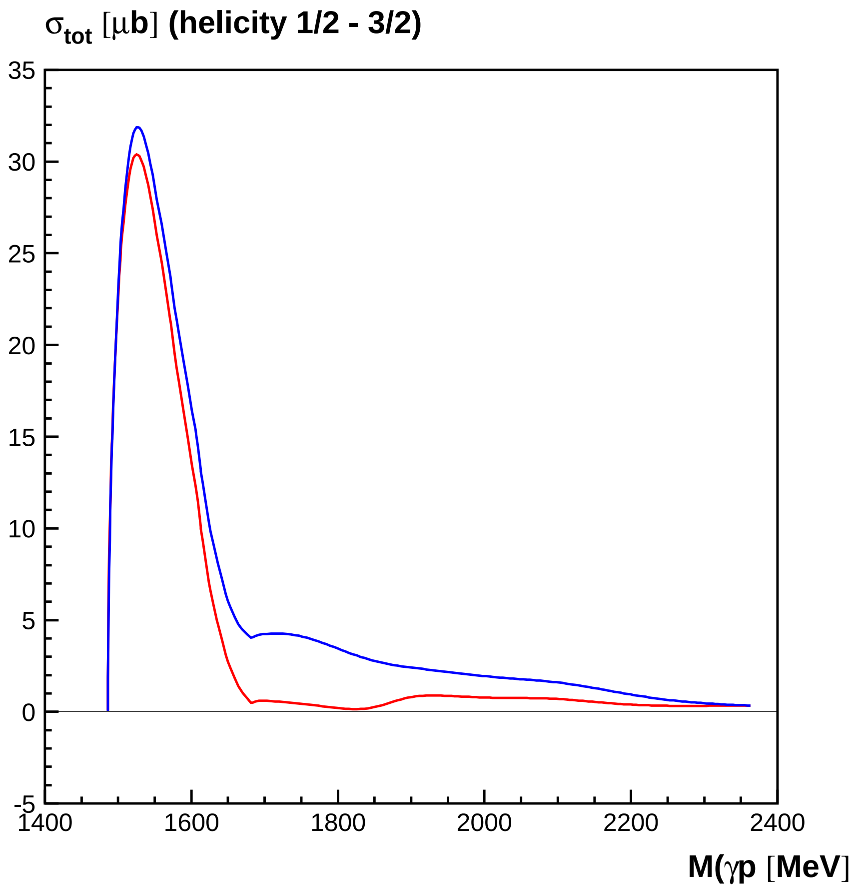




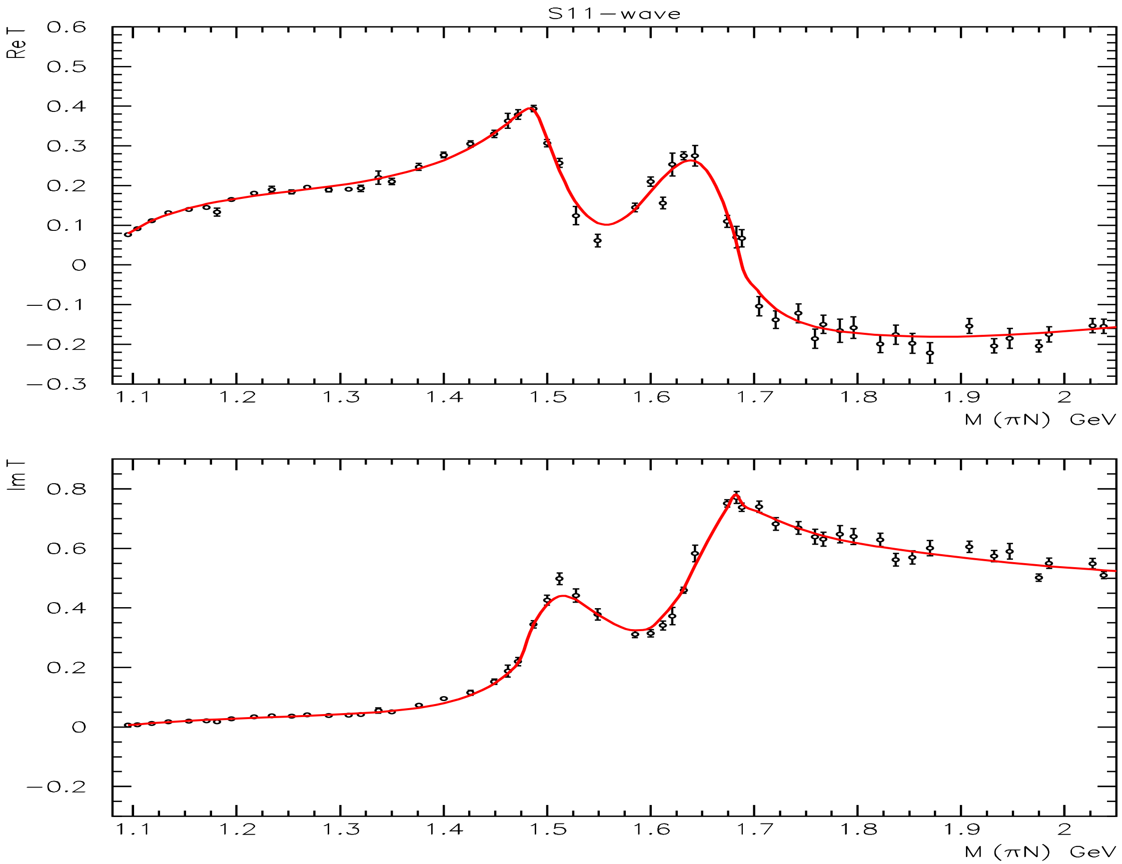
Observable	N_{data}	$\frac{\chi^2}{N_{data}}$		Observable	N_{data}	$\frac{\chi^2}{N_{data}}$	
$\sigma(\gamma p \to p\eta)$	667	0.92 (0.85)	CB-ELSA	$\sigma(\gamma p \to p\eta)$	100	2.72 (1.97)	TAPS
$\Sigma(\gamma p \to p\eta)$	51	2.06 (1.81)	GRAAL 98	$\Sigma(\gamma p \to p\eta)$	100	2.01 (1.43)	GRAAL 04

The target asymmetry $\gamma p \to \eta p$ data reduce coupling of the $P_{13}(1720)$ state to the ηN channel by factor $\sim$ 1.7.



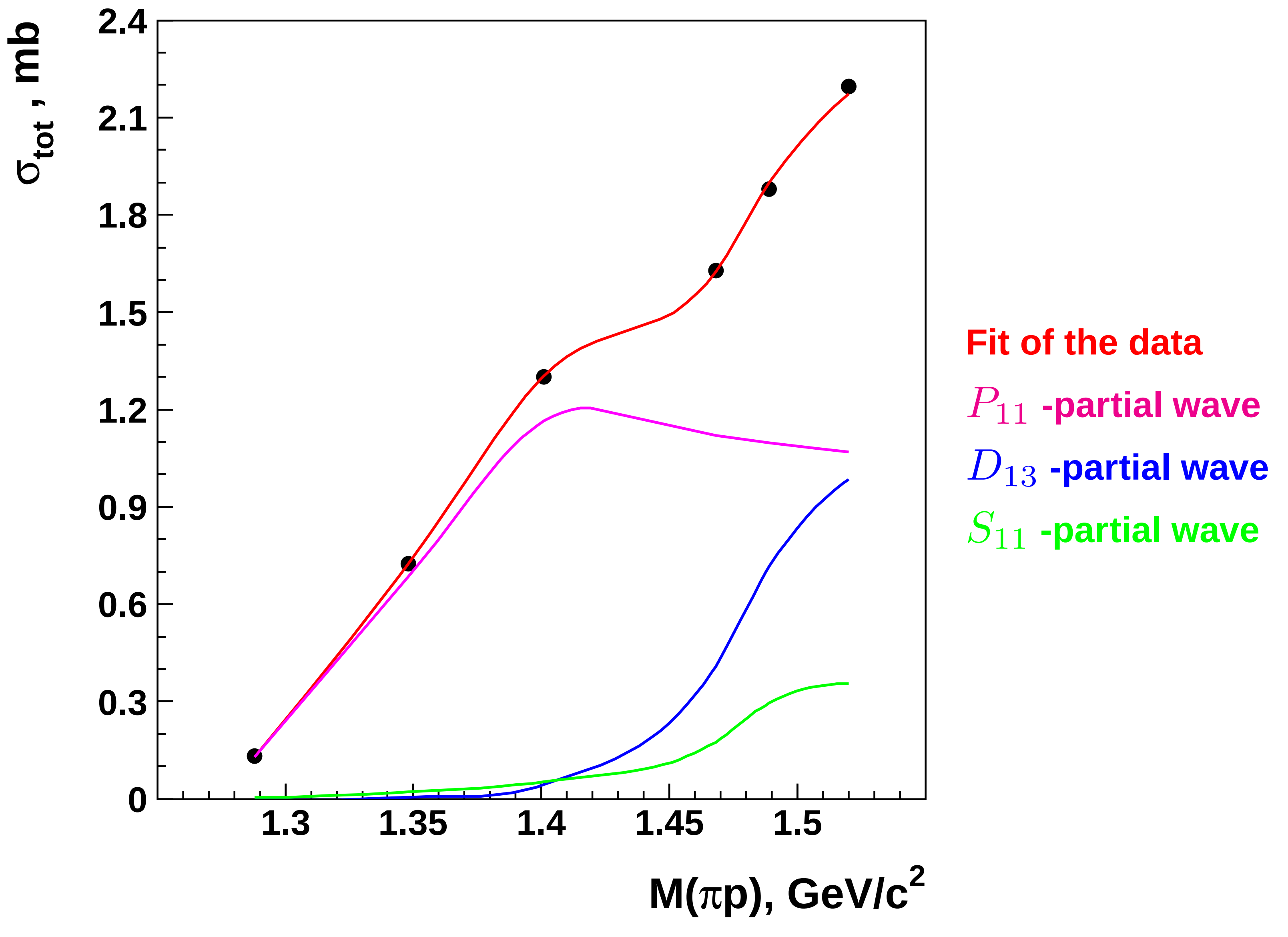


$N\pi \to N\pi$, S_{11} **wave** (2 pole 5 channel K-matrix)



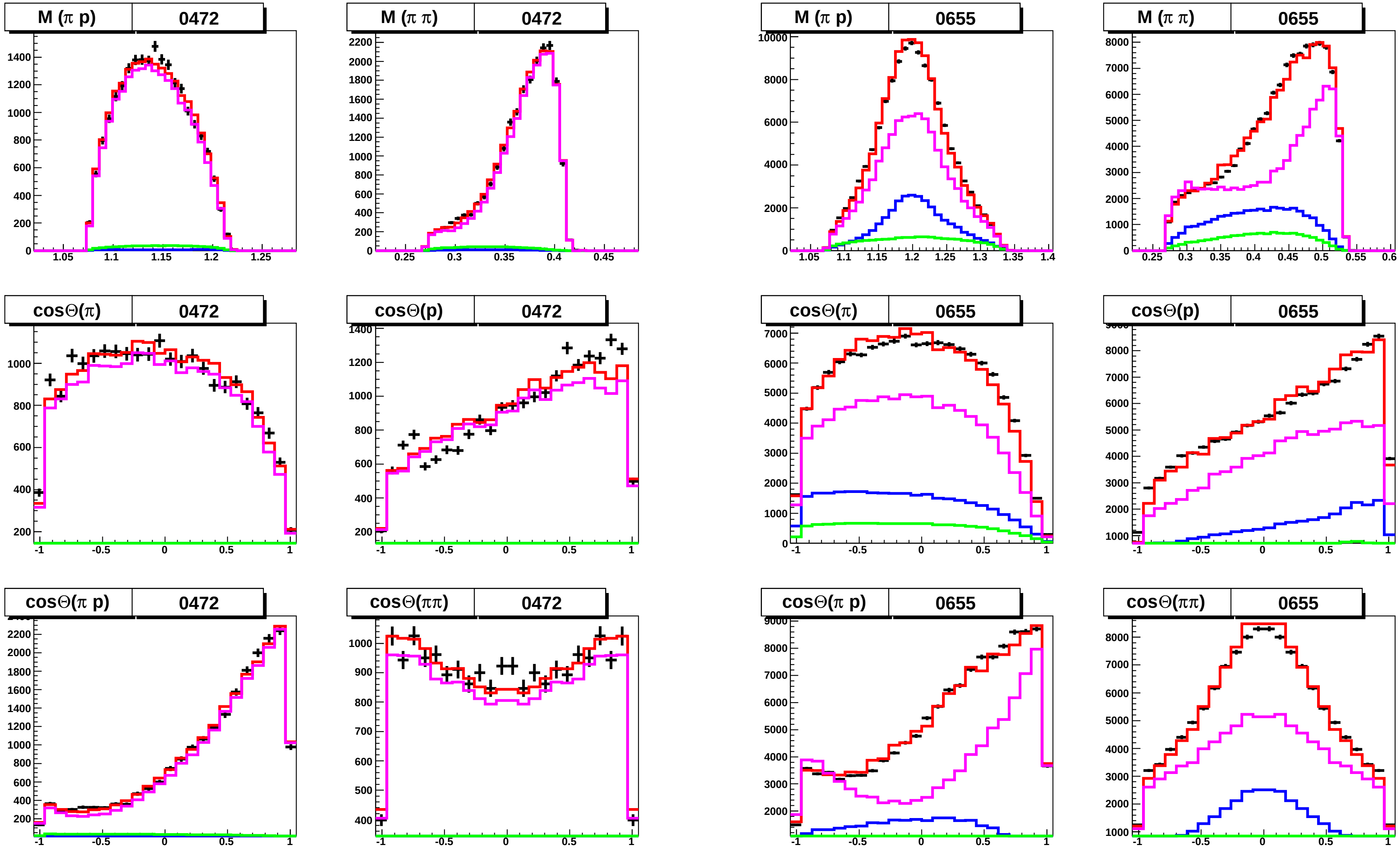
T-matrix poles: $M = 1508^{+10}_{-30}$ **MeV,** $2\ Im = 165 \pm 15$ **MeV;**
$M = 1645 \pm 15$ **MeV,** $2\ Im = 187 \pm 20$ **MeV**

$\pi^- p \to n\pi^0\pi^0$ (Crystal Ball) total cross section

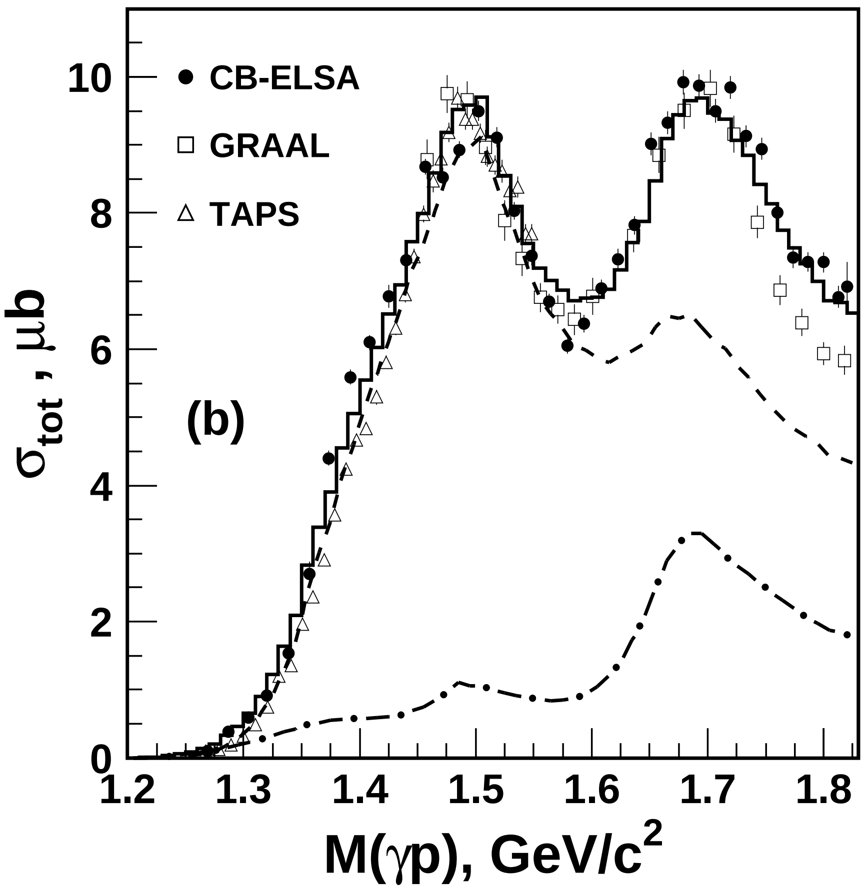


$\pi^- p \rightarrow n\pi^0\pi^0$ (Crystal Ball)

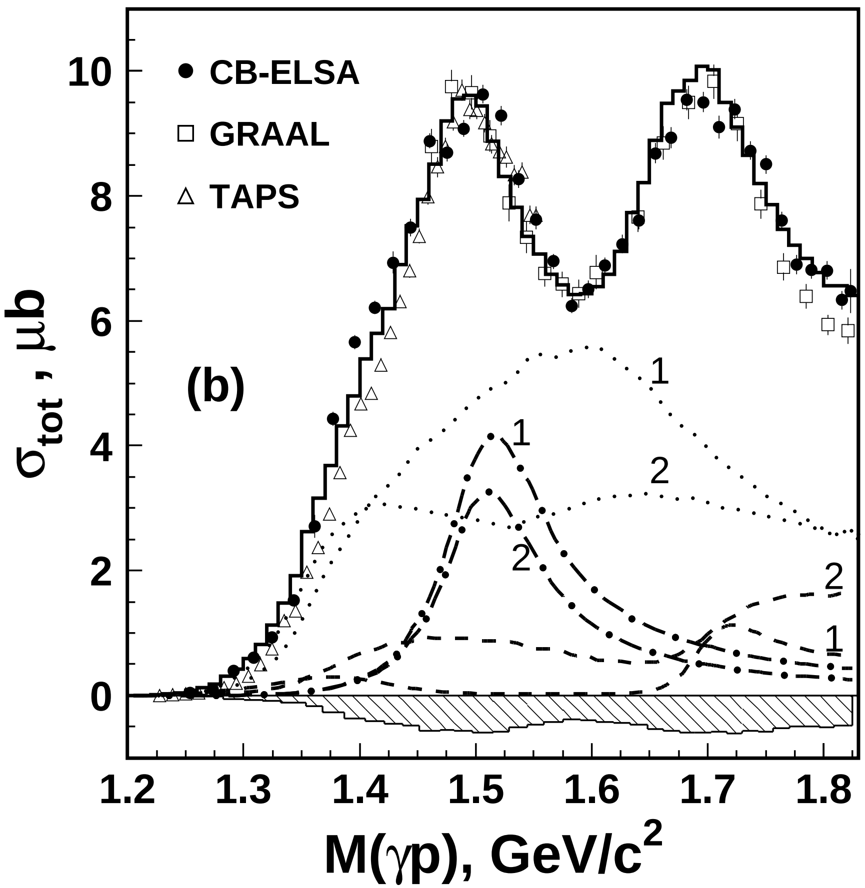
Differential cross sections for 472 and 665 MeV/c data.



$\gamma p \to p\pi^0\pi^0$ (CB-ELSA) M.Fuchs et al.



PWA corrected cross section and contributions from $\Delta(1232)\pi$ (dashed) and $N\sigma$ (dashed-dotted) final states.



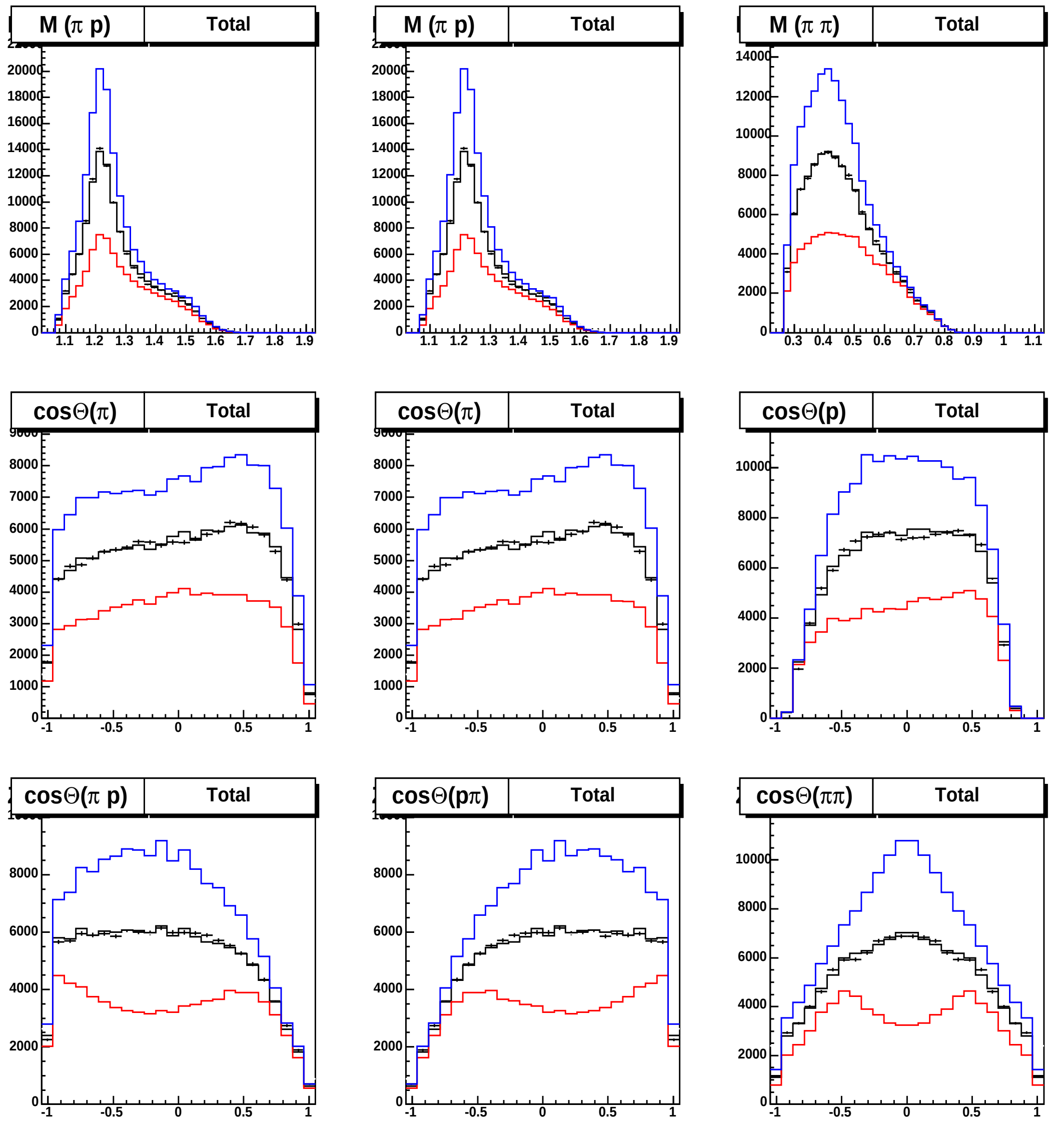
Contributions from D_{33} (dotted), P_{11} (dashed) and D_{13} (dashed-dotted) partial waves.

The $\gamma p \to \pi^0\pi^0 p$ differential cross section for the total energy region.

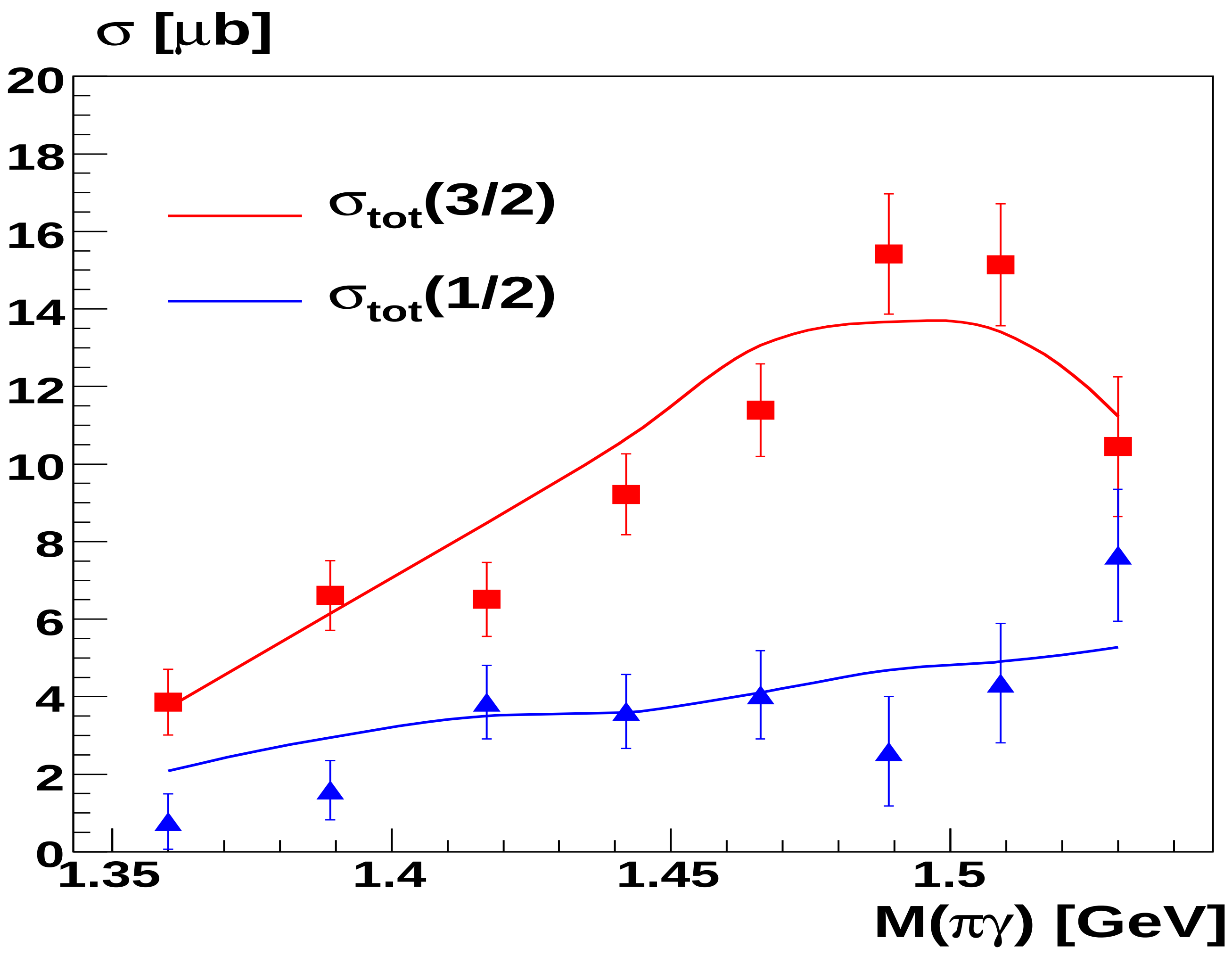
The fit of the unpolarized data and prediction for the double polarization measurements.

Red curve: only helicity 1/2 amplitudes contributed to the cross section.

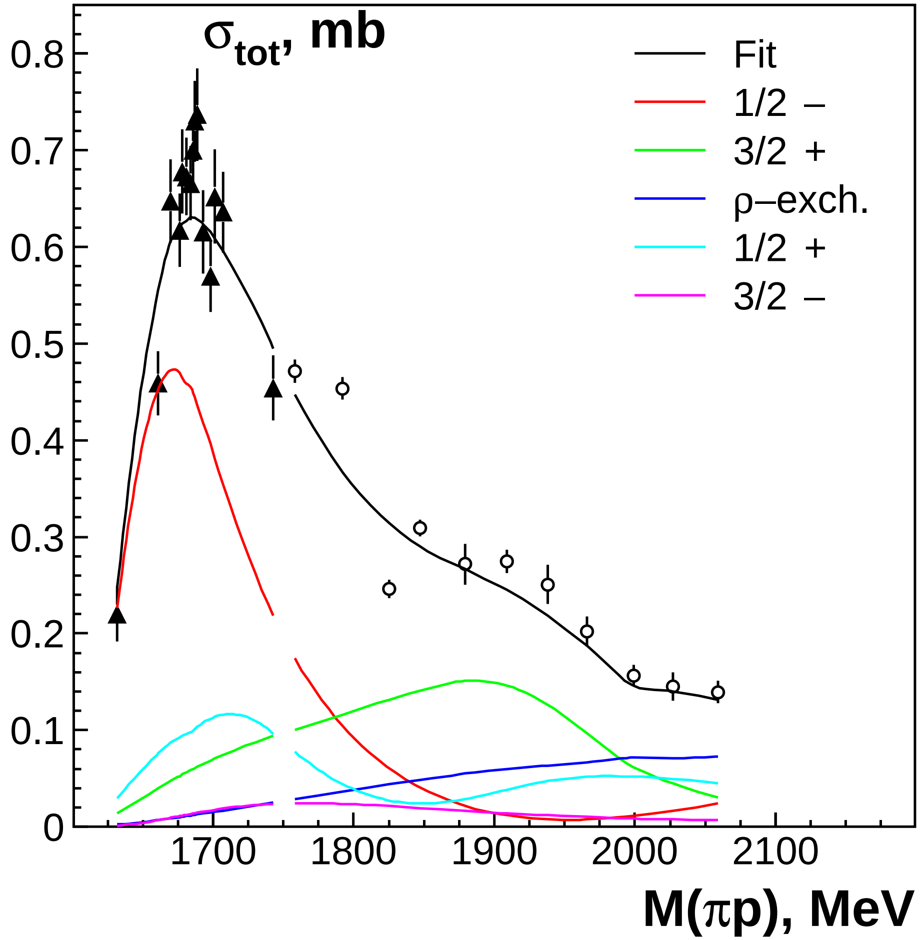
Blue curve: only helicity 3/2 amplitudes.



The $\gamma p \to \pi^0\pi^0 p$ helicity $3/2$ and $1/2$ differential cross sections



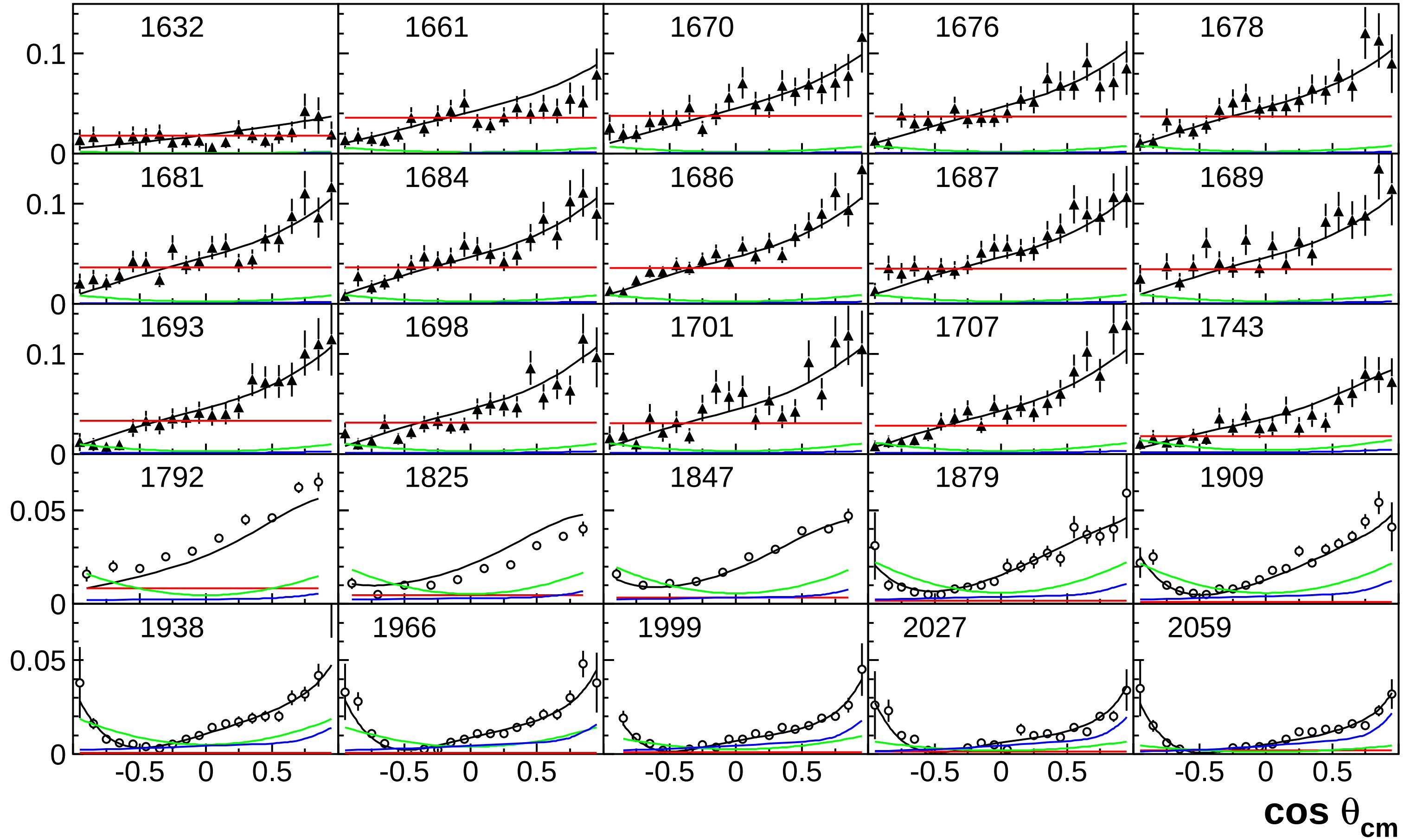
The total cross section for the $\pi^- p \rightarrow K\Lambda$ reaction also shows a clear contribution from this state:



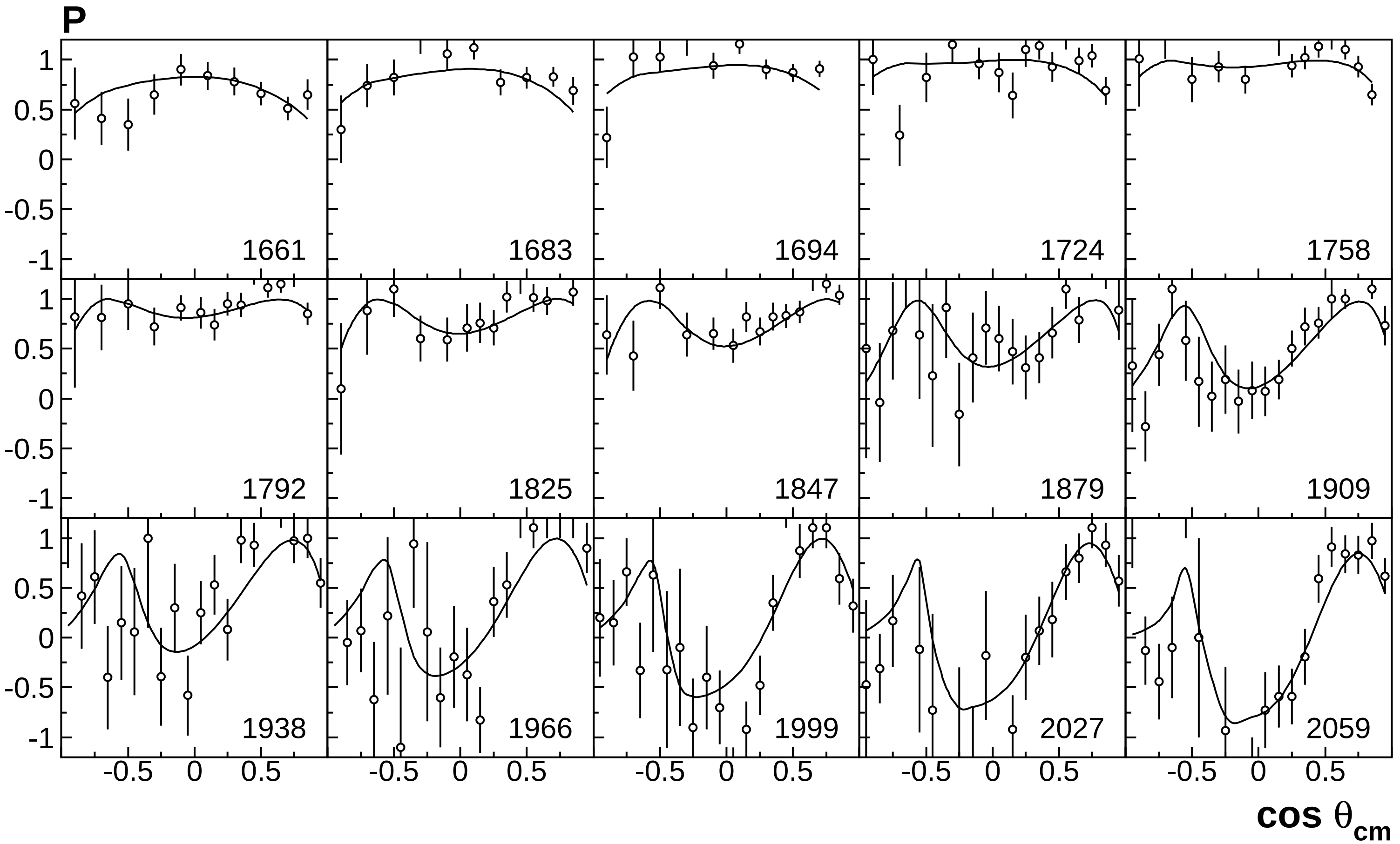
The same result was obtained before by the Giessen group:

V. Shklyar,H. Lenske, and U. Mosel, May 2005.

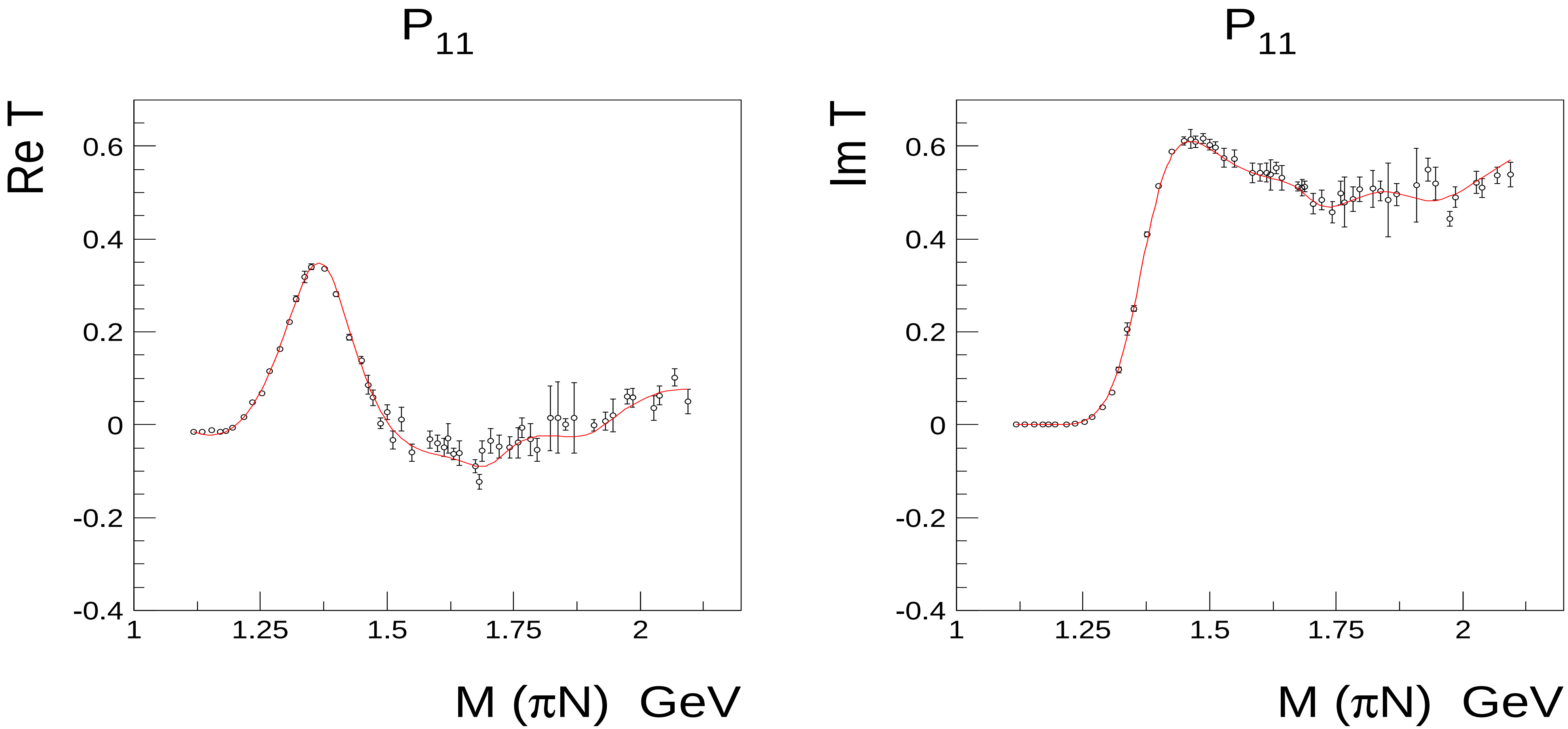
The differential cross section for the $\pi^- p \to K\Lambda$ reaction. also shows a clear contribution from this state:



The recoil asymmetry for the $\pi^- p \to K\Lambda$ reaction. also shows a clear contribution from this state:



$N\pi \to N\pi$ P_{11} wave (3 pole 4 channel K-matrix)

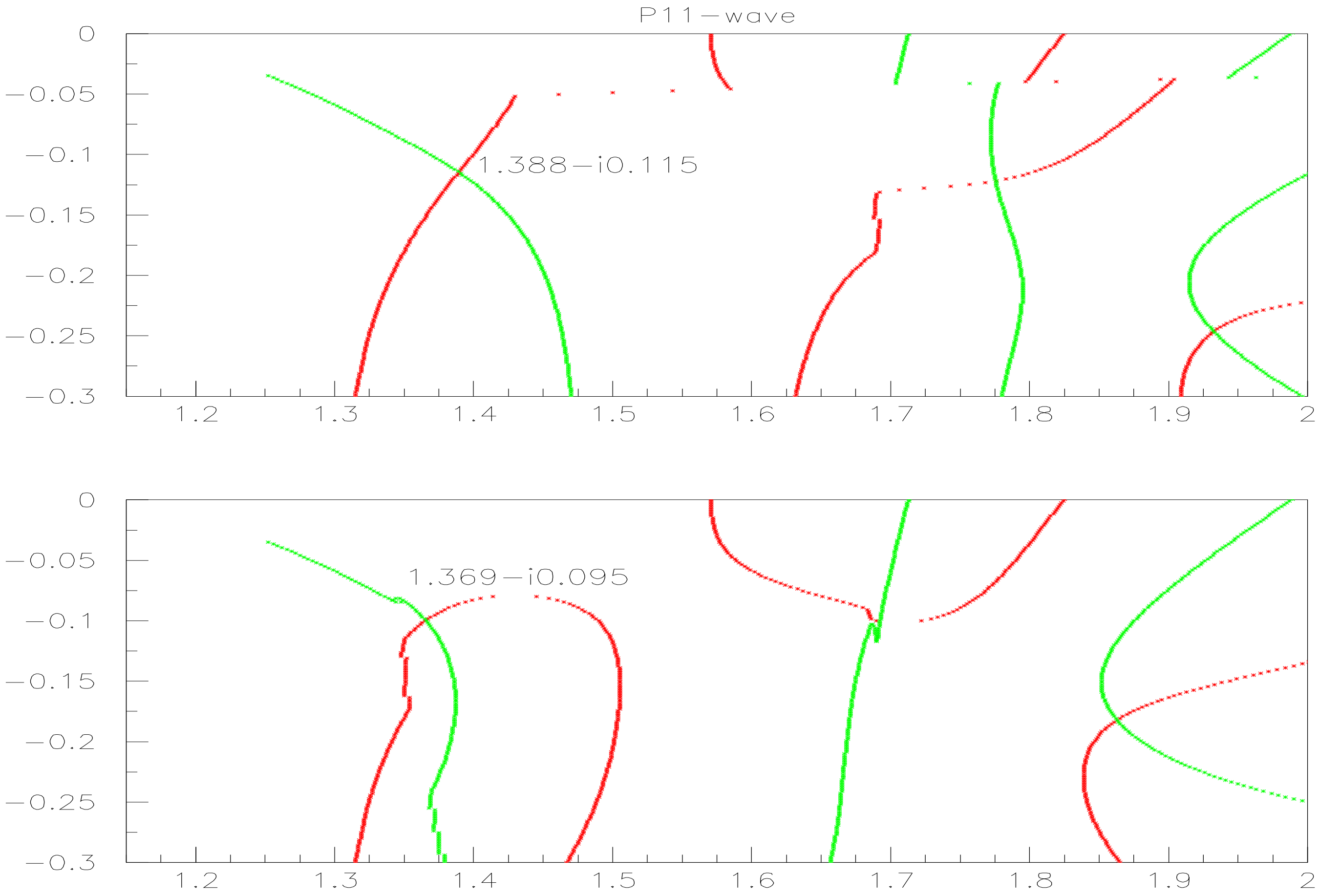


T-matrix poles: $M = 1368 \pm 7$ **MeV,** $2\ Im = 190 \pm 10$ **MeV;**

$M = 1685 \pm 20$ **MeV,** $2\ Im = 160 \pm 45$ **MeV**

$M = 1870 \pm 30$ **MeV,** $2\ Im = 280 \pm 80$ **MeV**

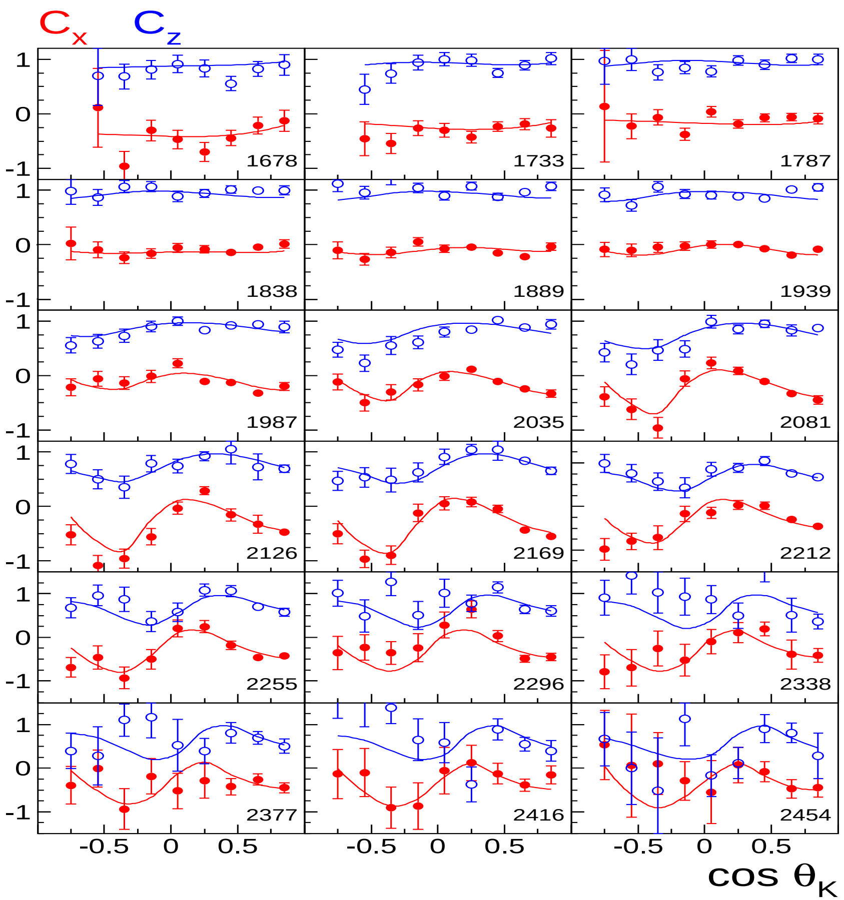
Position of zeros, for the determinant $(I - i\rho K)^{-1}$. Red points - real part and Green points -imaginary part.

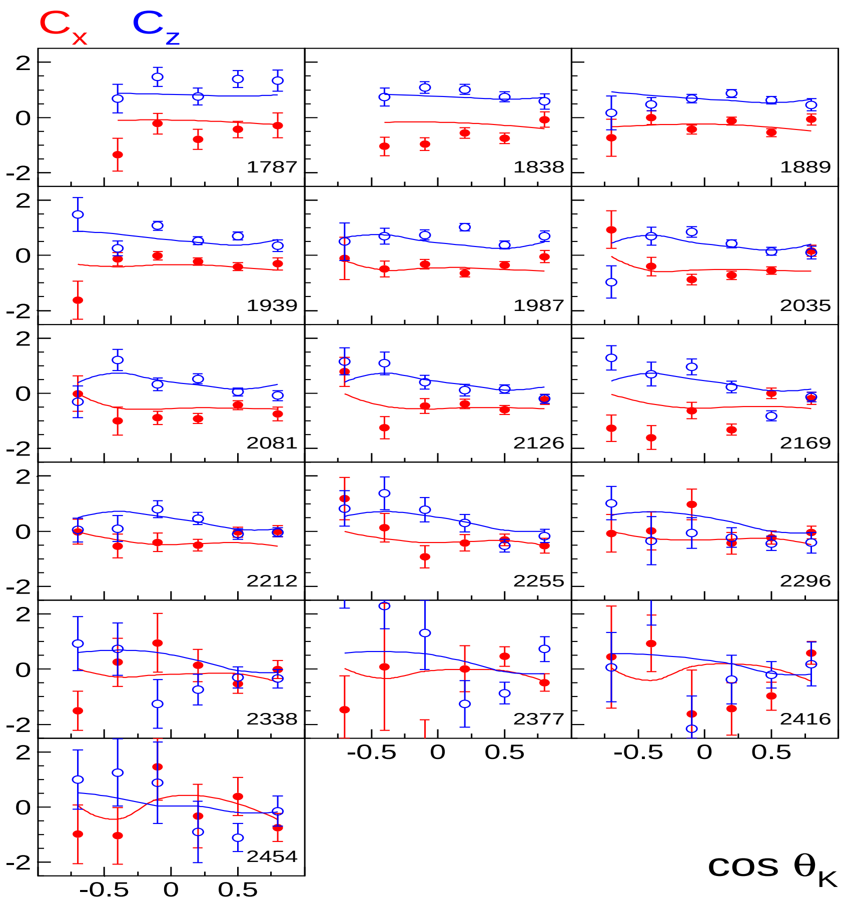


Properties of $N(1440)P_{11}$. The left column lists mass, width, partial widths of the Breit-Wigner resonance; the right column pole position and squared couplings to the final state at the pole position.

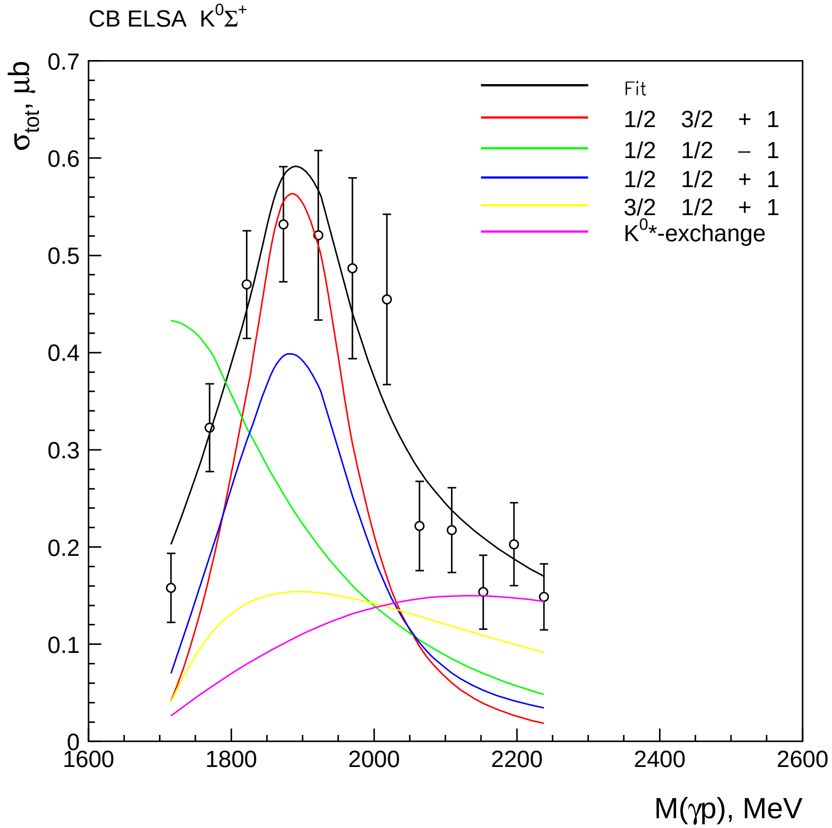
M	**=**	1436 ± 15 **MeV**	**M**$_{\rm pole}$	**=**	1371 ± 7 **MeV**
Γ	**=**	335 ± 40 **MeV**	$\Gamma_{\rm pole}$	**=**	192 ± 20 **MeV**
$\Gamma_{\pi N}$	**=**	205 ± 25 **MeV**	$g_{\pi N}$	**=**	$(0.51 \pm 0.05) \cdot e^{-i\pi\frac{(35\pm5)}{180}}$
$\Gamma_{\sigma N}$	**=**	71 ± 17 **MeV**	$g_{\sigma N}$	**=**	$(0.82 \pm 0.16) \cdot e^{-i\pi\frac{(20\pm13)}{180}}$
$\Gamma_{\pi\Delta}$	**=**	59 ± 15 **MeV**	$g_{\pi\Delta}$	**=**	$(-0.57 \pm 0.08) \cdot e^{i\pi\frac{(25\pm20)}{180}}$
T-matrix: $A_{1/2} = 0.055 \pm 0.020$ **GeV**			$\phi = (70 \pm 30)^\circ$		

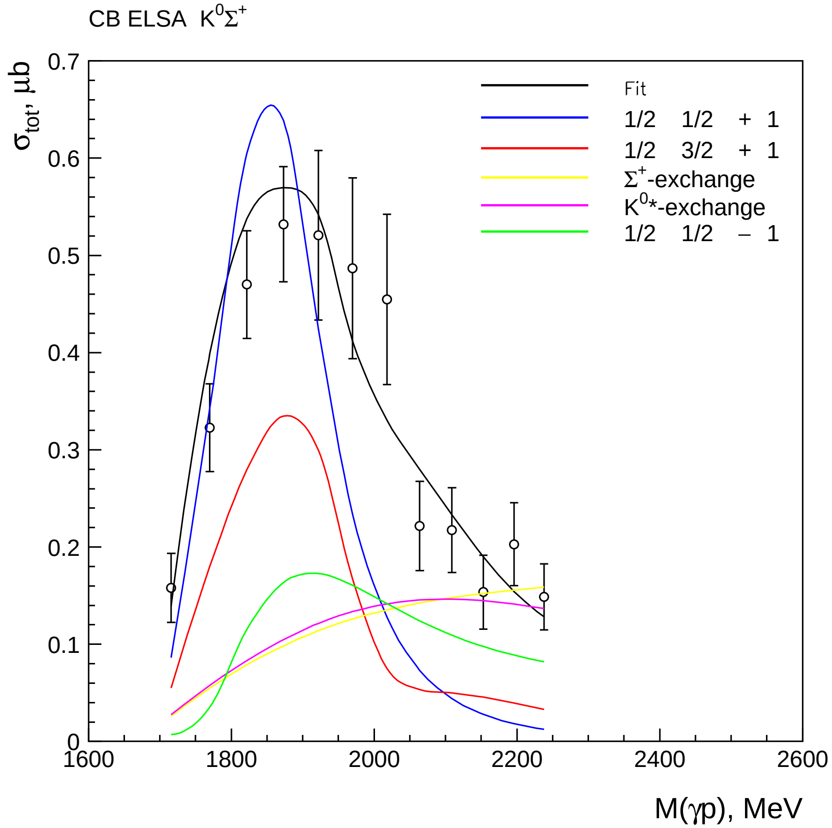
For $\gamma p \to K\Lambda$ and $\gamma p \to K\Sigma$ we have almost complete photoproduction experiment:
σ (CLAS, SAPHIR), Σ (GRAAL, LEP), P (CLAS), C_x, C_z (CLAS), T, O_x, O_z (GRAAL).
The C_x and C_z data can be explained with $P_{13}(1900)$.





$\sigma_{tot}(\gamma p \to K^0\Sigma^+)$ from CB-ELSA

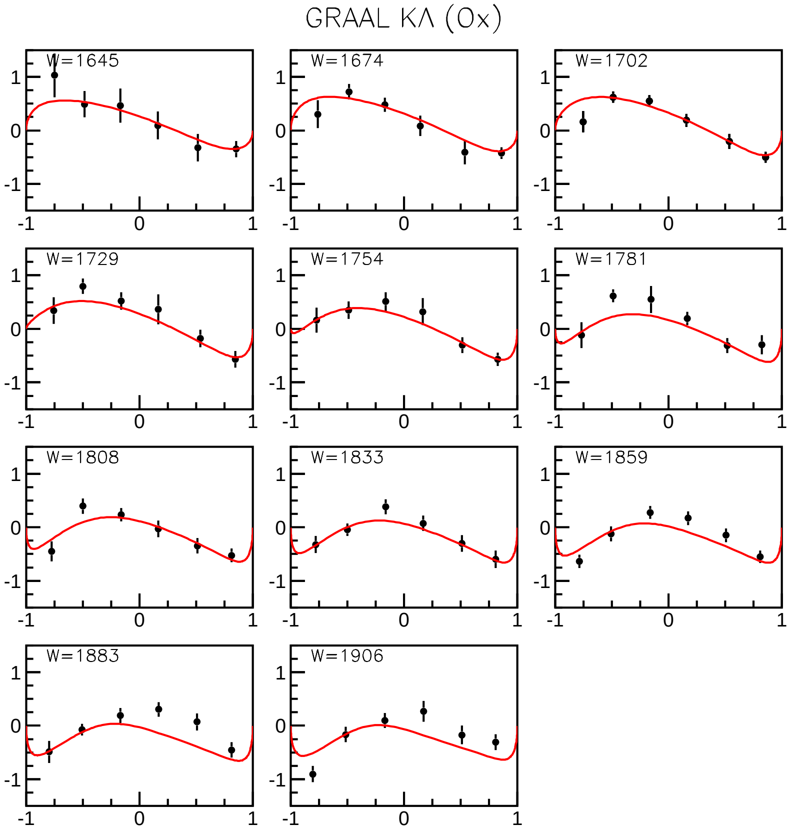


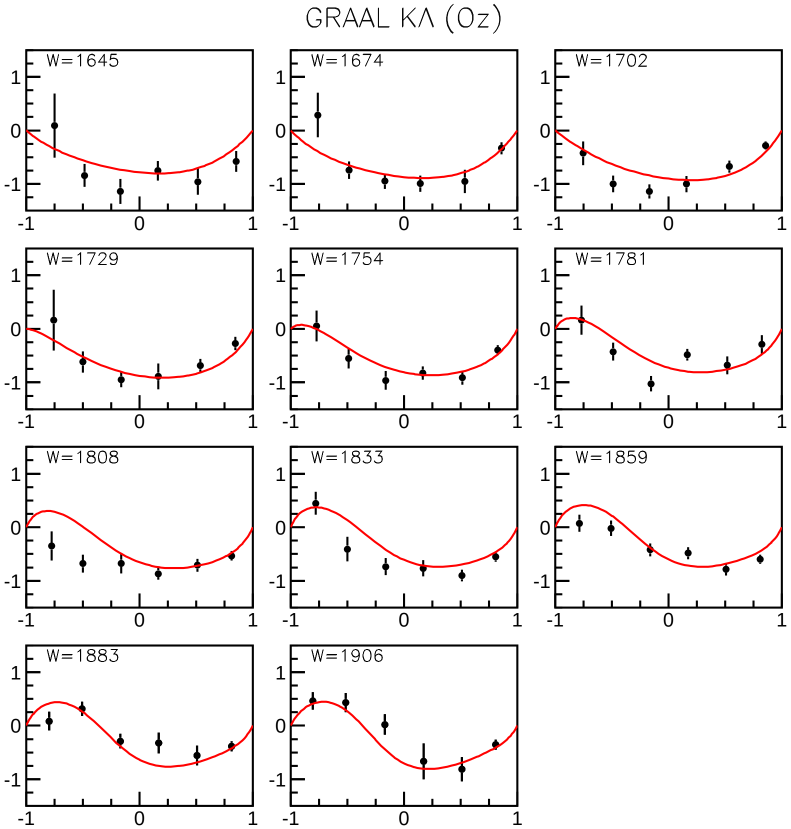


Red line – $P_{13}(1900)$

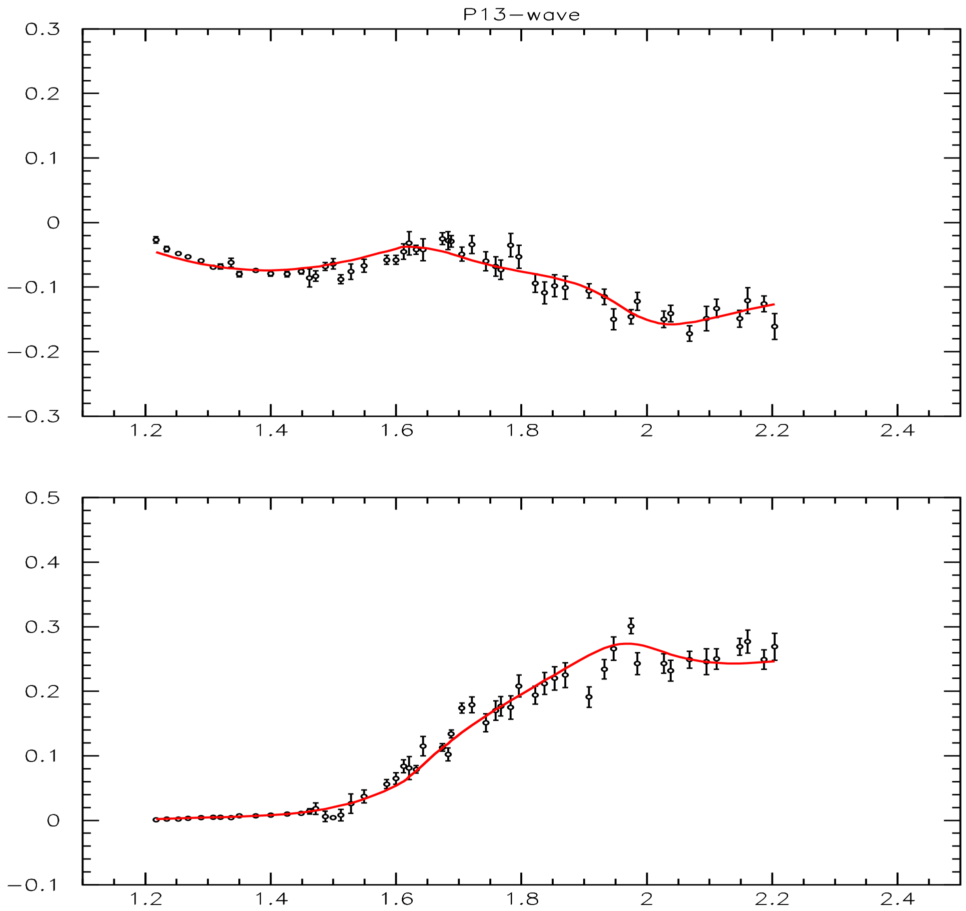
Blue line – $P_{11}(1860)$ **(improved** P **in** $K\Lambda$ **and** $K\Sigma$ **data)**

The solution is supported by the new GRALL data on O_x O_z and T-observables: an important step to a complete experiment.



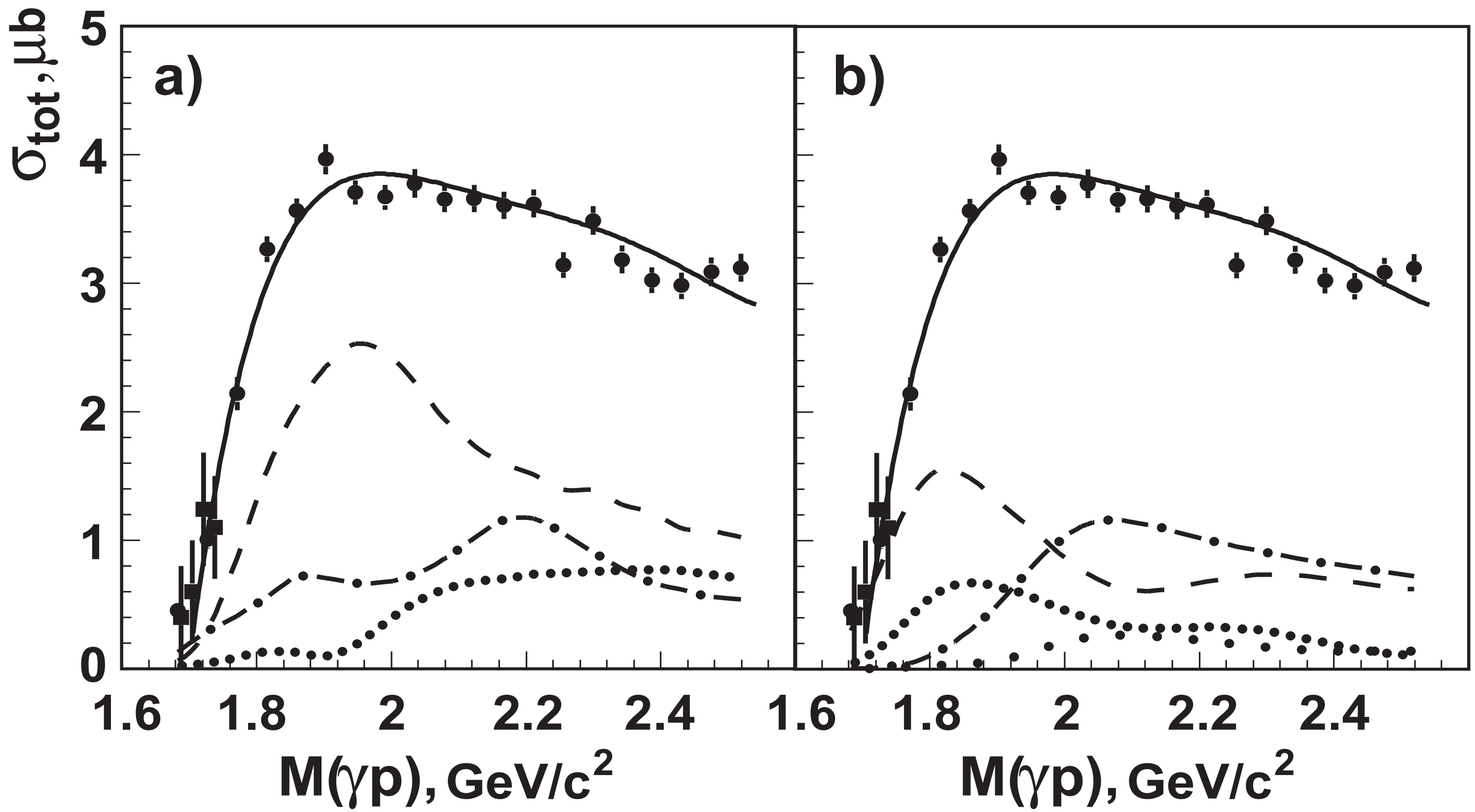


$\mathrm{N}\pi \rightarrow \mathrm{N}\pi$, P_{13} wave (3 pole 8 channel K-matrix)



2nd T-matrix poles: $M = 1960 \pm 20$ **MeV,** $2\ Im = 195 \pm 45$ **MeV;**

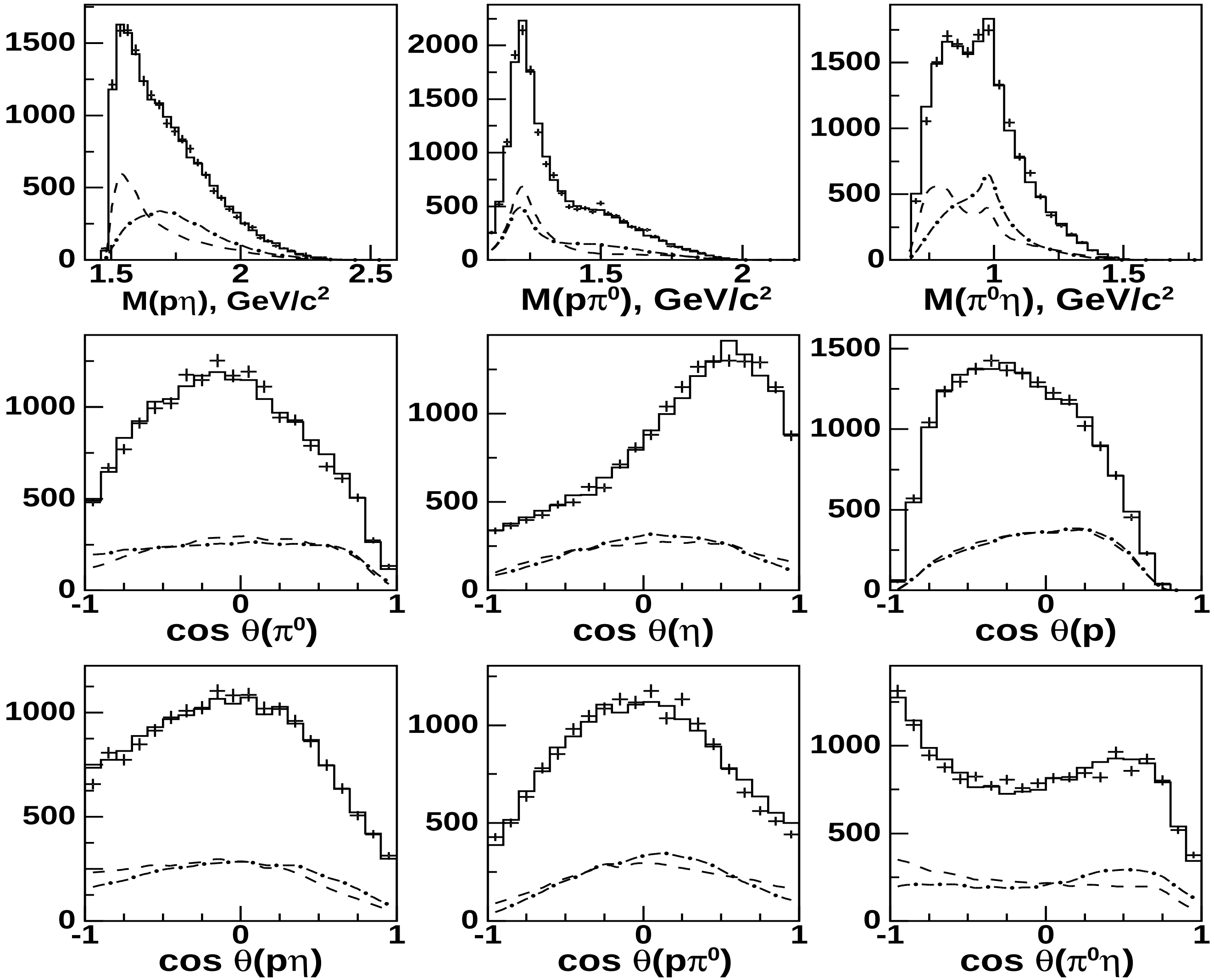
$\gamma p \rightarrow p\pi^0\eta$ (CB-ELSA)



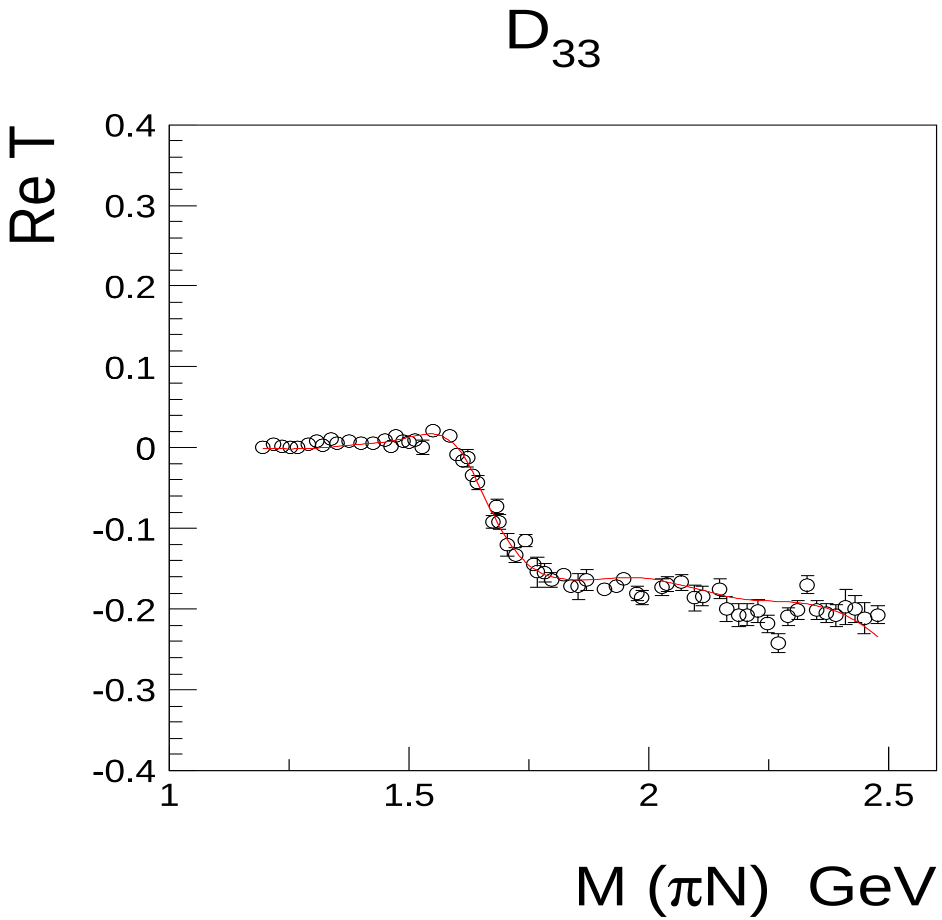
Left panel : contributions from $\Delta(1232)\eta$ (dashed), $S_{11}(1535)\pi$ (dashed-dotted) and $N\,a_0(980)$ final states.

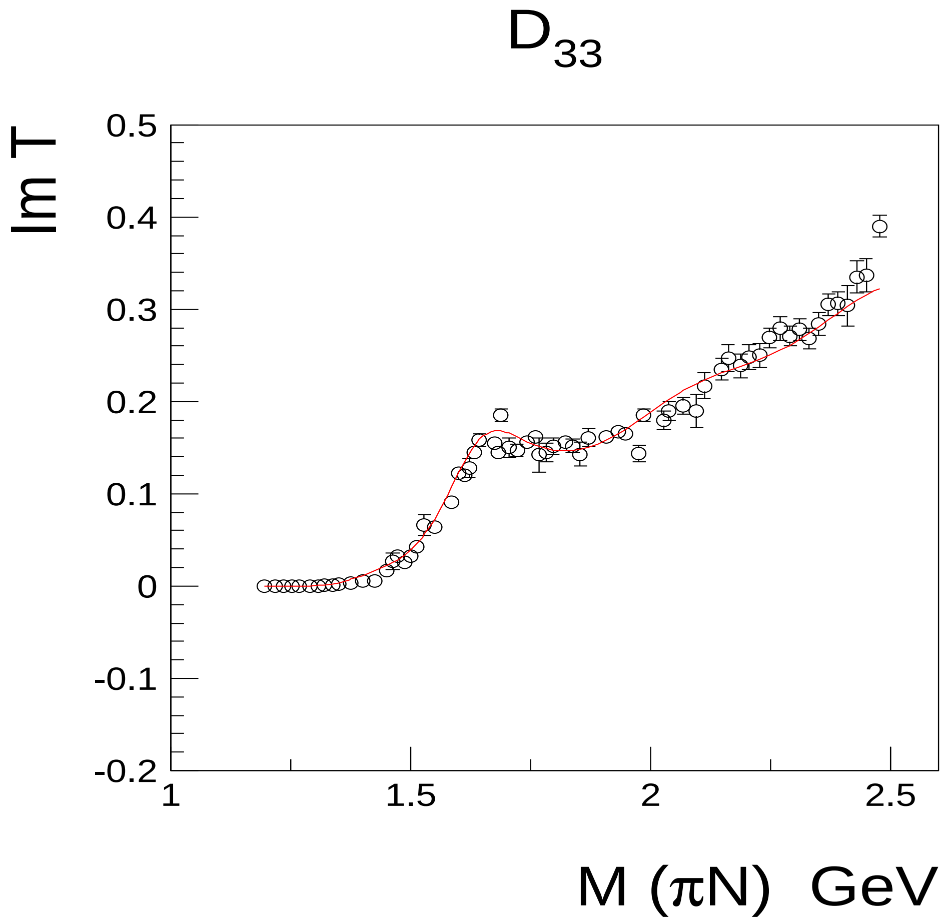
Right panel: D_{33} partial wave (dashed), P_{33} partial wave (dashed-dotted), $D_{33} \rightarrow \Delta(1232)\eta$ (dotted) and $D_{33} \rightarrow N\,a_0(980)$ (wide dotted).

The $\gamma p \to \pi^0 \eta p$ differential cross section for the total energy region.



$\mathrm{N}\pi \rightarrow \mathrm{N}\pi\ D_{33}$ wave (3 pole 5 channel K-matrix)





D_{33}**-wave:** πN, $\Delta(1232)\pi$ **(**S**- and** D**-waves)),** $\Delta(1232)\eta$, $S_{11}(1535)\pi$

Properties of the $\Delta(1920)P_{33}$ and $\Delta(1940)D_{33}$ resonances.

	M_{pole}	Γ_{pole}	M_{BW}	Γ_{tot}^{BW}
$\Delta(1920)P_{33}$	1940 ± 40	350^{+35}_{-55}	1970 ± 35	375 ± 50
$\Delta(1940)D_{33}$	1995 ± 30	420 ± 50	2000 ± 40	410 ± 70

	$\mathrm{Br}_{N\pi}$	$\mathrm{Br}_{\Delta\eta}$	$\mathrm{Br}_{N(1535)\pi}$	$\mathrm{Br}_{Na_0(980)}$
$\Delta(1920)P_{33}$	15 ± 8	18 ± 8	7 ± 4	4 ± 2
$\Delta(1940)D_{33}$	9 ± 4	5 ± 2	2 ± 1	2 ± 1

Parity doublets of N and Δ resonances at high mass region

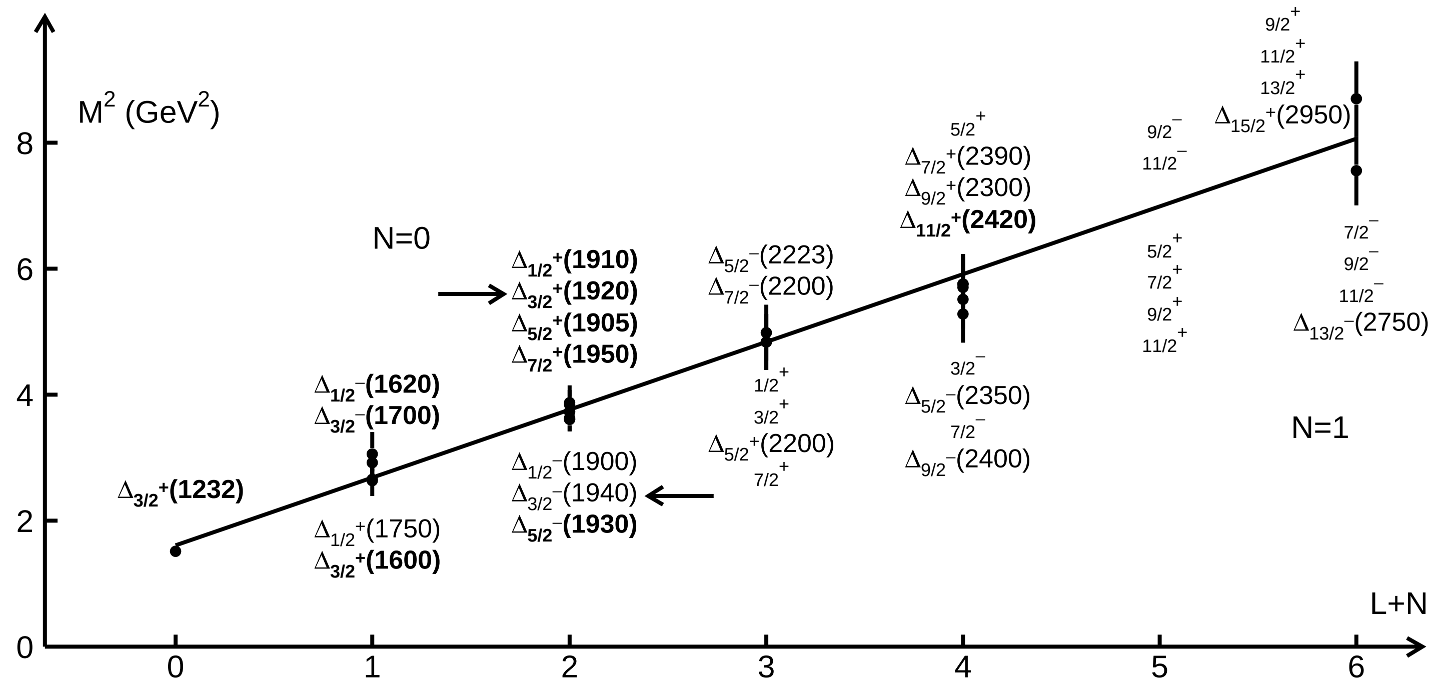
Glozman suggested a restoration of chiral symmetry in high-mass excitations. Parity doublets must not interact by pion emission and could have a small coupling to πN.

$J=\frac{1}{2}$	$N_{1/2+}(2100)^a$	*	$N_{1/2-}(2090)^a$	*	$\Delta_{1/2+}(1910)$	****	$\Delta_{1/2-}(1900)^a$	**
$J=\frac{3}{2}$	$N_{3/2+}(1900)^a$	**	$N_{3/2-}(2080)^a$	**	$\Delta_{3/2+}(1920)^a$	***	$\Delta_{3/2-}(1940)^a$	*
$J=\frac{5}{2}$	$N_{5/2+}(2000)^a$	**	$N_{5/2-}(2200)^a$	**	$\Delta_{5/2+}(1905)$	****	$\Delta_{5/2-}(1930)^a$	***
$J=\frac{7}{2}$	$N_{7/2+}(1990)^a$	**	$N_{7/2-}(2190)$	****	$\Delta_{7/2+}(1950)$	****	$\Delta_{7/2-}(2200)^a$	*
$J=\frac{9}{2}$	$N_{9/2+}(2220)$	****	$N_{9/2-}(2250)$	****	$\Delta_{9/2+}(2300)$	**	$\Delta_{9/2-}(2400)^a$	**

$J=\frac{3}{2}$	$N_{3/2+}(1900)$	$N_{3/2-}(1875)$	$\Delta_{3/2+}(1980)$	$\Delta_{3/2-}(1985)$
$J=\frac{5}{2}$	$N_{5/2+}(1960)$	$N_{5/2-}(2070)$	$\Delta_{5/2+}(1945)$	$\Delta_{5/2-}(1930)$
$J=\frac{7}{2}$	$N_{7/2+}(1990)$	$N_{7/2-}(????)$	$\Delta_{7/2+}(1910)$	$\Delta_{7/2-}(????)$

Holographic QCD (AdS/QCD)

Soft-wall model prediction: $M^2_{N,L} = 4\lambda^2 \left(N + L + \frac{3}{2}\right)$



$$M^2_{N,L} = 4\lambda^2 \left(N + L + \frac{3}{2}\right) - 2\left(M^2_\Delta - M^2_N\right)\kappa_{gd}$$

κ_{gd} **is the fraction of most attractive color-antitriplet isosinglet diquark.**

κ_{gd}**=0 for Δ and N(S=3/2) states,** $\frac{1}{2}$ **for** $S = 1/2\ (70SU_6)$ **and** $\frac{1}{4}$ **for** $S = 1/2\ (56SU_6)$**.**

Hilmar Forkel and Eberhard Klempt, hep-ph:0810.2959v1

L, S, N	κ_{gd}	Resonance					Pred.
0, $\frac{1}{2}$ **,0**	$\frac{1}{2}$	$N(940)$				**input:**	**0.94**
0, $\frac{3}{2}$ **,0**	**0**	$\Delta(1232)$					**1.27**
0, $\frac{1}{2}$ **,1**	$\frac{1}{2}$	$N(1440)$					**1.40**
1, $\frac{1}{2}$ **,0**	$\frac{1}{4}$	$N(1535)$	$N(1520)$				**1.53**
1, $\frac{3}{2}$ **,0**	**0**	$N(1650)$	$N(1700)$	$N(1675)$			**1.64**
1, $\frac{1}{2}$ **,0**	**0**	$\Delta(1620)$	$\Delta(1700)$		L, S, N**=0,** $\frac{3}{2}$ **,1:**	$\Delta(1600)$	**1.64**
2, $\frac{1}{2}$ **,0**	$\frac{1}{2}$	$N(1720)$	$N(1680)$		L, S, N**=0,** $\frac{1}{2}$ **,2:**	$N(1710)$	**1.72**
1, $\frac{1}{2}$ **,1**	$\frac{1}{4}$	$N(????)$	$N(1875)$				**1.82**
1, $\frac{3}{2}$ **,1**	**0**	$\Delta(1900)$	$\Delta(1940)$	$\Delta(1930)$			**1.92**
2, $\frac{3}{2}$ **,0**	**0**	$\Delta(1910)$	$\Delta(1920)$	$\Delta(1905)$	$\Delta(1950)$		**1.92**
2, $\frac{3}{2}$ **,0**	**0**	$N(1880)$	$N(1900)$	$N(1990)$	$N(2000)$		**1.92**
0, $\frac{1}{2}$ **,3**	$\frac{1}{2}$	$N(2100)$					**2.03**
3, $\frac{1}{2}$ **,0**	$\frac{1}{4}$	$N(2070)$	$N(2190)$	L, S, N**=1,** $\frac{1}{2}$ **,2:**	$N(2080)$	$N(2090)$	**2.12**
3, $\frac{3}{2}$ **,0**	**0**	$N(2200)$	$N(2250)$	L, S, N**=1,** $\frac{1}{2}$ **,2:**	$\Delta(2223)$	$\Delta(2200)$	**2.20**
4, $\frac{1}{2}$ **,0**	$\frac{1}{2}$	$N(2220)$					**2.27**
4, $\frac{3}{2}$ **,0**	**0**	$\Delta(2390)$	$\Delta(2300)$	$\Delta(2420)$	L, N**=3,1:**	$\Delta(2400)$ $\Delta(2350)$	**2.43**
5, $\frac{1}{2}$ **,0**	$\frac{1}{4}$	$N(2600)$					**2.57**

Summary

1. **An approach for the combined analysis of the pion and photo induced reaction with two and multi particle final states is developed.**

2. **The combined analysis of more them 65 different reactions helped to identify the properties of known baryons.**

3. **The new data support the two new baryon states observed in hyperon photoproduction $P_{11}(1880)$ and $P_{13}(1900)$.**

4. **The η-photoproduction data reveal the baryon resonance $D_{15}(2070)$.**

5. **The $D_{33}(1940)$ state is needed for the description of the $\gamma p \to \pi^0 \eta p$ data.**

6. **The data on $\pi^- p \to \eta n$ and $\pi^- p \to K^0 \Sigma$ support an existence of $P_{11}(1710)$.**

7. **The spectrum of observed states is in direct contradiction with a classical quark model. The best explanations are chiral symmetry restoration or AdS/QCD soft-wall model.**