Bonn-Gatchina partial wave analysis

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The main objective: the search for new baryon states.

1. The P-vector based analysis of single and multi-meson photoproduction data.
   The combined analysis of CB-ELSA data on $\gamma p \rightarrow \pi^0 p$, $\gamma p \rightarrow \eta p$, $\gamma p \rightarrow \pi^0 \pi^0 p$, $\gamma p \rightarrow \pi^0 \eta p$, $\gamma n \rightarrow \eta n$, $\gamma n \rightarrow \pi^0 n$ and $\gamma p \rightarrow \omega p$ with data from other collaborations: CLAS, GRAAL, LEPS.

2. The K-matrix analysis of pion-induced data on the reactions $\pi N \rightarrow \pi N$ (on the basis of SAID energy-fixed partial wave analysis), $\pi p \rightarrow \pi^0 \pi^0 n$ and recently included: $\pi p \rightarrow \eta n$, $\pi^- p \rightarrow K^0 \Lambda$ and $\pi^+ p \rightarrow K^+ \Sigma$.

3. Analysis of $NN$ interaction. The analysis of $pp \rightarrow K \Lambda p$ data and analysis of the $pp \rightarrow \pi^0 p$ and $np \rightarrow pp \pi^-$ data.
For combined analysis of a larger data set a new approach is needed:

1. Fully relativistically invariant.

2. Convenient for combined analysis of single and multi-meson photoproduction.

3. Energy dependent, which allow us to apply directly the unitarity and analyticity conditions.

4. Convenient for calculation of the triangle and box diagrams or projection of the t and u-channel exchange amplitudes to the partial waves in s-channel.


1 Bosons

1.1 Propagator – Green function

Let us introduce a function which is a solution of the following equation ($t = x_0$):

$$(\frac{\partial^2}{\partial x_\mu \partial x_\mu} + m^2)G(x - x') = \delta^4(x - x')$$

Properties of the Green function:

If the Green function is a solution of the equation:

$$R(x)G(x - x') = \delta^4(x - x')$$

where $R(x)$ is some relativistically invariant operator, then the solution of the equation:

$$R(x)\Psi(x) = V(x)\Psi(x)$$

is:

$$\Psi(x) = \int G(x - x')V(x')d^4x'$$
In the case of equation for a free particle the Green function is called a propagator and is equal to:

\[ G(x - x') = \langle 0 | T \Psi(x) \Psi^*(x') | 0 \rangle \]

If wave function is a spinor or tensor with rank \( n \) in Lorentz space then:

\[ G^{\mu_1 \mu_2 \cdots \mu_n}_{\nu_1 \nu_2 \cdots \nu_n}(x - x') = \langle 0 | T \Psi_{\mu_1 \mu_2 \cdots \mu_n}(x) \Psi_{\nu_1 \nu_2 \cdots \nu_n}^*(x') | 0 \rangle \]

And equation is:

\[ R(x) G^{\mu_1 \mu_2 \cdots \mu_n}_{\nu_1 \nu_2 \cdots \nu_n}(x - x') = \delta^4(x - x') \prod_{i=1}^{n} \delta_{\mu_i \nu_i} \]

For spinless particle in momentum representation we have:

\[ (p^2 - m^2) G = 1 \quad G = \frac{1}{p^2 - m^2} \]
Particles with spin 1

A particle with spin 1 is described by a 4-vector $\Psi_\mu$ and obeys the K-G equation:

$$\left(\frac{\partial^2}{\partial x_\mu \partial x_\mu} + m^2\right)\Psi_\mu(x) = 0$$

1) In c.m. system such wave function has only three components. It means that in any system only three components are independent.

2) A free propagating particle with fixed spin can not became a particle with another quantum numbers without interaction.

If $\Psi_\mu(x)$ is a wave function of particle with spin 1 and $\Psi(x)$ is a wave function of a particle with spin 0:

$$\int \Psi_\mu(x)\Psi^*(x)d^4x = \alpha p_\mu$$

Where $\alpha$ - is a scalar function of $p^2$. If such integral is equal to 0 for any wave functions and any $p_\mu$ it means that $\Psi_\mu$ has no projection on this vector and:

$$p_\mu \Psi_\mu = 0$$
In momentum representation:

\[ \Psi_\mu = \frac{1}{\sqrt{2\varepsilon}} u_\mu e^{ipx} \]

and for wave function \( u_\mu \) we obtain:

\[ p_\mu u_\mu = 0 \]

A spin 2 particle in c.m. system has 5 independent components and described by symmetrical tensor \( \Psi_{\mu\nu} \). For the transition between spin 2 and spin 0 state we have:

\[ \int \Psi_{\mu\nu}(x)\Psi^*(x)d^4x = \beta \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \]

Then an additional condition is:

\[ g_{\mu\nu} \Psi_{\mu\nu} = 0 \quad g_{\mu\nu} u_{\mu\nu} = 0 \]
Such procedure can be repeated for any spin, providing that the tensor have only $2s + 1$ independent components. Thus for any integer spin:

$$p^2 u_{\mu_1 \mu_2 \ldots \mu_n} = m^2 u_{\mu_1 \mu_2 \ldots \mu_n}$$

$$p_{\mu_i} u_{\mu_1 \mu_2 \ldots \mu_n} = 0$$

$$g_{\mu_i \mu_j} u_{\mu_1 \mu_2 \ldots \mu_n} = 0$$

$$u_{\mu_1 \ldots \mu_i \ldots \mu_j \ldots \mu_n} = u_{\mu_1 \ldots \mu_j \ldots \mu_i \ldots \mu_n}$$

These conditions are the main basis for the construction of the projection operators, which are defined as:

$$G_{\nu_1 \nu_2 \ldots \nu_n}^{\mu_1 \mu_2 \ldots \mu_n} = (-1)^n O_{\nu_1 \nu_2 \ldots \nu_n}^{\mu_1 \mu_2 \ldots \mu_n} \frac{1}{p^2 - m^2}$$
2  Boson projection operators

In momentum representation:

\[ O_{\nu_1 \nu_2 \ldots \nu_n}^{\mu_1 \mu_2 \ldots \mu_n} = (-1)^n \sum_{i=1}^{2n+1} u^{(i)}_{\mu_1 \mu_2 \ldots \mu_n} u^{(i)*}_{\nu_1 \nu_2 \ldots \nu_n} \]

The projection operator can depend only on the total momentum and the metrical tensor. For spin 0 it is a unit operator. For spin 1 the only possible combination is:

\[ O^\mu_\nu = g^\perp_{\mu \nu} = g_{\mu \nu} - \frac{P^\mu P^\nu}{P^2} \]

The propagator for the particle with spin \( S > 2 \) must be constructed from the tensors \( g^\perp_{\mu \nu} \): this is the only combination which satisfies:

\[ P^\mu g^\perp_{\mu \nu} = 0. \]

Then for spin 2 state we obtain:

\[ O^{\mu_1 \mu_2}_{\nu_1 \nu_2} = \frac{1}{2} (g^\perp_{\mu_1 \nu_1} g^\perp_{\mu_2 \nu_2} + g^\perp_{\mu_1 \nu_2} g^\perp_{\mu_2 \nu_1} \frac{1}{3} g^\perp_{\mu_1 \mu_2} g^\perp_{\nu_1 \nu_2} \]
There is a recurrent expression for the boson projector operator:

\[
O_{\nu_1...\nu_L}^{\mu_1...\mu_L} = \frac{1}{L^2} \left( \sum_{i,j=1}^{L} g_{\mu_i \nu_j} O_{\nu_1...\nu_{j-1} \nu_{j+1}...\nu_L}^{\mu_1...\mu_{i-1} \mu_{i+1}...\mu_L} - \right)
\]

\[
\frac{4}{(2L-1)(2L-3)} \sum_{i<j,k<m} g_{\mu_i \mu_j} g_{\nu_k \nu_m} O_{\nu_1...\nu_{k-1} \nu_{k+1}...\nu_{m-1} \nu_{m+1}...\nu_L}^{\mu_1...\mu_{i-1} \mu_{i+1}...\mu_{j-1} \mu_{j+1}...\mu_L}
\]
Orbital momentum operator

Let us consider decay of a resonance into two scalar (pseudoscalar) particles. The angular momentum operator can be constructed only from momenta of particles $k_1, k_2$ and metric tensor $g_{\mu\nu}$.

The wave function of a scalar resonance is a const: $X^0 = 1$

The wave function of vector particle is a vector $X^{(1)}_\mu$, constructed from: $k_\mu = \frac{1}{2}(k_{1\mu} - k_{2\mu})$ and $P_\mu = (k_{1\mu} + k_{2\mu})$.

Orthogonality: no transition between vector and scalar states:

$$\int \frac{d^4k}{4\pi} X^{(1)}_\mu X^{(0)} = 0$$
Then:

$$\int \frac{d^4 k}{4\pi} X_{\mu_1 \ldots \mu_n}^{(n)} X_{\mu_2 \ldots \mu_n}^{(n-1)} = 0$$

The integral over internal momentum can be only proportional to the external momentum. Then:

$$\int \frac{d^4 k}{4\pi} X_{\mu_1 \ldots \mu_n}^{(n)} X_{\mu_2 \ldots \mu_n}^{(n-1)} = \xi P_{\mu_1}$$

$$X_{\mu}^{(1)} P_{\mu} = 0 \quad X_{\mu_1 \ldots \mu_n}^{(n)} P_{\mu_j} = 0$$

Then:

$$X_{\mu}^{(1)} = k_{\mu} \perp = k_{\nu} g_{\nu \mu}; \quad g_{\nu \mu} = \left( g_{\nu \mu} - \frac{P_{\nu} P_{\nu}}{p^2} \right)$$

The production of two operators different by even number of indices is proportional to $g_{\mu \nu} \perp$. To satisfy orthogonality we need the traceless condition.

$$X_{\mu_1 \mu_2 \ldots \mu_n}^{(n)} g_{\mu_i \mu_j} = 0$$
The orthogonality and symmetry properties can be written as the set of following conditions:

1. \[ X_{\mu_1 \ldots \mu_i \ldots \mu_j \ldots \mu_n}^{(n)} = X_{\mu_1 \ldots \mu_j \ldots \mu_i \ldots \mu_n}^{(n)} \] (symmetry)

2. \[ P_{\mu_i} X_{\mu_1 \ldots \mu_i \ldots \mu_n}^{(n)} = 0 \] (\(P\)-orthogonality)

3. \[ g_{\mu_1 \mu_2} X_{\mu_1 \mu_2 \ldots \mu_n}^{(n)} = 0 \] (tracelessness)

For low orbital momenta:

\[ X^0 = 1 ; \quad X_\mu^1 = k_\mu^\perp ; \quad X_\mu^2 = \frac{3}{2} \left( k_{\mu}^\perp k_{\nu}^\perp - \frac{1}{3} k_{\perp}^2 g_{\mu \nu}^\perp \right) ; \]

\[ X_{\mu \nu \alpha}^3 = \frac{5}{2} \left[ k_{\mu}^\perp k_{\nu}^\perp k_{\alpha}^\perp - \frac{k_{\perp}^2}{5} \left( g_{\mu \nu}^\perp k_{\alpha}^\perp + g_{\mu \alpha}^\perp k_{\nu}^\perp + g_{\nu \alpha}^\perp k_{\mu}^\perp \right) \right] , \]
The recurrent expression for the operators $X_{\mu_1 \ldots \mu_n}^{(n)}$

$$X_{\mu_1 \ldots \mu_n}^{(n)} = \frac{2n-1}{n^2} \sum_{i=1}^{n} k_{\mu_i}^\perp X_{\mu_1 \ldots \mu_{i-1} \mu_{i+1} \ldots \mu_n}^{(n-1)} - \frac{2k_{\perp}^2}{n^2} \sum_{\begin{smallmatrix} i, j = 1 \\ i < j \end{smallmatrix}}^{n} g_{\mu_i \mu_j} X_{\mu_1 \ldots \mu_{i-1} \mu_{i+1} \ldots \mu_{j-1} \mu_{j+1} \ldots \mu_n}^{(n-2)}$$

Taking into account the traceless property of $X^{(n)}$ we have:

$$X_{\mu_1 \ldots \mu_n}^{(n)} X_{\mu_1 \ldots \mu_n}^{(n)} = \alpha(n) (k_{\perp}^2)^n$$

$$\alpha(n) = \prod_{i=1}^{n} \frac{2i-1}{i} = \frac{(2n-1)!}{n!}.$$

From the recursive procedure one can get the following expression for the operator $X^{(n)}$:

$$X_{\mu_1 \ldots \mu_n}^{(n)} = \left( \prod_{k=1}^{n} \frac{2k-1}{k} \right) \left[ k_{\mu_1}^\perp k_{\mu_2}^\perp \ldots k_{\mu_n}^\perp - \frac{k_{\perp}^2}{2n-1} (g_{\mu_1 \mu_2} k_{\mu_3}^\perp \ldots k_{\mu_n}^\perp + \cdots) + \right. \\
\left. \frac{k_{\perp}^4}{(2n-1)(2n-3)} (g_{\mu_1 \mu_2} g_{\mu_3 \mu_4} k_{\mu_5}^\perp \ldots k_{\mu_4}^\perp + \cdots) + \cdots \right].$$
2.1 Scattering amplitude of two pseudoscalar (scalar) particles.

If we denote the relative momentum of the system before and after interaction as $k$ and $q$, then amplitude in the partial wave $L$ ($L = n$) is:

$$A = a(m_1^2, m_2^2, P^2)X^{(n)}_{\mu_1...\mu_n}(k)X^{(n)}_{\mu_1...\mu_n}(q)$$

The $a(k_\perp^2, q_\perp^2, P^2)$ is a kinematical factor which depends only on invariants.

$$X^{(n)}_{\mu_1...\mu_n}(k)X^{(n)}_{\mu_1...\mu_n}(q) = \left(\sqrt{k_\perp^2}\right)^n \left(\sqrt{q_\perp^2}\right)^nP_n(z), \quad z = \frac{(k_\perp q_\perp)}{\sqrt{k_\perp^2 \sqrt{q_\perp^2}}}$$

Then:

$$A = a(m_1^2, m_2^2, P^2) \left(\sqrt{k_\perp^2}\right)^n \left(\sqrt{q_\perp^2}\right)^nP_n(z)$$
Structure of the fermion propagator

The orthogonality condition has a different form in a fermion case:

\[ \int \Psi_\mu(x) \Psi^*(x) d^4x = A p_\mu + B \gamma_\mu \]

where \( A \) and \( B \) are matrices in spinor space.

It means that we have an additional condition:

\[ \gamma_\mu \Psi_\mu = 0 \quad \gamma_\mu u_\mu = 0 \]

Thus in momentum space we have:

\[ (\hat{p} - m) u_{\mu_1 \cdots \mu_n} = 0 \]
\[ p_{\mu_i} u_{\mu_1 \cdots \mu_n} = 0 \]
\[ u_{\mu_1 \cdots \mu_i \cdots \mu_j \cdots \mu_n} = u_{\mu_1 \cdots \mu_j \cdots \mu_i \cdots \mu_n} \]
\[ g_{\mu_i \mu_j} u_{\mu_1 \cdots \mu_n} = 0 \]
\[ \gamma_{\mu_i} \Psi_{\mu_1 \cdots \mu_n} = 0 \]
These properties define the structure of the fermion projection operator \( P_{\nu_1...\nu_n}^{\mu_1...\mu_n} \):

\[
G_{\nu_1...\nu_n}^{\mu_1...\mu_n} = \frac{(-1)^n}{2m} \frac{m + \hat{p}}{m^2 - p^2} P_{\nu_1...\nu_n}^{\mu_1...\mu_n}
\]

The \( P_{\nu_1...\nu_n}^{\mu_1...\mu_L} \) is equal to 1 for \( J = 1/2 \) particle. For particle with spin \( 3/2 \) it has form:

\[
\frac{1}{2} \left( g_{\mu\nu} - \frac{\gamma_{\mu} \gamma_{\nu}}{3} \right)
\]

where

\[
\gamma_{\mu} = g_{\mu\nu} \gamma_{\nu}.
\]

The boson projector operator projects any operator to one which satisfies all boson properties. It means that we can write:

\[
P_{\nu_1...\nu_n}^{\mu_1...\mu_n} = O_{\alpha_1...\alpha_n}^{\mu_1...\mu_n} T_{\beta_1...\beta_n}^{\alpha_1...\alpha_n} O_{\nu_1...\nu_n}^{\beta_1...\beta_n}
\]
The $T_{\beta_1...\beta_n}^{\alpha_1...\alpha_n}$ operator can have a rather simple form: all symmetry and orthogonality conditions will be imposed by $O$-operators. First of all $T$-operator can be constructed only out of the metrical tensor and $\gamma$-matrices. Second a construction like $\gamma_{\alpha i} \gamma_{\alpha j}$:

$$\gamma_{\alpha i} \gamma_{\alpha j} = \frac{1}{2} g_{\alpha i \alpha j} + \sigma_{\alpha i \alpha j},$$

where

$$\sigma_{\alpha i \alpha j} = \frac{1}{2} (\gamma_{\alpha i} \gamma_{\alpha j} - \gamma_{\alpha j} \gamma_{\alpha i})$$

gives zero after production with $O$-operator (first term due to traceless and second due to symmetrical properties). Then the only one structure which can be constructed out of gamma matrices is $g_{\alpha_i \beta_j}$ and $\sigma_{\alpha_i \beta_j}$. Moreover, taking into account symmetrical properties of $O$-operator the latest can be used as for example $\sigma_{\alpha_1 \beta_1}$:

$$T_{\beta_1...\beta_L}^{\alpha_1...\alpha_L} = \frac{L + 1}{2L + 1} \left( g_{\alpha_1 \beta_1} - \frac{L}{L + 1} \sigma_{\alpha_1 \beta_1} \right) \prod_{i=2}^{L} g_{\alpha_i \beta_i}$$

Here the coefficients are calculated to satisfy the orthogonality condition with $\gamma$-matrices.
\( \pi N \) interaction

Pion has quantum numbers \( J^{PC} = 0^{-+} \), proton \( 1/2^+ \). Then in S-wave the only state can be formed is \( 1/2^- \). P-wave can form two states \( 1/2^+ \) and \( 3/2^+ \).

In PDG states are defined by the quantum numbers in \( \pi N \) decay: \( L_{2I,2J} \). For example \( D_{13} \) means \( 3/2^- N^* \) state.

States where \( J = L - 1/2 \) called ’-’ states \( (1/2^+, 3/2^-, 5/2^+, \ldots) \) and states with \( J = L + 1/2 \) called ’+’ states \( (1/2^-, 3/2^+, 5/2^-, \ldots) \).

For ’+’ states and for ’-’ states:

\[
N^{+}_{\mu_1 \ldots \mu_n} = X^{(n)}_{\mu_1 \ldots \mu_n} \quad \quad \quad N^{-}_{\mu_1 \ldots \mu_n} = i \gamma_\nu \gamma_5 X^{(n+1)}_{\nu \mu_1 \ldots \mu_n}
\]

\[
A = \bar{u}(k_1) \tilde{N}^{\pm}_{\mu_1 \ldots \mu_n} F^{\mu_1 \ldots \mu_n}_{\nu_1 \ldots \nu_n}(P) N^{\pm}_{\nu_1 \ldots \nu_n} u(q_1) BW_{L^\pm}(s)
\]

where

\[
F^{\mu_1 \ldots \mu_L}_{\nu_1 \ldots \nu_L}(p) = (m + \hat{p}) O^{\mu_1 \ldots \mu_L}_{\alpha_1 \ldots \alpha_L} \frac{L + 1}{2L + 1} \left( g^{\perp}_{\alpha_1 \beta_1} - \frac{L}{L + 1} \sigma_{\alpha_1 \beta_1} \right) \prod_{i=2}^{L} g_{\alpha_i \beta_i} O^{\beta_1 \ldots \beta_L}_{\nu_1 \ldots \nu_L}
\]
In c.m.s. of the reaction

\[ A_{\pi N} = \omega^* \left[ G(s, t) + H(s, t)i(\vec{\sigma}\vec{n}) \right] \omega' , \]

\[ G(s, t) = \sum_L \left[ (L+1)F^+_L(s) - LF^-_L(s) \right] P_L(z) , \]

\[ H(s, t) = \sum_L \left[ F^+_L(s) + F^-_L(s) \right] P'_L(z) . \]

\[ F^+_L = - (|\vec{k}||\vec{q}|)^L \sqrt{\chi_i\chi_f} \frac{\alpha(L)}{2L+1} BW^+_L(s) , \]

\[ F^-_L = (|\vec{k}||\vec{q}|)^L \sqrt{\chi_i\chi_f} \frac{\alpha(L)}{L} BW^-_L(s) . \]

\[ \chi_i = m_i + k_{i0} \quad \alpha(L) = \prod_{l=1}^L \frac{2l - 1}{l} = \frac{(2L - 1)!!}{L!} . \]
\( \gamma N \) interaction

Photon has quantum numbers \( J^{PC} = 1^{--} \), proton \( 1/2^+ \). Then in S-wave two states can be formed is \( 1/2^- \) and \( 3/2^- \).

Then P-wave \( 1/2^+ \), \( 3/2^+ \) and \( 1/2^+, 3/2^+, 5/2^+ \).

In general case: \( 1/2^- \), \( 1/2^+ \) described by two amplitudes and higher states by three amplitudes.

\[
\begin{align*}
V_{\alpha_1...\alpha_n}^{(1+)} &= \gamma_\mu i\gamma_5 X^{(n)}_{\alpha_1...\alpha_n}, \\
V_{\alpha_1...\alpha_n}^{(2+)} &= \gamma_\mu i\gamma_5 X^{(n+2)}_{\mu\nu\alpha_1...\alpha_n}, \\
V_{\alpha_1...\alpha_n}^{(3+)} &= \gamma_\mu i\gamma_5 X^{(n+1)}_{\nu\alpha_1...\alpha_n} g_{\mu\alpha_n},
\end{align*}
\]

\[
\begin{align*}
V_{\alpha_1...\alpha_n}^{(1-)} &= \gamma_\xi \gamma_\mu X^{(n+1)}_{\xi\alpha_1...\alpha_n}, \\
V_{\alpha_1...\alpha_n}^{(2-)} &= X^{(n+1)}_{\mu\alpha_1...\alpha_n}, \\
V_{\alpha_1...\alpha_n}^{(3-)} &= X^{(n-1)}_{\alpha_2...\alpha_n} g_{\alpha_1\mu}.
\end{align*}
\]

Gauge invariance: \( \varepsilon_\mu q_1^\mu = 0 \) where \( q_1^\mu \)-photon momentum.

\[
\varepsilon_\mu V_{\alpha_1...\alpha_n}^{(2\pm)} = C^{\pm} \varepsilon_\mu V_{\alpha_1...\alpha_n}^{(3\pm)}
\]

where \( C^{\pm} \) do not depend on angles.

\[
A = \bar{u}(k_1) N_{\mu_1...\mu_n}^{\pm} F_{\nu_1...\nu_n}^{\mu_1...\mu_n} (P) V_{\nu_1...\nu_n}^{(i\pm)} u(q_1) BW_L^{\pm}(s) \varepsilon_\mu
\]
The amplitude for the photoproduction of a single pseudoscalar has structure

\[ A = \omega^* J_\mu \varepsilon_\mu \omega', \]

\[ J_\mu = i F_1 \sigma_\mu + F_2 (\vec{\sigma} \vec{q}) \frac{\varepsilon_{\mu ij} \sigma^i k^j}{|k| |q|} \right. \left. + i F_3 \frac{(\vec{\sigma} k)}{|k| |q|} q_\mu + i F_4 \frac{(\vec{\sigma} q)}{q^2} q_\mu . \]

\[ F_1(z) = \sum_{L=0}^{\infty} \left[ LM_L^+ + E_L^+ \right] P_{L+1}(z) + [(L + 1) M_L^- + E_L^-] P_{L-1}'(z), \]

\[ F_2(z) = \sum_{L=1}^{\infty} \left[ (L + 1) M_L^+ + LM_L^- \right] P_L'(z), \]

\[ F_3(z) = \sum_{L=1}^{\infty} \left[ E_L^+ - M_L^+ \right] P_{L+1}(z) + [E_L^- + M_L^-] P_{L-1}''(z), \]

\[ F_4(z) = \sum_{L=2}^{\infty} \left[ M_L^+ - E_L^+ - M_L^- - E_L^- \right] P_L''(z). \]
Our amplitudes can be algebraically rewritten to multipole representation:

\[
E_L^{+(1/2)} = \sqrt{\chi_i \chi_f} \frac{\alpha(L)}{2L+1} \frac{(|\vec{k}| |\vec{q}|)^L}{L+1} BW^+(s),
\]

\[
M_L^{+(1/2)} = E_L^{+(1/2)}.
\]

\[
E_L^{+(3/2)} = \sqrt{\chi_i \chi_f} \frac{\alpha(L)}{2L+1} \frac{(|\vec{k}| |\vec{q}|)^L}{L+1} BW^+(s),
\]

\[
M_L^{+(3/2)} = -\frac{E_L^{+(3/2)}}{L}.
\]
The resonance amplitudes for meson photoproduction

\[ \gamma \ p \rightarrow R_1 \rightarrow R_2 \ \pi \rightarrow p \ \pi \ \pi \]

The general form of the angular dependent part of the amplitude:

\[ \bar{u}(q_1) \tilde{N}_{\alpha_1 \ldots \alpha_n} (R_2 \rightarrow \mu N) F_{\beta_1 \ldots \beta_n}^{\alpha_1 \ldots \alpha_n} (q_1 + q_2) \tilde{N}_{\gamma_1 \ldots \gamma_m}^{(j)\beta_1 \ldots \beta_n} (R_1 \rightarrow \mu R_2) \]

\[ \prod_{\nu_1 \ldots \nu_L}^{\mu_1 \ldots \mu_L} \langle p \rangle \sigma_{\alpha_1 \beta_1} \prod_{i=2}^{L} g_{\alpha_i \beta_i} O_{\nu_1 \ldots \nu_L}^{\beta_1 \ldots \beta_L} \]

\[ \sigma_{\alpha_i \alpha_j} = \frac{1}{2} (\gamma_{\alpha_i} \gamma_{\alpha_j} - \gamma_{\alpha_j} \gamma_{\alpha_i}) \]
Diagram approach to the calculation of the scattering amplitude

Let us consider a state which is produced from two interacting particles, then propagates and decays into same two particles in the final state. The amplitude for such process can be represented as a sum of the diagrams:

\[
\begin{align*}
A &= 1/M_0^2 - s - B(s)/M_0^2 - s
\end{align*}
\]

The amplitude can be found as direct sum of the diagrams:
This amplitude also can be found by solving the following equation:

\[
A = A \frac{B(s)}{M_0^2 - s} + \frac{g^2}{M_0^2 - s}
\]

\[
A = \frac{g^2}{M_0^2 - s} \left( 1 - \frac{B(s)}{M_0^2 - s} \right) = \frac{g^2}{M_0^2 - s - B(s)}
\]

Here \( M_0 \) is a bare mass of the state and \( B(s) \) is the two body loop diagram:

\[
B^F = \int \frac{d^4 k}{i(2\pi)^4} \frac{g^2}{(m^2 - k^2)(m^2 - (P - k)^2)},
\]
Let us assume that the vertex (coupling) $g$ has no singularities and is a smooth function in the physical region. The imaginary part of the loop diagram appears at the energy $s > 4m^2$ and can be calculated by:

$$(m^2 - k^2)^{-1}(m^2 - (P - k)^2)^{-1} \rightarrow (-2\pi)^2 i \delta(m^2 - k^2)\Theta(k_0)$$

$$\delta(m^2 - (P - k)^2)\Theta(P_0 - k_0)$$

Then loop diagram can be rewritten in the dispersion representation:

$$B(s) = \int_{4m^2}^{\infty} \frac{ds'}{\pi} \frac{g(s')\rho(s')g(s')}{s' - s - i0} = Re \int_{4m^2}^{\infty} \frac{ds'}{\pi} \frac{g(s')\rho(s')g(s')}{s' - s} - i\rho(s)g^2(s)$$

The real part of the B-function does not have any singularities in the physical region. It is a smooth function and can be used for renormalization of the bare state mass (or neglected in some cases). Then:

$$A(s) = \frac{g^2}{M^2 - s - i\rho(s)g^2} \quad M^2 = M_0^2 - ReB(M^2)$$

And we obtain a Breit-Wigner expression with $M\Gamma = g^2 \rho(M^2)$.
Let us calculate analytically the loop diagram assuming that $g$ is a constant.

First we need a renormalization:

$$B(s) = B(M^2) + (s - M^2) \int_{4m^2}^{\infty} \frac{ds'}{\pi} \frac{g^2}{(s' - s)(s' - M^2)} \sqrt{\frac{s' - 4m^2}{s'}}$$

Such integral is equal to:

$$B(s) = \text{Re}B(M^2) + \frac{g^2}{\pi}[\rho(s) \ln \frac{1 - \rho(s)}{1 + \rho(s)} - \rho(M^2) \ln \frac{1 - \rho(M^2)}{1 + \rho(M^2)}] + i\rho(s)g^2$$

At $s \to 0$:

$$i\rho(s) = i\sqrt{\frac{s - 4m^2}{s}} \to -\infty$$

$$i\rho(s)(1. - \frac{i}{\pi}ln \frac{1 - \rho(s)}{1 + \rho(s)}) = i\sqrt{\frac{s - 4m^2}{s}}(1 - \frac{2}{\pi\arctg \frac{4m^2 - s}{s}}) \to \text{const}$$
Black curve - BW amplitude, red curve - full B(s) calculation, blue curve - BW amplitude with reduced width, magenta - dispersion correction of the real part.
P-vector approach

Let us consider photoproduction of two pions. This case is different from the $\pi\pi$ scattering by the first interaction:

The first interaction can be the direct production of K-matrix poles or nonresonant production: $\gamma\gamma \rightarrow \pi\pi$, $\gamma\gamma \rightarrow K\bar{K}$ and so on.
Combined analysis of the different reactions:

For pion induced reactions the transition partial wave amplitude can be written as:

\[ A_{1i} = K_{1j} (I - i\rho K)^{-1}_{ji} \]

and

\[ K_{ij} = \sum_{\alpha} \frac{g_i^\alpha g_j^\alpha}{M_\alpha^2 - s} + f_{ij}(s) \quad f_{ij} = \frac{f_{ij}^{(1)} + f_{ij}^{(2)} \sqrt{s}}{s - s_{ij}^0} . \]

where \( f_{ij} \) is nonresonant transition part.

For the photoproduction:

\[ A_k = P_j (I - i\rho K)^{-1}_{jk} \]

The vector of the initial interaction has the form:

\[ P_j = \sum_{\alpha} \frac{\Lambda^\alpha g_j^\alpha}{M_\alpha^2 - s} + F_j(s) \]

Here \( F_j \) is nonresonant production of the final state \( j \).
D-vector approach

For $\pi N$ transition into channel ’a’ the amplitude can be written as:

$$A_a = \hat{D}_a + [\hat{K}(\hat{I} - i\hat{\rho}\hat{K})^{-1} \hat{\rho}]_{ab} \hat{D}_b,$$

For strong channels:

$$D_a = K_{1a} \quad A_a = K_{1j} (I - i\rho K)^{-1}_{ji}$$

For weak channels:

$$D_a = \sum_{\alpha} g_1^\alpha \frac{\Lambda_a^{dec}}{M_\alpha^2 - s} + d_{1a}(s)$$

For photoproduction of ’weak channels’

$$A_{ab} = \hat{G}_{ab} + P_a(\hat{I} - i\hat{\rho}\hat{K})^{-1} \hat{\rho}\hat{D}_b \quad G_{ab} = \sum_{\alpha} \frac{\Lambda_b \Lambda_a^{dec}}{M_\alpha^2 - s} + b_{ab}(s)$$
Phase volumes

Two body phase volume:

\[ \rho(s, m_1, m_2) = \frac{2k}{s} = \frac{\sqrt{(s - (m_1 + m_2)^2)(s - (m_1 - m_2)^2)}}{s} \]

Three body phase volume:

\[ \rho_3(s) = \frac{(\sqrt{s} - m_1)^2}{(m_2 + m_3)^2} \int \frac{ds_{23}}{\pi} \frac{\rho(s, \sqrt{s_{23}}, m_1) M_R \Gamma_R^{R \text{tot}}}{(M_R^2 - s_{23})^2 + (M_R \Gamma_R^{R \text{tot}})^2} , \]

\[ M_R \Gamma_R^{R \text{tot}} = \rho(s_{23}, m_2, m_3) g^2(s_{23}) , \]
Pole position in the complex plane.

Two sheets are defined as:

\[ \rho(s, m_1, m_2) = \frac{\sqrt{(s - (m_N + m_\eta)^2)(s - (m_N - m_\eta)^2)}}{s} \] \hspace{1cm} I sheet

\[ \rho(s, m_1, m_2) = i \frac{\sqrt{(m_N + m_\eta)^2 - s)(s - (m_N - m_\eta)^2)}}{s} \] \hspace{1cm} II sheet

We search for

\[ \text{Im } \det(I - i\rho K) \prod_{\alpha} (M^2_{\alpha} - s) = 0 \]

\[ \text{Re } \det(I - i\rho K) \prod_{\alpha} (M^2_{\alpha} - s) = 0 \]

For BW approximation (1pole K-matrix)

\[ \det(I - i\rho K)(M^2_{\alpha} - s) = M^2_{\alpha} - s - i \sum_i g_i^2 \rho_i \]
One pole one channel ($\eta N$) K-matrix
One pole one channel ($\pi\Delta$) K-matrix
Two poles one channel ($\pi N$) K-matrix
Calculation of the residues

For any function of complex variables $F(s)$:

$$\int ds F(s) = \sum_\alpha 2\pi i \text{Res}_\alpha$$

This does not depend on the form of counter.

K-matrix amplitude $A_{ij}$, full factorization:

$$\int ds A_{ij}(s) = 2\pi i g_i g_j$$

In our article:

$$A_{BW} = \frac{X}{M_{BW}^2 - s - i\beta \sum_i g_i^2 \rho_i}$$

where $M_{BW}$ and $\beta$ are fitted to reconstruct pole position.
The fitted reactions. **Recently included data sets. New points added**

<table>
<thead>
<tr>
<th>Observable</th>
<th>(N_{\text{data}})</th>
<th>(\chi^2)</th>
<th>Source</th>
<th>Observable</th>
<th>(N_{\text{data}})</th>
<th>(\chi^2)</th>
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<td>(\sigma(\gamma p \rightarrow p\pi^0))</td>
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<td>Phoenics</td>
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The fitted reactions. Recently included data sets.

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<th>(\chi^2_{N_{\text{data}}})</th>
<th>Observable</th>
<th>(N_{\text{data}})</th>
<th>(\chi^2_{N_{\text{data}}})</th>
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<td>(P(\pi^+p \to K^+\Sigma))</td>
<td>420</td>
<td>2.74</td>
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</table>
\[ \gamma p \rightarrow \pi^0 p \text{ from Crystal Barrel at ELSA } (E_\gamma \leq 3.2 \text{ GeV}) \]

\[ \Delta(1232)P_{33} \]
\[ N(1520)D_{13} S_{11} \]
\[ N(1680)F_{15} \]
\[ \Delta(1700)D_{33} \]
\[ \Delta(1920)P_{33} \]

Non-resonance contribution:

- t-channel \( \rho - \omega \) exchange,
- u-exchange and non-resonance production in \( J^P = 3/2^+ \) wave
$N\pi \rightarrow N\pi$, $P_{33}$ wave (3 pole 4 channel K-matrix)
The $\Delta$-states decaying into $K\Sigma$ can be fixed from the $\pi^+ \rightarrow K^+\Sigma$ data. The main contribution comes from $P_{33}(1920)$ - red curves and $F_{37}(1900)$ - blue curves.
The recoil asymmetry for the $\pi^+ p \rightarrow K\Sigma$ reaction also shows a clear contribution from this state:
\( \gamma p \rightarrow \eta p \) from Crystal Barrel at ELSA \((E_\gamma \leq 3.2 \text{ GeV})\)

Main resonance contributions:
- \(N(1535)S_{11}\)
- \(N(1650)S_{11}\)
- \(N(1720)P_{13}\)
- \(\text{new } N(2070)D_{15}\)

Non-resonance contribution: \(\text{reggezed t-channel } \rho - \omega \) exchange.

No evidence for third \(N(1800)S_{11}\)
The data on $\pi^- p \rightarrow \eta n$ and the target asymmetry $\gamma p \rightarrow \eta p$ fix the position and couplings of $P_{11}(1710)$ state and reduce $\eta N$ coupling of the $P_{13}(1720)$ state.

<table>
<thead>
<tr>
<th>Observable</th>
<th>$N_{\text{data}}$</th>
<th>$\frac{\chi^2}{N_{\text{data}}}$</th>
<th>Source</th>
<th>Observable</th>
<th>$N_{\text{data}}$</th>
<th>$\frac{\chi^2}{N_{\text{data}}}$</th>
<th>Source</th>
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</thead>
<tbody>
<tr>
<td>$\sigma(\gamma p \rightarrow p\eta)$</td>
<td>667</td>
<td>0.92 (0.85)</td>
<td>CB-ELSA</td>
<td>$\sigma(\gamma p \rightarrow p\eta)$</td>
<td>100</td>
<td>2.72 (1.97)</td>
<td>TAPS</td>
</tr>
<tr>
<td>$\Sigma(\gamma p \rightarrow p\eta)$</td>
<td>51</td>
<td>2.06 (1.81)</td>
<td>GRAAL 98</td>
<td>$\Sigma(\gamma p \rightarrow p\eta)$</td>
<td>100</td>
<td>2.01 (1.43)</td>
<td>GRAAL 04</td>
</tr>
</tbody>
</table>
The target asymmetry $\gamma p \rightarrow \eta p$ data reduce coupling of the $P_{13}(1720)$ state to the $\eta N$ channel by factor $\sim 1.7$. 

![Graph showing total cross-section $\sigma_{tot}$ in mb vs. $M(\gamma p)$ in MeV.](image)

![Graph showing $\sigma_{tot}$ in mb vs. $M(\gamma p)$ in MeV.](image)
$N\pi \rightarrow N\pi$, $S_{11}$ wave (2 pole 5 channel K-matrix)

T-matrix poles: $M = 1508^{+10}_{-30}$ MeV, $2\, Im = 165 \pm 15$ MeV;
$M = 1645 \pm 15$ MeV, $2\, Im = 187 \pm 20$ MeV
\( \pi^- p \rightarrow n\pi^0\pi^0 \) (Crystal Ball) total cross section

\[ \sigma_{tot}, \text{mb} \]

\[ M(\pi p), \text{GeV/c}^2 \]

Fit of the data
- \( P_{11} \) - partial wave
- \( D_{13} \) - partial wave
- \( S_{11} \) - partial wave
$\pi^- p \rightarrow n\pi^0\pi^0$ (Crystal Ball)

Differential cross sections for 472 and 665 MeV/c data.
\[ \gamma p \rightarrow p\pi^0\pi^0 \] (CB-ELSA) M. Fuchs et al.

PWA corrected cross section and contributions from $\Delta (1232) \pi$ (dashed) and $N\sigma$ (dashed-dotted) final states.

Contributions from $D_{33}$ (dotted), $P_{11}$ (dashed) and $D_{13}$ (dashed-dotted) partial waves.
The $\gamma p \rightarrow \pi^0\pi^0 p$ differential cross section for the total energy region.

The fit of the unpolarized data and prediction for the double polarization measurements. Red curve: only helicity 1/2 amplitudes contributed to the cross section. Blue curve: only helicity 3/2 amplitudes.
The $\gamma p \rightarrow \pi^0\pi^0 p$ helicity $3/2$ and $1/2$ differential cross sections
The total cross section for the $\pi^- p \rightarrow K\Lambda$ reaction also shows a clear contribution from this state:

The same result was obtained before by the Giessen group:
The differential cross section for the $\pi^- p \rightarrow K\Lambda$ reaction also shows a clear contribution from this state:
The recoil asymmetry for the $\pi^- p \rightarrow K\Lambda$ reaction also shows a clear contribution from this state:
\( \pi N \rightarrow \pi N P_{11} \) wave (3 pole 4 channel K-matrix)

\[ \text{T-matrix poles: } M = 1368 \pm 7 \text{ MeV}, \quad 2 \text{Im} = 190 \pm 10 \text{ MeV;} \]
\[ M = 1685 \pm 20 \text{ MeV}, \quad 2 \text{Im} = 160 \pm 45 \text{ MeV} \]
\[ M = 1870 \pm 30 \text{ MeV}, \quad 2 \text{Im} = 280 \pm 80 \text{ MeV} \]
Position of zeros, for the determinant $(I - i\rho K)^{-1}$. Red points - real part and Green points - imaginary part.
Properties of $N(1440)_{P_{11}}$. The left column lists mass, width, partial widths of the Breit-Wigner resonance; the right column pole position and squared couplings to the final state at the pole position.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
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<tbody>
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<tr>
<td>$M_{\text{pole}}$</td>
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<tr>
<td>$\Gamma$</td>
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<td>$\Gamma_{\text{pole}}$</td>
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<td>$\Gamma_{\pi N}$</td>
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<td>$g_{\pi N}$</td>
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<td>$(-0.57 \pm 0.08) \cdot e^{i\pi \frac{25 \pm 20}{180}}$</td>
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</table>

**T-matrix:** $A_{1/2} = 0.055 \pm 0.020$ GeV \quad $\phi = (70 \pm 30)^\circ$
For $\gamma p \to K\Lambda$ and $\gamma p \to K\Sigma$ we have almost complete photoproduction experiment:

$\sigma$ (CLAS, SAPHIR), $\Sigma$ (GRAAL, LEP), $P$ (CLAS), $C_x, C_z$ (CLAS), $T, Ox, Oz$ (GRAAL).

The $C_x$ and $C_z$ data can be explained with $P_{13}(1900)$. 
\[ \sigma_{\text{tot}}(\gamma p \rightarrow K^0 \Sigma^+) \text{ from CB-ELSA} \]
The solution is supported by the new GRALL data on $O_x$, $O_z$ and $T$-observables: an important step to a complete experiment.
$N\pi \rightarrow N\pi$, $P_{13}$ wave (3 pole 8 channel K-matrix)

2nd T-matrix poles: $M = 1960 \pm 20$ MeV, $2 Im = 195 \pm 45$ MeV;
\[ \gamma p \rightarrow p\pi^0\eta \text{ (CB-ELSA)} \]

Left panel: contributions from \( \Delta(1232)\eta \) (dashed), \( S_{11}(1535)\pi \) (dashed-dotted) and \( N a_0(980) \) final states.

Right panel: \( D_{33} \) partial wave (dashed), \( P_{33} \) partial wave (dashed-dotted), \( D_{33} \rightarrow \Delta(1232)\eta \) (dotted) and \( D_{33} \rightarrow N a_0(980) \) (wide dotted).
The $\gamma p \rightarrow \pi^0 \eta p$ differential cross section for the total energy region.
$N\pi \rightarrow N\pi$ $D_{33}$ wave (3 pole 5 channel K-matrix)

$D_{33}$-wave: $\pi N$, $\Delta(1232)\pi$ ($S$- and $D$-waves), $\Delta(1232)\eta$, $S_{11}(1535)\pi$
Properties of the $\Delta(1920)P_{33}$ and $\Delta(1940)D_{33}$ resonances.

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<th>$M_{BW}$</th>
<th>$\Gamma_{BW_{tot}}$</th>
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<td>1940 ± 40</td>
<td>350$^{+35}_{-55}$</td>
<td>1970 ± 35</td>
<td>375 ± 50</td>
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<td>$\Delta(1940)D_{33}$</td>
<td>1995 ± 30</td>
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<th>$Br_{N\pi}$</th>
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<td>5 ± 2</td>
<td>2 ± 1</td>
<td>2 ± 1</td>
</tr>
</tbody>
</table>
Parity doublets of $N$ and $\Delta$ resonances at high mass region

Glozman suggested a restoration of chiral symmetry in high-mass excitations. Parity doublets must not interact by pion emission and could have a small coupling to $\pi N$.

\[
\begin{array}{cccccc}
J=\frac{1}{2} & N_{1/2}^+ (2100)^a & N_{1/2}^- (2090)^a & \Delta_{1/2}^+ (1910) & \Delta_{1/2}^- (1900)^a & \\
J=\frac{3}{2} & N_{3/2}^+ (1900)^a & N_{3/2}^- (2080)^a & \Delta_{3/2}^+ (1920)^a & \Delta_{3/2}^- (1940)^a & \\
J=\frac{5}{2} & N_{5/2}^+ (2000)^a & N_{5/2}^- (2200)^a & \Delta_{5/2}^+ (1905) & \Delta_{5/2}^- (1930)^a & \\
J=\frac{7}{2} & N_{7/2}^+ (1990)^a & N_{7/2}^- (2190) & \Delta_{7/2}^+ (1950) & \Delta_{7/2}^- (2200)^a & \\
J=\frac{9}{2} & N_{9/2}^+ (2220) & N_{9/2}^- (2250) & \Delta_{9/2}^+ (2300) & \Delta_{9/2}^- (2400)^a & \\
\end{array}
\]

\[
\begin{array}{cccc}
J=\frac{3}{2} & N_{3/2}^+ (1900) & N_{3/2}^- (1875) & \Delta_{3/2}^+ (1980) & \Delta_{3/2}^- (1985) \\
J=\frac{5}{2} & N_{5/2}^+ (1960) & N_{5/2}^- (2070) & \Delta_{5/2}^+ (1945) & \Delta_{5/2}^- (1930) \\
J=\frac{7}{2} & N_{7/2}^+ (1990) & N_{7/2}^- (????) & \Delta_{7/2}^+ (1910) & \Delta_{7/2}^- (????) \\
\end{array}
\]
Holographic QCD (AdS/QCD)

Soft-wall model prediction: \( M_{N,L}^2 = 4 \lambda^2 \left( N + L + \frac{3}{2} \right) \)

\[M_{N,L}^2 = 4 \lambda^2 \left( N + L + \frac{3}{2} \right) - 2 \left( M_\Delta^2 - M_N^2 \right) \kappa_{gd}\]

\( \kappa_{gd} \) is the fraction of most attractive color-antitriplet isosinglet diquark.

\( \kappa_{gd} = 0 \) for \( \Delta \) and \( N(S=3/2) \) states, \( \frac{1}{2} \) for \( S = 1/2 \) (70SU6) and \( \frac{1}{4} \) for \( S = 1/2 \) (56SU6).

Hilmar Forkel and Eberhard Klempt, hep-ph:0810.2959v1
<table>
<thead>
<tr>
<th>$L, S, N$</th>
<th>$\kappa_{gd}$</th>
<th>Resonance</th>
<th>Pred.</th>
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<tr>
<td>$0, \frac{1}{2}, 0$</td>
<td>$\frac{1}{2}$</td>
<td>$N(940)$</td>
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<tr>
<td>$0, \frac{3}{2}, 0$</td>
<td>$0$</td>
<td>$\Delta(1232)$</td>
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<td>$\frac{1}{2}$</td>
<td>$N(1440)$</td>
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<td>$1, \frac{1}{2}, 0$</td>
<td>$\frac{1}{4}$</td>
<td>$N(1535)$, $N(1520)$</td>
<td>1.53</td>
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<tr>
<td>$1, \frac{3}{2}, 0$</td>
<td>$0$</td>
<td>$N(1650)$, $N(1700)$, $N(1675)$</td>
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</tr>
<tr>
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<td>$0$</td>
<td>$\Delta(1620)$, $\Delta(1700)$ $L, S, N=0, \frac{3}{2}, 1$: $\Delta(1600)$</td>
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<tr>
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<td>$N(1720)$, $N(1680)$ $L, S, N=0, \frac{1}{2}, 2$: $N(1710)$</td>
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<tr>
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<tr>
<td>$1, \frac{3}{2}, 1$</td>
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<td>$\Delta(1900)$, $\Delta(1940)$, $\Delta(1930)$</td>
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<tr>
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<td>$\Delta(1910)$, $\Delta(1920)$, $\Delta(1905)$, $\Delta(1950)$</td>
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<tr>
<td>$2, \frac{3}{2}, 0$</td>
<td>$0$</td>
<td>$N(1880)$, $N(1900)$, $N(1990)$, $N(2000)$</td>
<td>1.92</td>
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<tr>
<td>$0, \frac{1}{2}, 3$</td>
<td>$\frac{1}{2}$</td>
<td>$N(2100)$</td>
<td></td>
</tr>
<tr>
<td>$3, \frac{1}{2}, 0$</td>
<td>$\frac{1}{4}$</td>
<td>$N(2070)$, $N(2190)$ $L, S, N=1, \frac{1}{2}, 2$: $N(2080)$, $N(2090)$</td>
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<tr>
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<td>$0$</td>
<td>$N(2200)$, $N(2250)$ $L, S, N=1, \frac{1}{2}, 2$: $\Delta(2223)$, $\Delta(2200)$</td>
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<tr>
<td>$4, \frac{1}{2}, 0$</td>
<td>$\frac{1}{2}$</td>
<td>$N(2220)$</td>
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</tr>
<tr>
<td>$4, \frac{3}{2}, 0$</td>
<td>$0$</td>
<td>$\Delta(2390)$, $\Delta(2300)$, $\Delta(2420)$ $</td>
<td>L, N=3, 1$: $\Delta(2400)$ $\Delta(2350)$</td>
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<tr>
<td>$5, \frac{1}{2}, 0$</td>
<td>$\frac{1}{4}$</td>
<td>$N(2600)$</td>
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</table>
Summary

1. An approach for the combined analysis of the pion and photo induced reaction with two and multi particle final states is developed.

2. The combined analysis of more than 65 different reactions helped to identify the properties of known baryons.

3. The new data support the two new baryon states observed in hyperon photoproduction $P_{11}(1880)$ and $P_{13}(1900)$.

4. The $\eta$-photoproduction data reveal the baryon resonance $D_{15}(2070)$.

5. The $D_{33}(1940)$ state is needed for the description of the $\gamma p \rightarrow \pi^0 \eta p$ data.

6. The data on $\pi^- p \rightarrow \eta n$ and $\pi^- p \rightarrow K^0 \Sigma$ support an existence of $P_{11}(1710)$.

7. The spectrum of observed states is in direct contradiction with a classical quark model. The best explanations are chiral symmetry restoration or AdS/QCD soft-wall model.