Let us consider $\pi\pi$ and $K\bar{K}$ channels. Then

$$\rho_1(s) = \sqrt{\frac{s-4m_\pi^2}{s}} \qquad \rho_2(s) = \sqrt{\frac{s-4m_K^2}{s}}$$

If $g_2^2 = 0$ the amplitude is one channel amplitude and has the Breit-Wigner form with $\Gamma M = g_1^2\rho_1(M^2)$. If the $g_2^2 > 0$ but K-matrix pole is situated much higher then the second threshold then again the amplitude has the Breit-Wigner form with

$$\Gamma M = g_1^2\rho_1(M^2) + g_2^2\rho_2(M^2).$$

Let us assume that pole is situated far from first threshold but close to the second threshold. Then we can put $\rho_1(s) = 1$ and the pole in amplitude is situated at:

$$s = M^2 - ig_1^2$$

Let us now increase g_2 coupling. On second sheet defined as:

$$i\rho(s) = i\sqrt{\frac{s-4m_K^2}{s}} = i(a-ib) = b+ia \qquad where \qquad a>0, \quad b>0.$$

Then in first approximation:

$$s \to M^2 - b - i(g_1^2 + a)$$

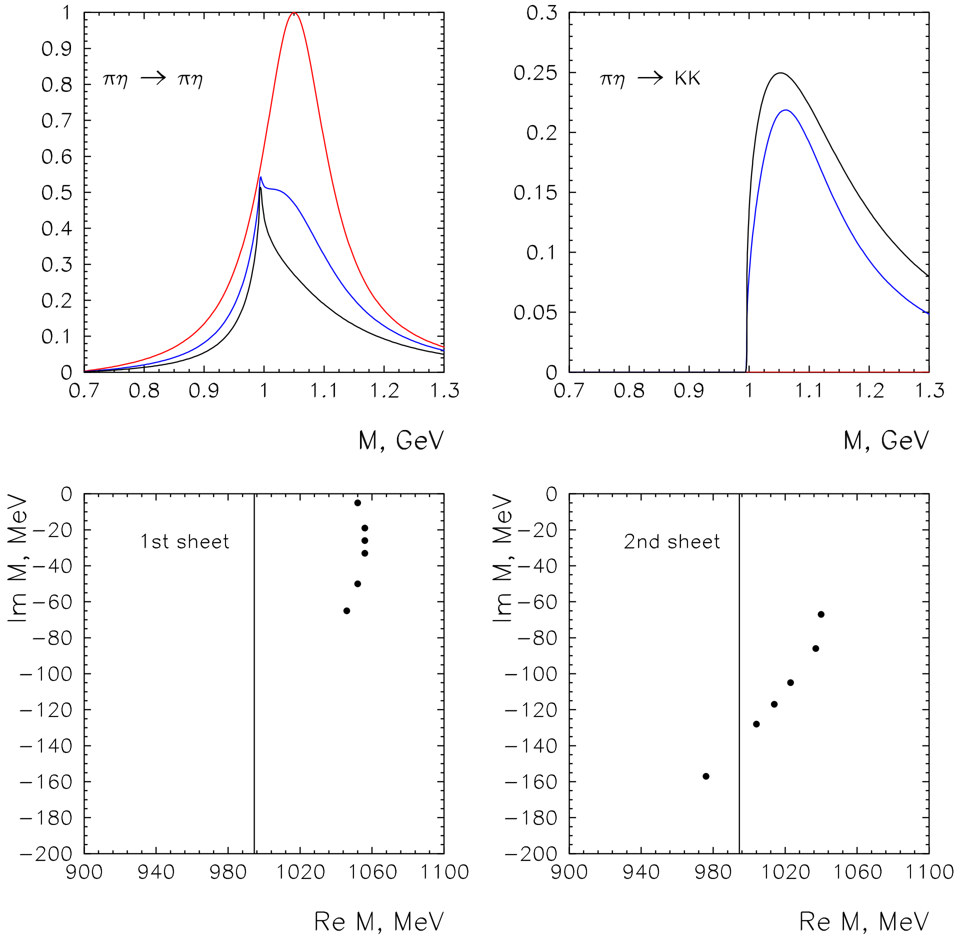
On the first sheet:

$$i\rho(s) = i(-a' + ib') = -ia' - b' \qquad where \qquad a'>0, \quad b'>0.$$

And

$$s \to M^2 + b' - i(g_1^2 - a')$$

Thus, the pole on second sheet moves to $K\bar{K}$ threshold and became broader. Pole on the first sheet gets away from $K\bar{K}$ threshold and up in the complex plane.



If pole was situated below $K\bar{K}$ threshold then the closest to physical region sheet is the first sheet:

$$i\rho(s) = i\sqrt{\frac{s-4m_K^2}{s}} = i(-a+ib) = -ia-b \qquad where \qquad a>0, \quad b>0.$$

and then:

$$s \to M^2 + b - i(g_1^2 - a)$$

On the second sheet we have:

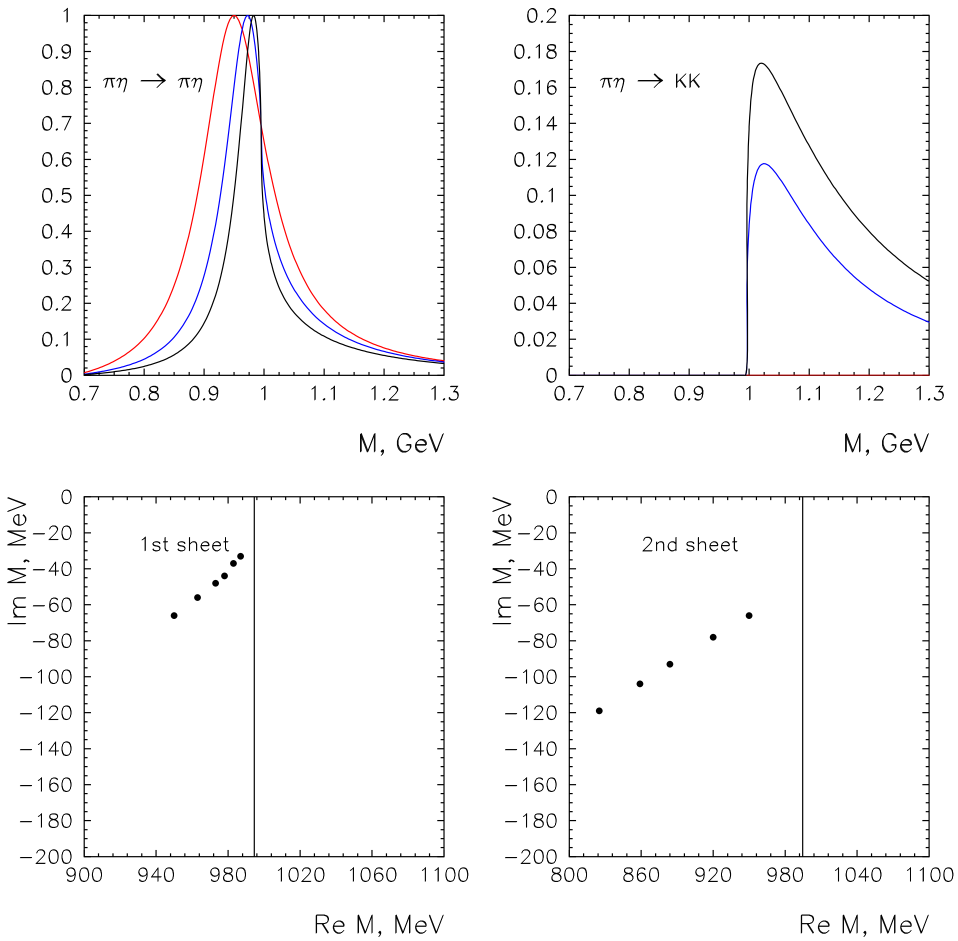
$$i\rho(s) = i\sqrt{\frac{s-4m_K^2}{s}} = i(a'-ib') = ia'+b' \qquad where \qquad a'>0, \quad b'>0.$$

and then:

$$s \to M^2 - b' - i(g_1^2 + a')$$

So the pole on first sheet moves to the $K\bar{K}$ threshold and became a narrow one while on second sheet pole moves out of $K\bar{K}$ and down in the complex plane.

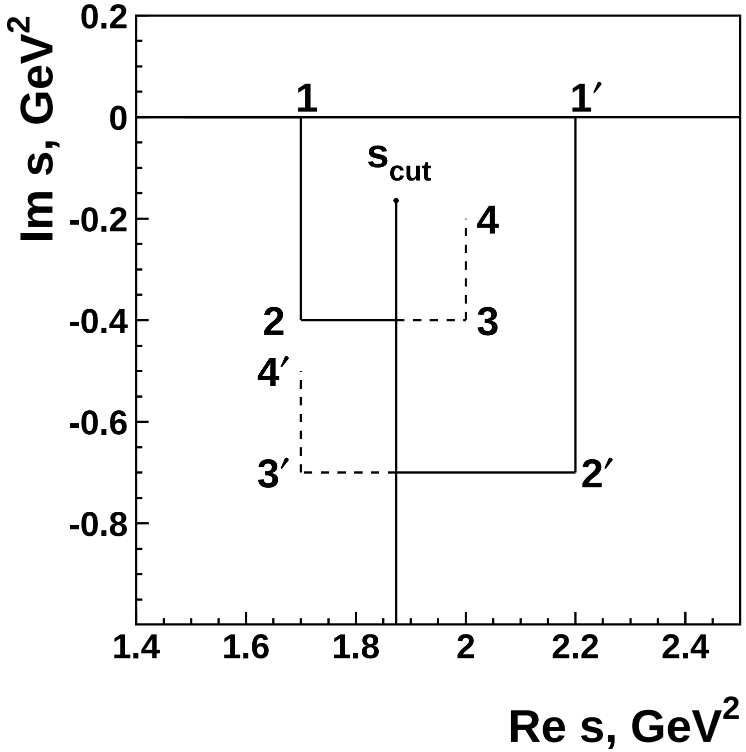
At large $K\bar{K}$ coupling the amplitude squared $|A_{11}|^2$ became effectively more narrow due to fast opening of the $K\bar{K}$ threshold.

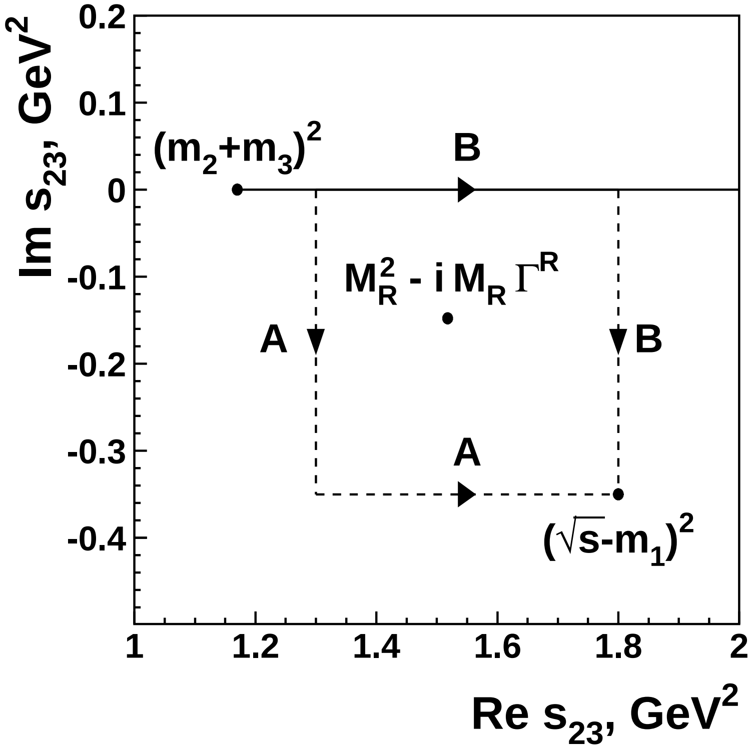


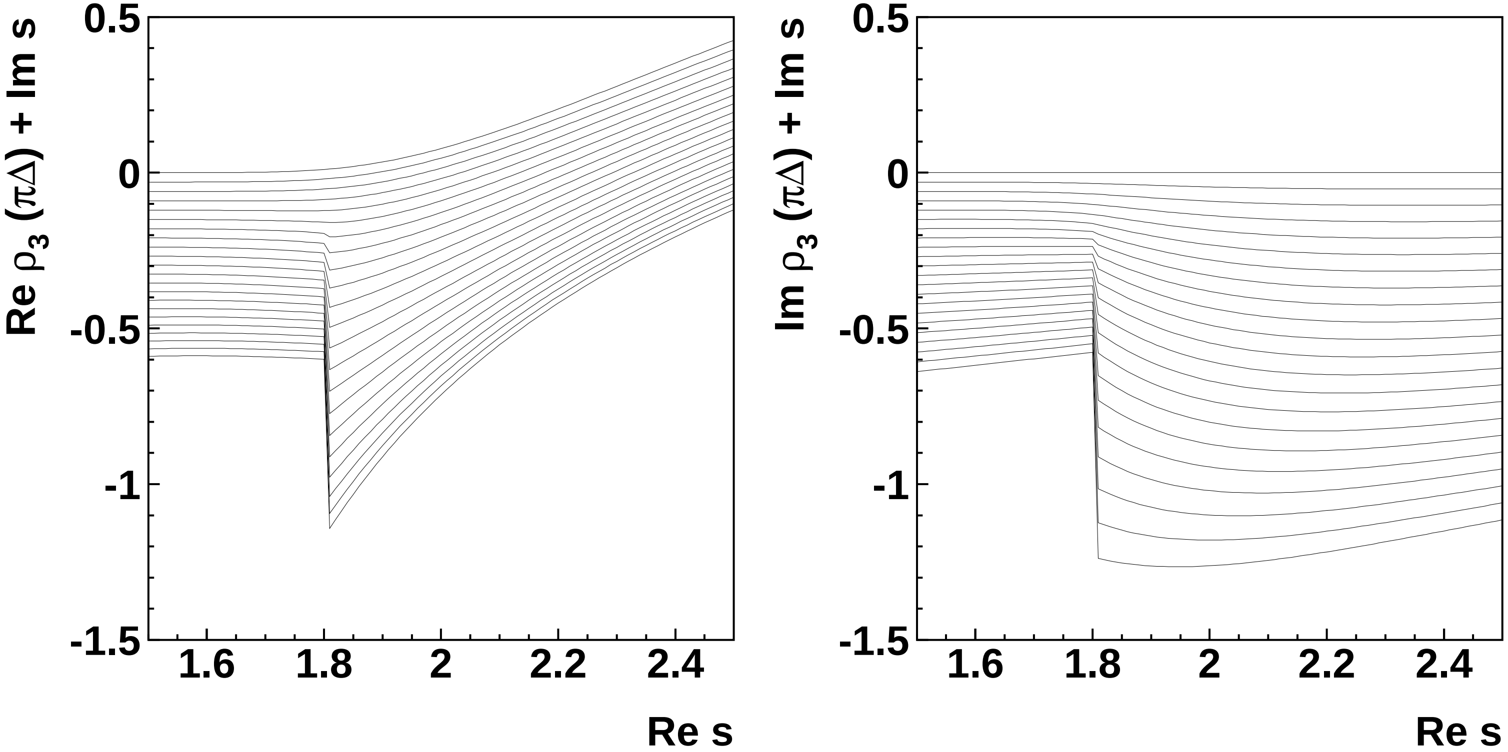
Three body phase volume:

$$\rho_3(s) = \int\limits_{(m_2+m_3)^2}^{(\sqrt{s}-m_1)^2} \frac{ds_{23}}{\pi} \frac{\rho(s,\sqrt{s_{23}},m_1)\, M_R\Gamma^R_{tot}}{(M_R^2-s_{23})^2+(M_R\Gamma^R_{tot})^2} \ ,$$

$$M_R\Gamma^R_{tot} = \rho(s_{23},m_2,m_3)g^2(s_{23}) \ ,$$







Re ρ3 (πΔ) + Im s
0.5
0
-0.5
-1
-1.5
1.6
1.8
2
2.2
2.4
Re s
Im ρ3 (πΔ) + Im s
0.5
0
-0.5
-1
-1.5
1.6
1.8
2
2.2
2.4
Re s

Gauge invariance

The result does not depend on shift: for the photon polarization vector:

$$\varepsilon_\mu^\alpha \to \varepsilon_\mu^\alpha + \chi q_\mu$$

where q_μ is photon momentum and χ is any scalar function.

It means that:

$$\tilde{A} = \varepsilon_\mu^\alpha J_\mu \qquad q_\mu J_\mu = 0$$

For real photon:

$$\varepsilon_\mu^\alpha q_\mu = 0 \quad q_\mu = (q_0, 0, 0, q_z) \quad \varepsilon_\mu^1 = (0,1,0,0), \varepsilon_\mu^2 = (0,0,1,0)$$

Structure of the projection operator for a massive particle:

$$O_{\mu\nu} = \sum_\alpha \varepsilon_\mu^\alpha \varepsilon_\nu^{\alpha *} = g_{\mu\nu} - \frac{P_\mu P_\nu}{M^2}$$

For photon propagator the projection operator only exist for the interaction

$$\gamma(q) + N(k) = N^*(P)$$

$$O^{\gamma}_{\mu\nu} = \sum_{\alpha} \varepsilon^{\alpha}_{\mu} \varepsilon^{\alpha *}_{\nu} = g_{\mu\nu} - \frac{q^{\perp}_{\mu} q^{\perp}_{\nu}}{(q^{\perp})^2} - \frac{P_{\mu} P_{\nu}}{M^2}$$

where

$$q^{\perp}_{\mu} = q_{\nu} \left(g_{\mu\nu} - \frac{P_{\mu} P_{\nu}}{M^2} \right)$$

Therefore:

$$P_{\mu} O^{\gamma}_{\mu\nu} = 0 \qquad q_{\mu} O^{\gamma}_{\mu\nu} = 0$$

The current conservation:

$$\tilde{J}_{\mu} = O^{\gamma}_{\mu\nu} J_{\nu}$$

General structure of the single–meson electro-production amplitude in c.m.s. of the reaction is given by

$$J_\mu = i\mathcal{F}_1 \tilde{\sigma}_\mu + \mathcal{F}_2 (\vec{\sigma}\vec{q}) \frac{\varepsilon_{\mu ij}\sigma_i k_j}{|\vec{k}||\vec{q}|} + i\mathcal{F}_3 \frac{(\vec{\sigma}\vec{k})}{|\vec{k}||\vec{q}|} \tilde{q}_\mu + i\mathcal{F}_4 \frac{(\vec{\sigma}\vec{q})}{\vec{q}^2} \tilde{q}_\mu$$
$$+ i\mathcal{F}_5 \frac{(\vec{\sigma}\vec{k})}{|\vec{k}|^2} k_\mu + i\mathcal{F}_6 \frac{(\vec{\sigma}\vec{q})}{|\vec{q}||\vec{k}|} k_\mu \,,$$

where $\vec{q}$ is the momentum of the nucleon in the πN channel and $\vec{k}$ the momentum of the nucleon in the γN channel calculated in the c.m.s. of the reaction. The σ_i are Pauli matrices.

$$\tilde{\sigma}_\mu = \sigma_\mu - \frac{\vec{\sigma}\vec{k}}{|\vec{k}|^2} k_\mu \qquad \mu = 1, 2, 3$$

$$\tilde{q}_\mu = q_\mu - \frac{\vec{q}\vec{k}}{|\vec{k}||\vec{q}|} k_\mu = q_\mu - z\, k_\mu$$

The functions $\mathcal{F}_i$ have the following angular dependence:

$$
\begin{aligned}
\mathcal{F}_1(z) &= \sum_{L=0}^{\infty} [LM_L^+ + E_L^+]P'_{L+1}(z) + [(L+1)M_L^- + E_L^-]P'_{L-1}(z), \\
\mathcal{F}_2(z) &= \sum_{L=1}^{\infty} [(L+1)M_L^+ + LM_L^-]P'_L(z) \ , \\
\mathcal{F}_3(z) &= \sum_{L=1}^{\infty} [E_L^+ - M_L^+]P''_{L+1}(z) + [E_L^- + M_L^-]P''_{L-1}(z) \ , \\
\mathcal{F}_4(z) &= \sum_{L=2}^{\infty} [M_L^+ - E_L^+ - M_L^- - E_L^-]P''_L(z) \ , \\
\mathcal{F}_5(z) &= \sum_{L=0}^{\infty} [(L+1)S_L^+ P'_{L+1}(z) - LS_L^- P'_{L-1}(z)] \ , \\
\mathcal{F}_6(z) &= \sum_{L=1}^{\infty} [LS_L^- - (L+1)S_L^+]P'_L(z)
\end{aligned}
$$

Here L corresponds to the orbital angular momentum in the πN system, $P'_L(z)$, $P''_L(z)$ are derivatives of Legendre polynomials $z = (\vec{k}\vec{q})/(|\vec{k}||\vec{q}|)$.

γN interaction

Photon has quantum numbers $J^{PC} = 1^{--}$, proton $1/2^+$. Then in S-wave two states can be formed is $1/2^-$ and $3/2^-$.

Then P-wave $1/2^+, 3/2^+$ and $1/2^+, 3/2^+, 5/2^+$.

In general case: $1/2^-$, $1/2^+$ are described by two amplitudes and higher states by three amplitudes.

$$
\begin{aligned}
V^{(1+)\mu}_{\alpha_1\ldots\alpha_n} &= \gamma_\mu i\gamma_5 X^{(n)}_{\alpha_1\ldots\alpha_n} \,, & V^{(1-)\mu}_{\alpha_1\ldots\alpha_n} &= \gamma_\xi \gamma_\mu X^{(n+1)}_{\xi\alpha_1\ldots\alpha_n} \,, \\
V^{(2+)\mu}_{\alpha_1\ldots\alpha_n} &= \gamma_\nu i\gamma_5 X^{(n+2)}_{\mu\nu\alpha_1\ldots\alpha_n} \,, & V^{(2-)\mu}_{\alpha_1\ldots\alpha_n} &= X^{(n+1)}_{\mu\alpha_1\ldots\alpha_n} \,, \\
V^{(3+)\mu}_{\alpha_1\ldots\alpha_n} &= \gamma_\nu i\gamma_5 X^{(n+1)}_{\nu\alpha_1\ldots\alpha_n} g^{\perp}_{\mu\alpha_n} \,, & V^{(3-)\mu}_{\alpha_1\ldots\alpha_n} &= X^{(n-1)}_{\alpha_2\ldots\alpha_n} g^{\perp}_{\alpha_1\mu} \,.
\end{aligned}
$$

Gauge invariance: $\varepsilon_\mu q_{1\mu} = 0$ where q_1-photon momentum.

$$
\varepsilon_\mu V^{(2\pm)\mu}_{\alpha_1\ldots\alpha_n} = C^{\pm} \varepsilon_\mu V^{(3\pm)\mu}_{\alpha_1\ldots\alpha_n}
$$

where $C^{\pm}$ do not depend on angles.

The functions $\mathcal{F}_i$ have the following angular dependence:

$$
\begin{aligned}
\mathcal{F}_1(z) &= \sum_{L=0}^{\infty} [LM_L^+ + E_L^+]P'_{L+1}(z) + [(L+1)M_L^- + E_L^-]P'_{L-1}(z), \\
\mathcal{F}_2(z) &= \sum_{L=1}^{\infty} [(L+1)M_L^+ + LM_L^-]P'_L(z) \ , \\
\mathcal{F}_3(z) &= \sum_{L=1}^{\infty} [E_L^+ - M_L^+]P''_{L+1}(z) + [E_L^- + M_L^-]P''_{L-1}(z) \ , \\
\mathcal{F}_4(z) &= \sum_{L=2}^{\infty} [M_L^+ - E_L^+ - M_L^- - E_L^-]P''_L(z) \ , \\
\mathcal{F}_5(z) &= \sum_{L=0}^{\infty} [(L+1)S_L^+ P'_{L+1}(z) - LS_L^- P'_{L-1}(z)] \ , \\
\mathcal{F}_6(z) &= \sum_{L=1}^{\infty} [LS_L^- - (L+1)S_L^+]P'_L(z)
\end{aligned}
$$

Here L corresponds to the orbital angular momentum in the πN system, $P'_L(z)$, $P''_L(z)$ are derivatives of Legendre polynomials $z = (\vec{k}\vec{q})/(|\vec{k}||\vec{q}|)$.

For the positive states $J = L + 1/2 \;\; (L\!=\!n)$:

$$A_\mu^{i+} \;\;=\;\; \bar{u}(q_N) X^{(n)}_{\alpha_1\ldots\alpha_n}(q^\perp) F^{\alpha_1\ldots\alpha_n}_{\beta_1\ldots\beta_n} V^{(i+)\mu}_{\beta_1\ldots\beta_n}(k^\perp) u(k_N)$$

$$\begin{aligned}
\mathcal{F}_1^{1+} &= \lambda_n\, P'_{n+1} \\
\mathcal{F}_2^{1+} &= \lambda_n\, P'_n \\
\mathcal{F}_3^{1+} &= 0 \\
\mathcal{F}_4^{1+} &= 0 \\
\mathcal{F}_5^{1+} &= +\lambda_n\, P'_{n+1} \\
\mathcal{F}_6^{1+} &= -\lambda_n\, P'_n
\end{aligned}$$

where

$$\lambda_n = \frac{\alpha_n}{2n+1}(|\vec{k}||\vec{q}|)^n \chi_i \chi_f \qquad \chi_{i,f} = \sqrt{m_{i,f} + k_{0i,f}}$$

Therefore

$$E_n^{1+} = M_n^{1+} = S_n^{1+} = \frac{\lambda_n}{n+1}$$

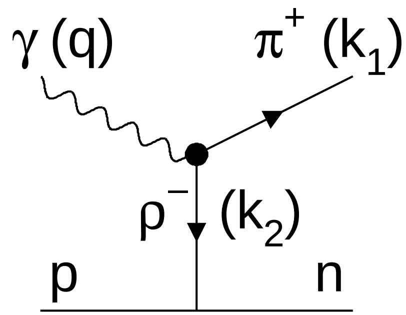
The correspondence of the vertices and multipoles ($J = n + \frac{1}{2}$):

	E	M	S
V_n^{1+}	$\frac{\lambda_n}{n+1}$	$\frac{\lambda_n}{n+1}$	$\frac{\lambda_n}{n+1}$
V_n^{2+}	$\frac{\lambda_n}{n+1}$	$-\frac{\lambda_n}{n(n+1)}$	$\frac{\lambda_n}{n+1}$
V_n^{3+}	ξ^n	**0**	$-\xi_n \frac{n+2}{n+1}$
V_n^{1-}	$-\frac{\zeta_{n+1}}{n+1}$	$\frac{\zeta_{n+1}}{n+1}$	$-\frac{\zeta_{n+1}}{n+1}$
V_n^{2-}	$-\Delta_n$	**0**	$-\Delta_n \frac{2n^2}{n+1}$
V_n^{3-}	$-\varrho_{n-1}$	**0**	$\varrho_{n-1} \frac{n-1}{n}$

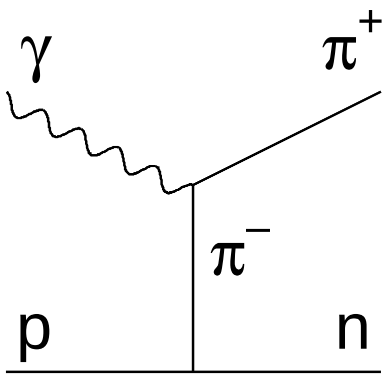
$$\lambda_n = \frac{\alpha_n}{2n+1}(|\vec{k}||\vec{q}|)^n \chi_i \chi_f \qquad \Delta_n = \frac{\alpha_n}{n(n+1)^2}(|\vec{k}||\vec{q}|)^{n+1} \chi_i \chi_f$$

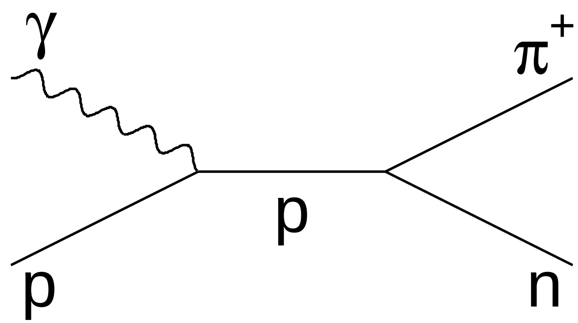
$$\zeta_n = \frac{\alpha_n}{n}(|\vec{k}||\vec{q}|)^n \chi_i \chi_f \qquad \varrho_n = \frac{\alpha_n}{(n+1)(n+2)}|\vec{k}|^n |\vec{q}|^{n+2} \chi_i \chi_f$$

$$\xi_n = \frac{\alpha_n}{(n+2)(n+1)}|\vec{k}|^{n+2} |\vec{q}|^n \chi_i \chi_f$$



$$J_\mu = \varepsilon_{\mu\nu\alpha\beta} k_{1\nu} k_{2\alpha} q_\beta$$





$$q_\mu \left[\frac{(q-p_\pi)^\mu \hat{q} \gamma^5}{t - m_\pi^2} - 2 m_N \gamma^5 \frac{\hat{p_s} + m_N}{s - m_N^2} \gamma^\mu \right] = 0$$

Partial wave amplitude:

transition amplitude with fixed initial and final states

Quantum numbers: mesons $I^G\, J^{PC}$, baryons: $I\, J^P$, decay LS basis: $^{2S+1}L_J$

$$I_1^{G_1} J_1^{P_1 C_1} + I_2^{G_2} J_2^{P_2 C_2} \left(^{2S+1}L_J\right) \to I^G J^{PC} \to I_1'^{\,G_1'} J_1'^{\,P_1' C_1'} + I_2'^{\,G_2'} J_2'^{\,P_2' C_2'} \left(^{2S'+1}L_J'\right)$$

$$G = G_1 G_2 \qquad G = G_1' G_2'$$

$$P = P_1 P_2 (-1)^L \qquad P = P_1' P_2' (-1)^{L'}$$

$$|I_1 - I_2| < I < I_1 + I_2 \qquad |I_1' - I_2'| < I < I_1' + I_2'$$

$$|J_1 - J_2| < S < J_1 + J_2 \qquad |J_1' - J_2'| < S' < J_1' + J_2'$$

$$|S - L| < J < S + L \qquad |S' - L'| < J < S' + L'$$

$$A(s,t) = V_{\mu_1 \ldots \mu_n}(S,L)\, P^{\mu_1 \ldots \mu_n}_{\nu_1 \ldots \nu_n}\, V'_{\nu_1 \ldots \nu_n}(S',L') A(s)$$

$n = J$ **mesons** $\qquad n = J - 1/2$ **baryons**

In momentum representation the particle with spin J ($n = J$):

$$\Psi_{\mu_1\ldots\mu_n} = \frac{1}{\sqrt{2p_0}} u_{\mu_1\ldots\mu_n} e^{ipx}$$

The spinor function $u_{\mu_1\ldots\mu_n}$ satisfies:

$$p^2 u_{\mu_1\mu_2\ldots\mu_n} = m^2 u_{\mu_1\mu_2\ldots\mu_n}$$

$$p_{\mu_i} u_{\mu_1\mu_2\ldots\mu_n} = 0$$

$$g_{\mu_i\mu_j} u_{\mu_1\mu_2\ldots\mu_n} = 0$$

$$u_{\mu_1\ldots\mu_i\ldots\mu_j\ldots\mu_n} = u_{\mu_1\ldots\mu_j\ldots\mu_i\ldots\mu_n}$$

These conditions are the main basis for construction of projection operators.

1 Boson projection operators

In momentum representation:

$$P^{\mu_1\mu_2\dots\mu_n}_{\nu_1\nu_2\dots\nu_n} = (-1)^n O^{\mu_1\mu_2\dots\mu_n}_{\nu_1\nu_2\dots\nu_n} = \sum_{i=1}^{2n+1} u^{(i)}_{\mu_1\mu_2\dots\mu_n} u^{(i)*}_{\nu_1\nu_2\dots\nu_n}$$

The projection operator can depends only on the total momentum and the metric tensor.

For spin 0 it is a unit operator. For spin 1 the only possible combination is:

$$O^{\mu}_{\nu} = g^{\perp}_{\mu\nu} = g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}$$

The propagator for the particle with spin $S > 2$ must be constructed from the tensors $g^{\perp}_{\mu\nu}$: this is the only combination which satisfies:

$$p_\mu g^{\perp}_{\mu\nu} = 0.$$

Then for spin 2 state we obtain:

$$O^{\mu_1\mu_2}_{\nu_1\nu_2} = \frac{1}{2}(g^{\perp}_{\mu_1\nu_1} g^{\perp}_{\mu_2\nu_2} + g^{\perp}_{\mu_1\nu_2} g^{\perp}_{\mu_2\nu_1}) - \frac{1}{3} g^{\perp}_{\mu_1\mu_2} g^{\perp}_{\nu_1\nu_2}$$

Recurrent expression for the boson projector operator

$$O^{\mu_1\dots\mu_L}_{\nu_1\dots\nu_L} = \frac{1}{L^2}\left(\sum_{i,j=1}^{L} g^{\perp}_{\mu_i\nu_j} O^{\mu_1\dots\mu_{i-1}\mu_{i+1}\dots\mu_L}_{\nu_1\dots\nu_{j-1}\nu j+1\dots\nu_L} - \right.$$

$$\left. \frac{4}{(2L-1)(2L-3)} \sum_{i<j,k<m}^{L} g^{\perp}_{\mu_i\mu_j} g^{\perp}_{\nu_k\nu_m} O^{\mu_1\dots\mu_{i-1}\mu_{i+1}\dots\mu_{j-1}\mu_{j+1}\dots\mu_L}_{\nu_1\dots\nu_{k-1}\nu_{k+1}\dots\nu_{m-1}\nu_{m+1}\dots\nu_L}\right)$$

Normalization condition:

$$O^{\mu_1\dots\mu_L}_{\nu_1\dots\nu_L} O^{\nu_1\dots\nu_L}_{\alpha_1\dots\alpha_L} = O^{\mu_1\dots\mu_L}_{\alpha_1\dots\alpha_L}$$

Orbital momentum operator

The angular momentum operator is constructed from momenta of particles k_1, k_2 and metric tensor $g_{\mu\nu}$.

For $L = 0$ this operator is a constant: $X^0 = 1$

The $L = 1$ operator is a vector $X^{(1)}_\mu$, constructed from: $k_\mu = \frac{1}{2}(k_{1\mu} - k_{2\mu})$ and $P_\mu = (k_{1\mu} + k_{2\mu})$. Orthogonality:

$$\int \frac{d^4k}{4\pi} X^{(1)}_{\mu_1} X^{(0)} = \int \frac{d^4k}{4\pi} X^{(n)}_{\mu_1\ldots\mu_n} X^{(n-1)}_{\mu_2\ldots\mu_n} = \xi P_{\mu_1} = 0$$

Then:

$$X^{(1)}_\mu P_\mu = 0 \qquad\qquad X^{(n)}_{\mu_1\ldots\mu_n} P_{\mu_j} = 0$$

and:

$$X^{(1)}_\mu = k^\perp_\mu = k_\nu g^\perp_{\nu\mu}; \qquad\qquad g^\perp_{\nu\mu} = \left(g_{\nu\mu} - \frac{P_\nu P_\nu}{p^2} \right) \;;$$

in c.m.s $k^\perp = (0, \vec{k})$

$$\int \frac{d^4k}{4\pi} X^{(n)}_{\mu_1\ldots\mu_n} X^{(n-2)}_{\mu_3\ldots\mu_n} = \beta g^{\perp}_{\mu_1\mu_2} = 0$$

The orthogonality and symmetry properties can be written as the set of following conditions:

1. $X^{(n)}_{\mu_1\ldots\mu_i\ldots\mu_j\ldots\mu_n} = X^{(n)}_{\mu_1\ldots\mu_j\ldots\mu_i\ldots\mu_n}$ **(symmetry)**
2. $P_{\mu_i} X^{(n)}_{\mu_1\ldots\mu_i\ldots\mu_n} = 0$ **(P-orthogonality)**
3. $g_{\mu_1\mu_2} X^{(n)}_{\mu_1\mu_2\ldots\mu_n} = 0$ **(tracelessness)**

For low orbital momenta:

$$X^0 = 1\,; \quad X^1_\mu = k^\perp_\mu\,; \quad X^2_{\mu\nu} = \frac{3}{2}\left(k^\perp_\mu k^\perp_\nu - \frac{1}{3}\,k^2_\perp g^\perp_{\mu\nu}\right);$$

$$X^3_{\mu\nu\alpha} = \frac{5}{2}\left[k^\perp_\mu k^\perp_\nu k^\perp_\alpha - \frac{k^2_\perp}{5}\left(g^\perp_{\mu\nu}k^\perp_\alpha + g^\perp_{\mu\alpha}k^\perp_\nu + g_{\nu\alpha}k^\perp_\mu\right)\right],$$

Recurrent expression for the orbital momentum operators $X^{(n)}_{\mu_1\ldots\mu_n}$

$$X^{(n)}_{\mu_1\ldots\mu_n} = \frac{2n-1}{n^2}\sum_{i=1}^{n} k^{\perp}_{\mu_i} X^{(n-1)}_{\mu_1\ldots\mu_{i-1}\mu_{i+1}\ldots\mu_n} - \frac{2k^2_{\perp}}{n^2}\sum_{\substack{i,j=1\\ i<j}}^{n} g_{\mu_i\mu_j} X^{(n-2)}_{\mu_1\ldots\mu_{i-1}\mu_{i+1}\ldots\mu_{j-1}\mu_{j+1}\ldots\mu_n}$$

Taking into account the traceless property of $X^{(n)}$ we have:

$$X^{(n)}_{\mu_1\ldots\mu_n} X^{(n)}_{\mu_1\ldots\mu_n} = \alpha(n)(k^2_{\perp})^n \qquad \alpha(n) = \prod_{i=1}^{n}\frac{2i-1}{i} = \frac{(2n-1)!!}{n!}.$$

From the recursive procedure one can get the following expression for the operator $X^{(n)}$:

$$X^{(n)}_{\mu_1\ldots\mu_n} = \alpha(n)\Bigg[k^{\perp}_{\mu_1}k^{\perp}_{\mu_2}\ldots k^{\perp}_{\mu_n} - \frac{k^2_{\perp}}{2n-1}\Big(g^{\perp}_{\mu_1\mu_2}k^{\perp}_{\mu_3}\ldots k^{\perp}_{\mu_n}+\cdots\Big) + \frac{k^4_{\perp}}{(2n-1)(2n-3)}\Big(g^{\perp}_{\mu_1\mu_2}g^{\perp}_{\mu_3\mu_4}k^{\perp}_{\mu_5}\cdots k_{\mu_4}+\cdots\Big)+\cdots\Bigg].$$

Scattering of two spinless particles

Denote relative momenta of particles before and after interaction as q and k, correspondingly. The structure of partial–wave amplitude with orbital momentum $L=J$ is determined by convolution of operators $X^{(L)}(k)$ and $X^{(L)}(q)$:

$$A_L = BW_L(s) X^{(L)}_{\mu_1\ldots\mu_L}(k) O^{\mu_1\ldots\mu_L}_{\nu_1\ldots\nu_L} X^{(L)}_{\nu_1\ldots\nu_L}(q) = BW_L(s) X^{(L)}_{\mu_1\ldots\mu_L}(k) X^{(L)}_{\mu_1\ldots\mu_L}(q)$$

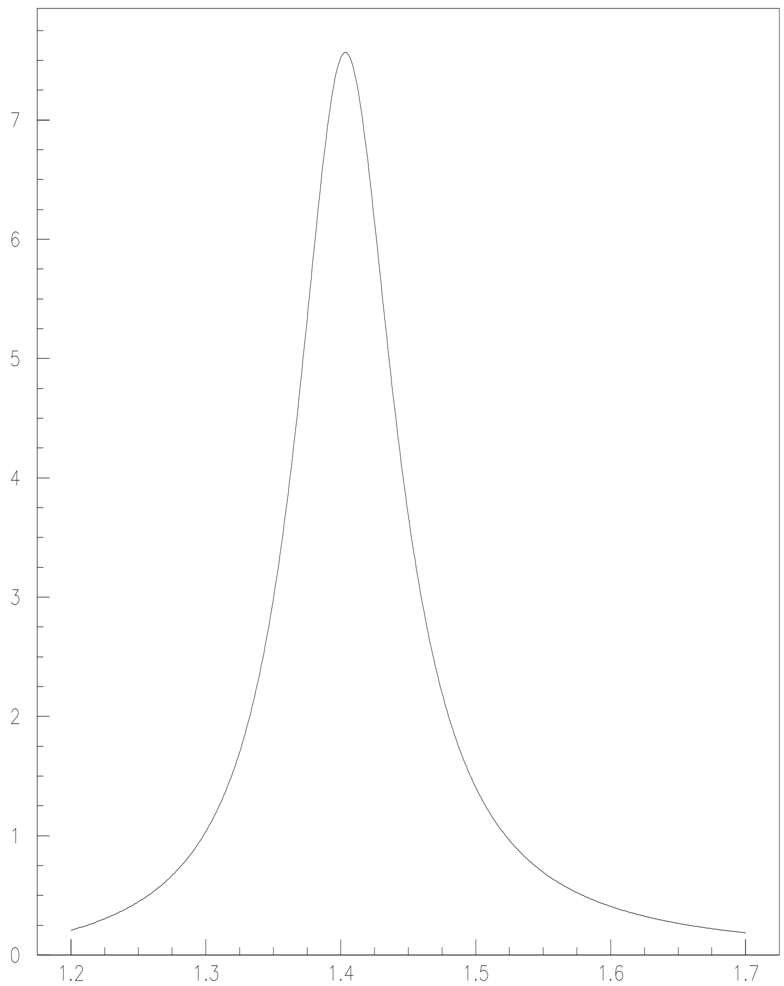
$BW_L(s)$ depends on the total energy squared only.

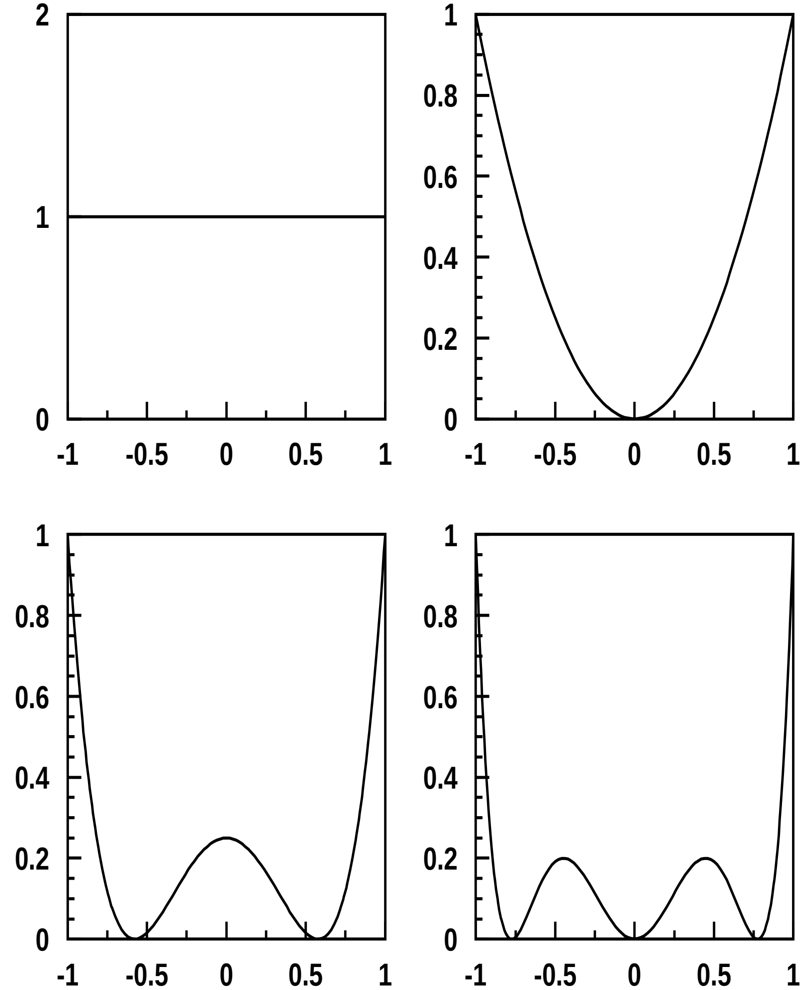
The convolution $X^{(L)}_{\mu_1\ldots\mu_L}(k) X^{(L)}_{\mu_1\ldots\mu_L}(q)$ can be written in terms of Legendre polynomials $P_L(z)$:

$$X^{(L)}_{\mu_1\ldots\mu_L}(k) X^{(L)}_{\mu_1\ldots\mu_L}(q) = \alpha(L) \left(\sqrt{k^2_\perp}\sqrt{q^2_\perp}\right)^L P_L(z) \,,$$

$$z = \frac{(k^\perp q^\perp)}{\sqrt{k^2_\perp}\sqrt{q^2_\perp}} \qquad \alpha(L) = \prod_{n=1}^{L} \frac{2n-1}{n}$$

Angular dependence of $|A_L|^2$ for $J = L = 0, 1, 2, 3$ states.





Structure of fermion propagator

The orthogonality condition has a different form in a fermion case:

$$\int \Psi_\mu(x)\Psi^*(x)d^4x = A\,p_\mu + B\,\gamma_\mu = 0$$

where A and B are matrices in spinor space.

It means that we have an additional condition:

$$\gamma_\mu \Psi_\mu = 0 \qquad \gamma_\mu u_\mu = 0 \qquad \Psi_\mu = \frac{1}{\sqrt{2p_0}} u_\mu e^{ipx}$$

$$\gamma_0 = \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \qquad \vec{\gamma} = \beta\vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}, \qquad \gamma_5 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix}$$

Here $\vec{\sigma}$ are 2×2 Pauly matrices:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$u^{(i)} = \frac{1}{\sqrt{p_0+m}} \begin{pmatrix} (p_0+m)\omega^{(i)} \\ (\vec{p}\vec{\sigma})\omega^{(i)} \end{pmatrix} \qquad \omega^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \omega^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\bar{u}^{(i)} = \frac{\left((p_0+m)\omega^{(i)*}, -(\vec{p}\vec{\sigma})\omega^{(i)*}\right)}{\sqrt{p_0+m}}$$

Summing over polarizations we obtain:

$$\sum_{i=1}^{2} u^{(i)}\bar{u}^{(i)} = m + \hat{p} \qquad \hat{p} = p_\mu\gamma_\mu$$

Orthogonality conditions for $J = n + \frac{1}{2}$ spinors:

$$(\hat{p}-m)u_{\mu_1\ldots\mu_n} = 0 \qquad \hat{p} = p_\mu\gamma_\mu$$

$$p_{\mu_i}u_{\mu_1\ldots\mu_n} = 0$$

$$u_{\mu_1\ldots\mu_i\ldots\mu_j\ldots\mu_n} = u_{\mu_1\ldots\mu_j\ldots\mu_i\ldots\mu_n}$$

$$g_{\mu_i\mu_j}u_{\mu_1\ldots\mu_n} = 0$$

$$\gamma_{\mu_i}u_{\mu_1\ldots\mu_n} = 0$$

These properties define structure of the fermion projection operator $P^{\mu_1\ldots\mu_n}_{\nu_1\ldots\nu_n}$:

$$G^{\mu_1\ldots\mu_n}_{\nu_1\ldots\nu_n} = (-1)^n \frac{m+\hat{p}}{m^2-p^2} F^{\mu_1\ldots\mu_n}_{\nu_1\ldots\nu_n}$$

The boson projector operator projects any operator to one which satisfies all boson properties. It means that we can write:

$$F^{\mu_1\ldots\mu_n}_{\nu_1\ldots\nu_n} = O^{\mu_1\ldots\mu_n}_{\alpha_1\ldots\alpha_n} T^{\alpha_1\ldots\alpha_n}_{\beta_1\ldots\beta_n} O^{\beta_1\ldots\beta_n}_{\nu_1\ldots\nu_n}$$

T-operator should be constructed from the metric tensor and γ-matrices.

$$\gamma_{\alpha_i}\gamma_{\alpha_j} = \frac{1}{2} g_{\alpha_i\alpha_j} + \sigma_{\alpha_i\alpha_j}, \qquad \text{where} \qquad \sigma_{\alpha_i\alpha_j} = \frac{1}{2}(\gamma_{\alpha_i}\gamma_{\alpha_j} - \gamma_{\alpha_j}\gamma_{\alpha_i})$$

$$\gamma_{\alpha_i}\gamma_{\alpha_j} O^{\ldots\alpha_i\ldots\alpha_j\ldots}_{\beta_1\beta_2\ldots} = 0$$

Therefore, the only nonzero structure is:

$$O^{\mu_1\mu_2\ldots}_{\alpha_1\alpha_2\ldots}\gamma_{\alpha_i}\gamma_{\beta_j} O^{\beta_1\beta_2\ldots}_{\nu_1\nu_2\ldots}$$

Then:

$$F^{\mu_1\ldots\mu_n}_{\nu_1\ldots\nu_n} = O^{\mu_1\ldots\mu_n}_{\alpha_1\ldots\alpha_n} T^{\alpha_1\ldots\alpha_n}_{\beta_1\ldots\beta_n} O^{\beta_1\ldots\beta_n}_{\nu_1\ldots\nu_n}$$

$$T^{\alpha_1\ldots\alpha_n}_{\beta_1\ldots\beta_n} = \frac{n+1}{2n+1}\left(g_{\alpha_1\beta_1} - \frac{n}{n+1}\sigma_{\alpha_1\beta_1}\right)\prod_{i=2}^{n} g_{\alpha_i\beta_i}$$

$$J = 1/2 \quad P = 1$$

$$J = 3/2 \quad P^{\mu}_{\nu} = \frac{1}{2}\Big(g^{\perp}_{\mu\nu} - \gamma^{\perp}_{\mu}\gamma^{\perp}_{\nu}/3\Big) \qquad \text{where} \qquad \gamma^{\perp}_{\mu} = g^{\perp}_{\mu\nu}\gamma_{\nu}.$$

πN interaction

Pion has quantum numbers $J^{PC} = 0^{-+}$, proton $1/2^+$. Then in S-wave the only state can be formed is $1/2^-$. P-wave can form two states $1/2^+$ and $3/2^+$.

In PDG review, states are defined by quantum numbers from the πN decay: $L_{2I,2J}$. For example D_{13} means $3/2^-$ N^* state.

States with $J = L - 1/2$ are called '–' states ($1/2^+$, $3/2^-$,$5/2^+$,$\ldots$) and states with $J = L + 1/2$ are called '+' states ($1/2^-$, $3/2^+$,$5/2^-$,$\ldots$).

For '+' states:

$$N^{+}_{\mu_1\ldots\mu_n} = X^{(n)}_{\mu_1\ldots\mu_n}$$

and for '-' states:

$$N^{-}_{\mu_1\ldots\mu_n} = i\gamma_\nu\gamma_5 X^{(n+1)}_{\nu\mu_1\ldots\mu_n}$$

$$A_{\pi N} = \bar{u}(k_1) N^{*\pm}_{\mu_1\ldots\mu_n} F^{\mu_1\ldots\mu_n}_{\nu_1\ldots\nu_n}(P) N^{\pm}_{\nu_1\ldots\nu_n} u(q_1) BW^{\pm}_{\substack{n\\n+1}}(s)$$

In c.m.s. of the reaction

$$A_{\pi N} = \omega^* \left[G(s,t) + H(s,t) i(\vec{\sigma}\vec{n})\right] \omega' \qquad n_i = \frac{1}{|\vec{k}||\vec{q}|} \epsilon_{ijm} k_j q_m \ ,$$

$$G(s,t) = \sum_L \left[(L+1) F_L^+(s) - L F_L^-(s)\right] P_L(z) \ ,$$

$$H(s,t) = \sum_L \left[F_L^+(s) + F_L^-(s)\right] P_L'(z) \ .$$

$$F_L^+ = (-1)^{L+1} (|\vec{k}||\vec{q}|)^L \sqrt{\chi_i \chi_f} \ \frac{\alpha(L)}{2L+1} BW_L^+(s) \ ,$$

$$F_L^- = (-1)^L (|\vec{k}||\vec{q}|)^L \sqrt{\chi_i \chi_f} \ \frac{\alpha(L)}{L} BW_L^-(s) \ .$$

$$\chi_i = m_i + k_{i0} \qquad \alpha(L) = \prod_{l=1}^{L} \frac{2l-1}{l} = \frac{(2L-1)!!}{L!} \ .$$

NN - scattering

Transition of two baryons with momenta p_1 and p_2 into two baryons with p'_1 and p'_2, $s=(p_1+p_2)^2=(p'_1+p'_2)^2$, $k=p_1-p_2$, $k'=p'_1-p'_2$. Two baryons with $J^P=\frac{1}{2}^+$ can have spin states $S=0,1$.

$$A=\left(\bar{u}(p'_1)V^{S',L'}_{\mu_1\ldots\mu_J}(k'_\perp)u^c(-p'_2)\right)O^{\mu_1\ldots\mu_n}_{\nu_1\ldots\nu_n}\left(\bar{u}^c(-p_2)V^{S,L}_{\nu_1\ldots\nu_J}(k_\perp)u(p_1)\right)A_{pw}(s)\,.$$

$$u^c_j(-p)\;=\;C\bar{u}^T_j(p)\qquad C=\gamma_2\gamma_0=\begin{pmatrix}0 & -\sigma_2\\ -\sigma_2 & 0\end{pmatrix}$$

Vertex operators:

$$V^{0,L}_{\mu_1\ldots\mu_J}=i\gamma_5X^{(J)}_{\mu_1\ldots\mu_J}(k^\perp)\qquad V^{1,L<J}_{\mu_1\ldots\mu_J}=\gamma_{\mu_1}X^{(n-1)}_{\mu_2\ldots\mu_J}(k^\perp)$$

$$V^{1,L=J}_{\mu_1\ldots\mu_J}=\varepsilon_{\mu_1\eta\xi\gamma}\gamma_\eta X^{(J)}_{\xi\mu_2\ldots\nu_J}(k^\perp)P_\gamma\qquad V^{1,L>J}_{\mu_1\ldots\mu_J}=\gamma_\alpha X_{\alpha\mu_1\ldots\mu_J}(k^\perp)$$

2 The cross section for photoproduction processes

The differential cross section for production of two or more particles has the form:

$$d\sigma = \frac{(2\pi)^4|A|^2}{4\sqrt{(k_1k_2)^2 - m_1^2m_2^2}} d\Phi_n(k_1 + k_2, q_1, \ldots, q_n) \ ,$$

where k_1 and k_2 are momenta of the initial particles (nucleon and γ in the case of photoproduction) and q_i are momenta of final state particles. The $d\Phi_n(k_1 + k_2, q_1, \ldots, q_n)$ is the element of the n-body phase volume given by

$$d\Phi_n(k_1 + k_2, q_1, \ldots, q_n) \quad = \quad \delta^4(k_1 + k_2 - \sum_{i=1}^{n} q_i) \prod_{i=1}^{n} \frac{d^3q_i}{(2\pi)^3 2q_{0i}} \ .$$

The photoproduction amplitude can be written as

$$A = \varepsilon_\mu \bar{u}_i A_\mu u_f \ ,$$

where ε_μ is the γ polarization vector and $\bar{u}_i$ and u_f are the bispinors of the initial and final state nucleon.

If particle polarizations are not known the amplitude squared is summed over polarizations of final particles and averaged over polarization of initial particles.

$$|A|^2 = \frac{1}{4}\sum_{\alpha j m} Tr\left[\varepsilon_\mu^{*\alpha}\varepsilon_\nu^\alpha \bar{u}_i^{(j)} A_\mu^* u_f^{(m)} \bar{u}_f^{(m)} A_\nu u_i^{(j)}\right] \ ,$$

$$\sum_{i=1}^{2} u^{(i)}(k_1)\bar{u}^{(i)}(k_1) = m + \hat{k}_1 \to (m+\hat{k}_1)\left(1 - i\gamma_5\hat{S}\right)$$

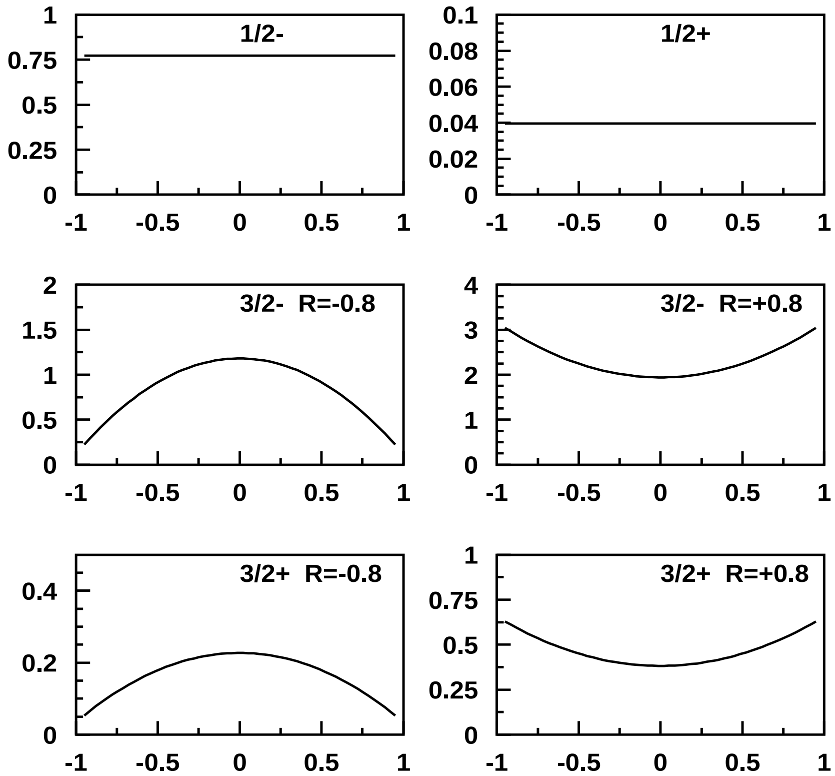
In the case of photon with momentum directed along z-axis

$$\varepsilon_\mu^1 = (0;1,0,0) \qquad\qquad \varepsilon_\mu^2 = (0;0,1,0)$$

For non-polarized case:

$$\frac{1}{2}\sum_\alpha \varepsilon_\nu^{*\alpha}\varepsilon_\mu^\alpha \quad = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \overset{\substack{Polarization \\ along\ y\ axis}}{\longrightarrow} \quad \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \ .$$

Single meson photoproduction



1) Meson-meson scattering:

only one observable is measured

2) The πN elastic scattering:

3 observables should be measured for

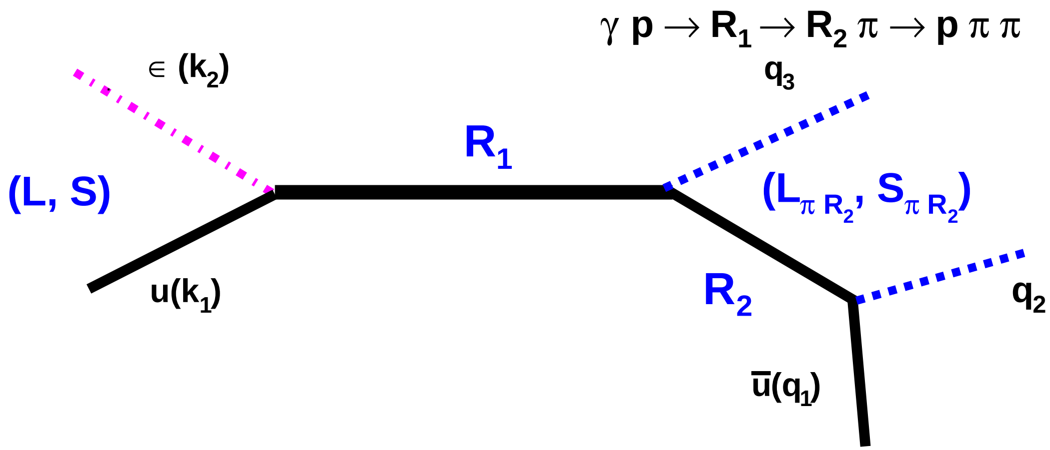
a complete experiment.

3) Meson photoproduction experiment:

8 observables should be measured for

a complete experiment.

The resonance amplitudes for meson photoproduction



The general form of the angular dependent part of the amplitude:

$$\bar{u}(q_1)\tilde{N}_{\alpha_1\ldots\alpha_n}(R_2\to\mu N)F^{\alpha_1\ldots\alpha_n}_{\beta_1\ldots\beta_n}(q_1+q_2)\tilde{N}^{(j)\beta_1\ldots\beta_n}_{\gamma_1\ldots\gamma_m}(R_1\to\mu R_2)$$
$$F^{\gamma_1\ldots\gamma_m}_{\xi_1\ldots\xi_m}(P)V^{(i)\mu}_{\xi_1\ldots\xi_m}(R_1\to\gamma N)u(k_1)\varepsilon_\mu$$

$$F^{\mu_1\ldots\mu_L}_{\nu_1\ldots\nu_L}(p)=(m+\hat{p})O^{\mu_1\ldots\mu_L}_{\alpha_1\ldots\alpha_L}\frac{L+1}{2L+1}\left(g^{\perp}_{\alpha_1\beta_1}-\frac{L}{L+1}\sigma_{\alpha_1\beta_1}\right)\prod_{i=2}^{L}g_{\alpha_i\beta_i}O^{\beta_1\ldots\beta_L}_{\nu_1\ldots\nu_L}$$

$$\sigma_{\alpha_i\alpha_j}=\frac{1}{2}(\gamma_{\alpha_i}\gamma_{\alpha_j}-\gamma_{\alpha_j}\gamma_{\alpha_i})$$