Bonn-Gatchina partial wave analysis: single energy fit

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Main approach: The energy dependent analysis of the data

In many cases an unambiguous partial wave decomposition at fixed energies is impossible. Then the energy and angular parts should be analyzed together:

$$A(s,t) \quad = \quad \sum_{\beta\beta' n} A_n^{\beta\beta'}(s) Q^{(\beta)+}_{\mu_1\ldots\mu_n} F^{\mu_1\ldots\mu_n}_{\nu_1\ldots\nu_n} Q^{(\beta')}_{\nu_1\ldots\nu_n}$$

1. **Correlations between angular part and energy part are under control.**
2. **Unitarity and analyticity can be introduced from the beginning.**
3. **Three or four body final state + FSI (triangle and box diagrams)**
4. **However, to fix simultaneously energy and angular dependencies of the amplitude a combined fit of many reactions is needed.**

Single energy fit

1. **First step: decomposition into partial amplitudes (model independent)**

   Limitations:

   (a) **The unique solution demands high precision polarization data.**

   (b) **The number of polarization measurements should correspond to a complete experiment.**

   (c) **Very difficult to apply to reactions with three or more final states.**

   But:

   i. **Unique decomposition of partial waves.**

   ii. **In many cases the resonance contributions are clearly seen.**

2. **Next step: analysis of the partial amplitudes and search for resonances (model dependent)**

Photoproduction amplitude in c.m.s. of the reaction

$$A = \sum_i u(k_1) V^{*(i\pm)\mu}_{\alpha_1\ldots\alpha_n} F^{\beta_1\ldots\beta_n}_{\alpha_1\ldots\alpha_n} N^{(\pm)}_{\beta_1\ldots\beta_n} u(q_1) \varepsilon_\mu BW^{\pm}_L(s) = \omega^* J_\mu \varepsilon_\mu \omega' \ ,$$

$$J_\mu = i\mathcal{F}_1 \sigma_\mu + \mathcal{F}_2 (\vec{\sigma}\vec{q}) \frac{\varepsilon_{\mu ij} \sigma_i k_j}{|\vec{k}||\vec{q}|} + i\mathcal{F}_3 \frac{(\vec{\sigma}\vec{k})}{|\vec{k}||\vec{q}|} q_\mu + i\mathcal{F}_4 \frac{(\vec{\sigma}\vec{q})}{\vec{q}^2} q_\mu \ .$$

$$\mathcal{F}_1(s,z) = \sum_{L=0}^{\infty} [L M^+_L + E^+_L] P'_{L+1}(z) + [(L+1) M^-_L + E^-_L] P'_{L-1}(z) \ ,$$

$$\mathcal{F}_2(s,z) = \sum_{L=1}^{\infty} [(L+1) M^+_L + L M^-_L] P'_L(z) \ ,$$

$$\mathcal{F}_3(s,z) = \sum_{L=1}^{\infty} [E^+_L - M^+_L] P''_{L+1}(z) + [E^-_L + M^-_L] P''_{L-1}(z) \ ,$$

$$\mathcal{F}_4(s,z) = \sum_{L=2}^{\infty} [M^+_L - E^+_L - M^-_L - E^-_L] P''_L(z).$$

1. Reconstruction 4 complex $F_i(s,z)$ from observables

$$Observables \sim \sum_{ij} \mathcal{F}_i \mathcal{F}_j^*$$

at least 8 observables are needed to find $F_i(s,z)$, BUT only up to factor $e^{i\phi(s,z)}$

$$E_L^{\pm}(s) = \int\limits_{-1}^{1} \frac{dz}{2} \sum_m \mathcal{F}_m D_m^{(L\pm)}\,, \tag{1}$$

However, there is no way to determine multipoles from $F_i(s,z)e^{i\phi(s,z)}$

$$E_L^{\pm}(s) \neq \int\limits_{-1}^{1} \frac{dz}{2} \sum_m \mathcal{F}_m D_m^{(L\pm)} e^{i\phi(s,z)}\,, \tag{2}$$

Scattering of spinless particles

$$A = \sum_L A_L(s)(2L+1)P_L(z)BW_L(s)$$

$$A_L(s) = \int\limits_{-1}^{1} \frac{dz}{2} A\, P_L(z)$$

In experiment only differential cross section can be measured, which is proportional to $|A|^2$:

$$A_L(s) \neq \int\limits_{-1}^{1} \frac{dz}{2} |A|\, P_L(z)$$

$$\text{BUT} \qquad |A|^2 = |A_0|^2 + |A_1|z^2 + \ldots + 2Re(A_0 A_1^*)z + \ldots$$

Single energy analysis is a multipole decomposition

Then the following questions should be answered:

1. **How many multipoles one needs to describe the full set of data (all polarization observables) at certain energy?**
2. **What additional ambiguities appear in multipole decomposition?**
3. **What are effects from statistics and acceptance (e.g. not full angular coverage).**

Limitations:

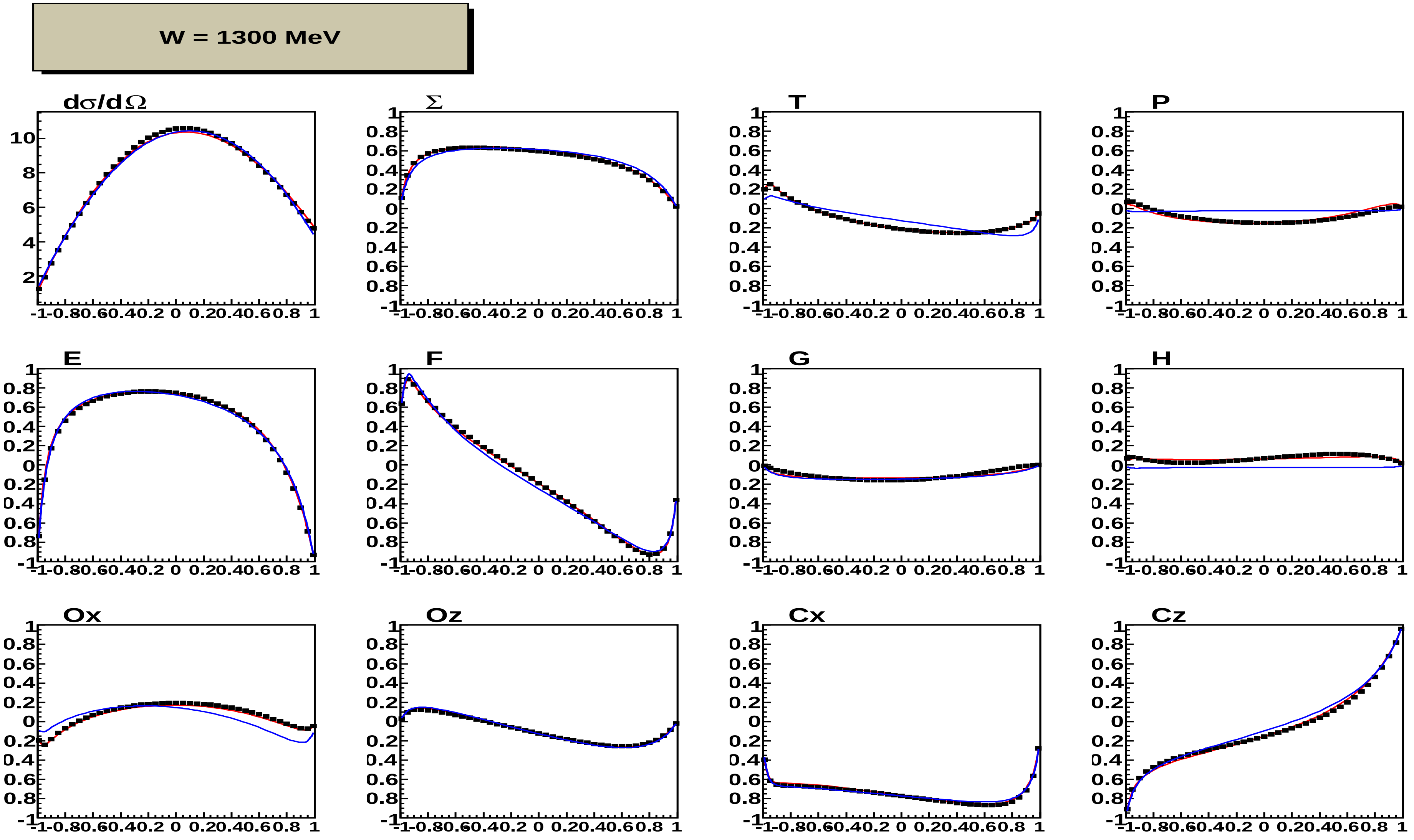
$$Observables(s,z) \sim \sum \mathcal{M}ultipole_i(s)\mathcal{M}ultipole^*_j(s)D(z)$$

Multipoles can be found up to the arbitrary phase $\phi(s)$. In our analysis only absolute value for M_1^+ is fitted.

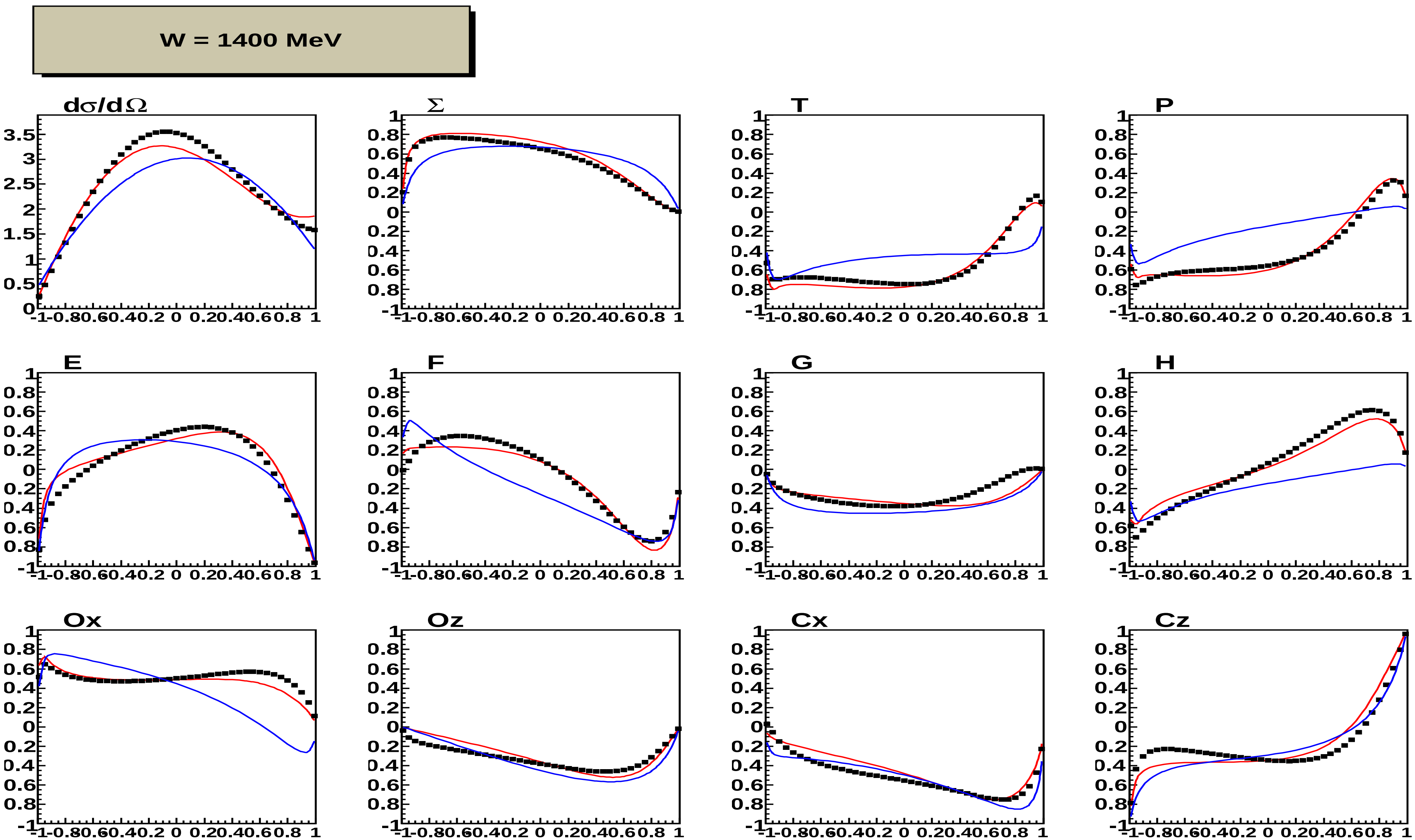
Single energy fit program.

- **Investigation of the $\gamma p \to \pi^0 p$ reaction: small contribution from t-channel exchanges, waves with low orbital momentum are dominant at low energies.**
  1. **Fit of pseudo data: observables predicted by the Bonn-Gatchina PWA solution taken with small errors and full angle coverage.**
  2. **Investigation of effects from statistic and acceptance. E.g., realistic errors, not full angular coverage.**
- **Investigation of the $\gamma p \to \pi^+ n$ reaction: large contribution from t-channel exchanges, waves with high orbital momentum are important already at low energies.**

  Same investigation as for $\gamma p \to \pi^0 p$

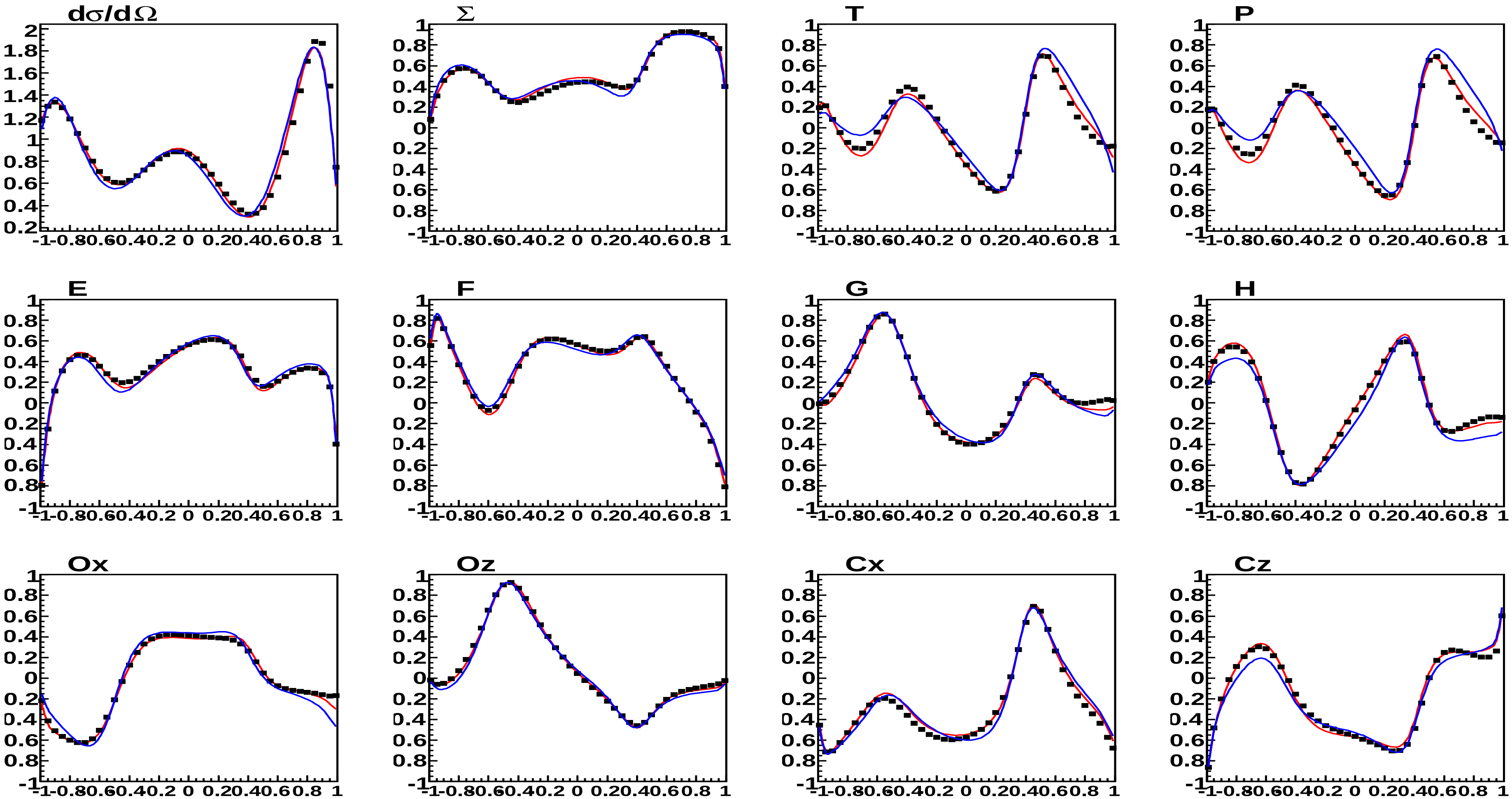


Description of the $\gamma p \rightarrow \pi^0 p$ pseudo data - multipoles up to $L = 1$ and $L = 2$



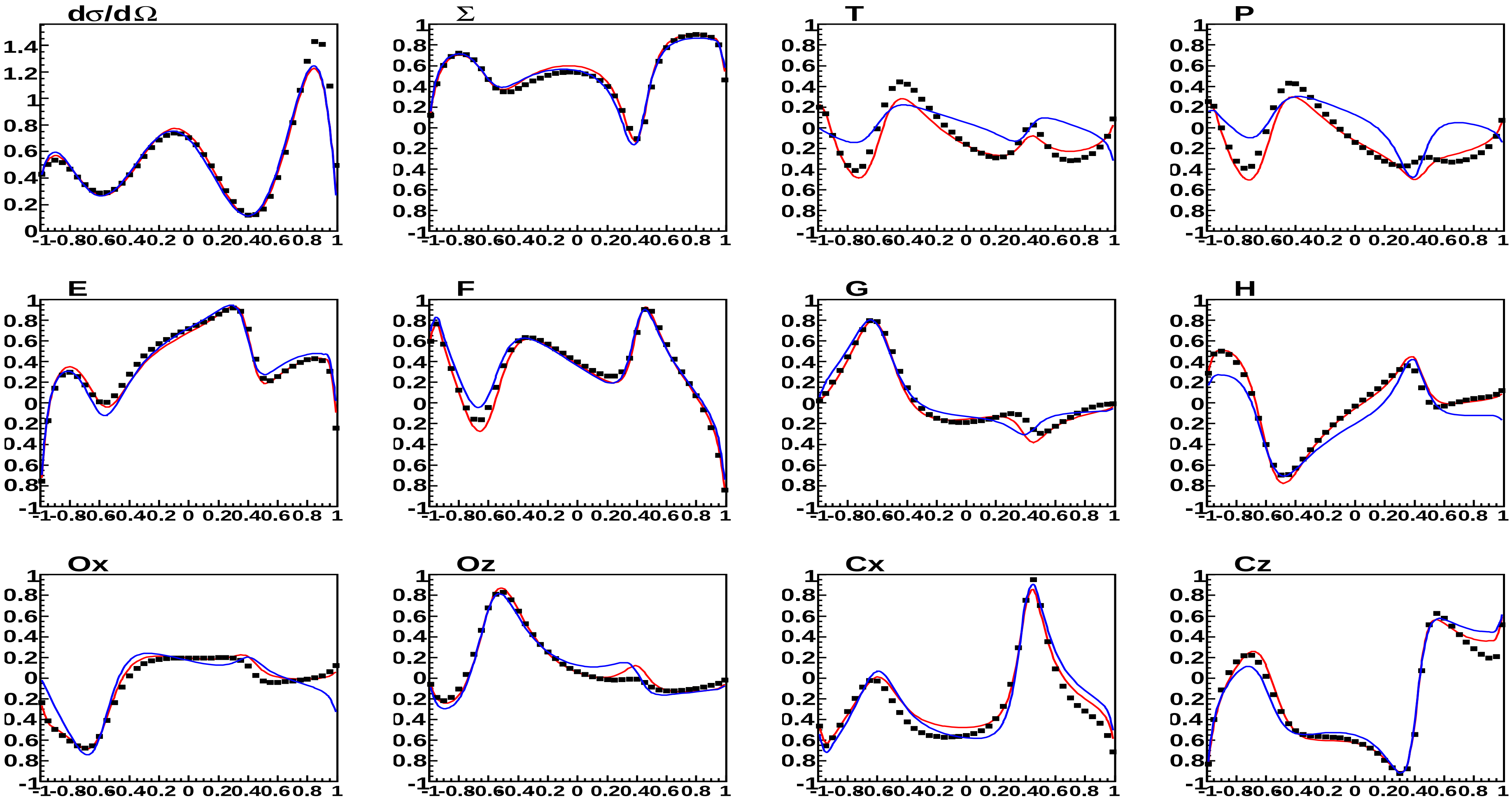
Description of the $\gamma p \to \pi^0 p$ pseudo data - multipoles up to $L = 1$ and $L = 2$

W = 1900 MeV

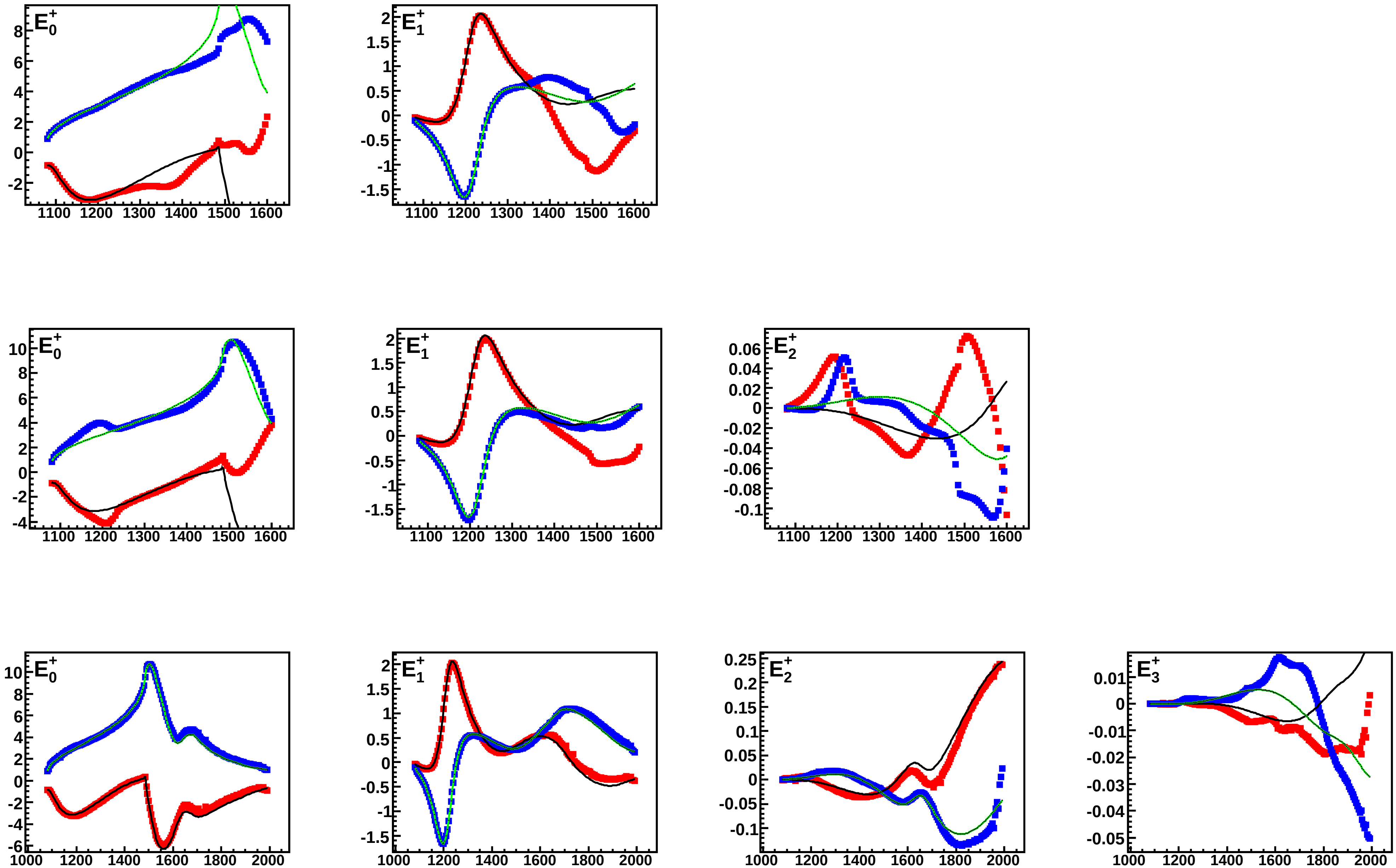


Description of the $\gamma p \rightarrow \pi^0 p$ pseudo data - multipoles up to $L = 3$ and $L = 4$

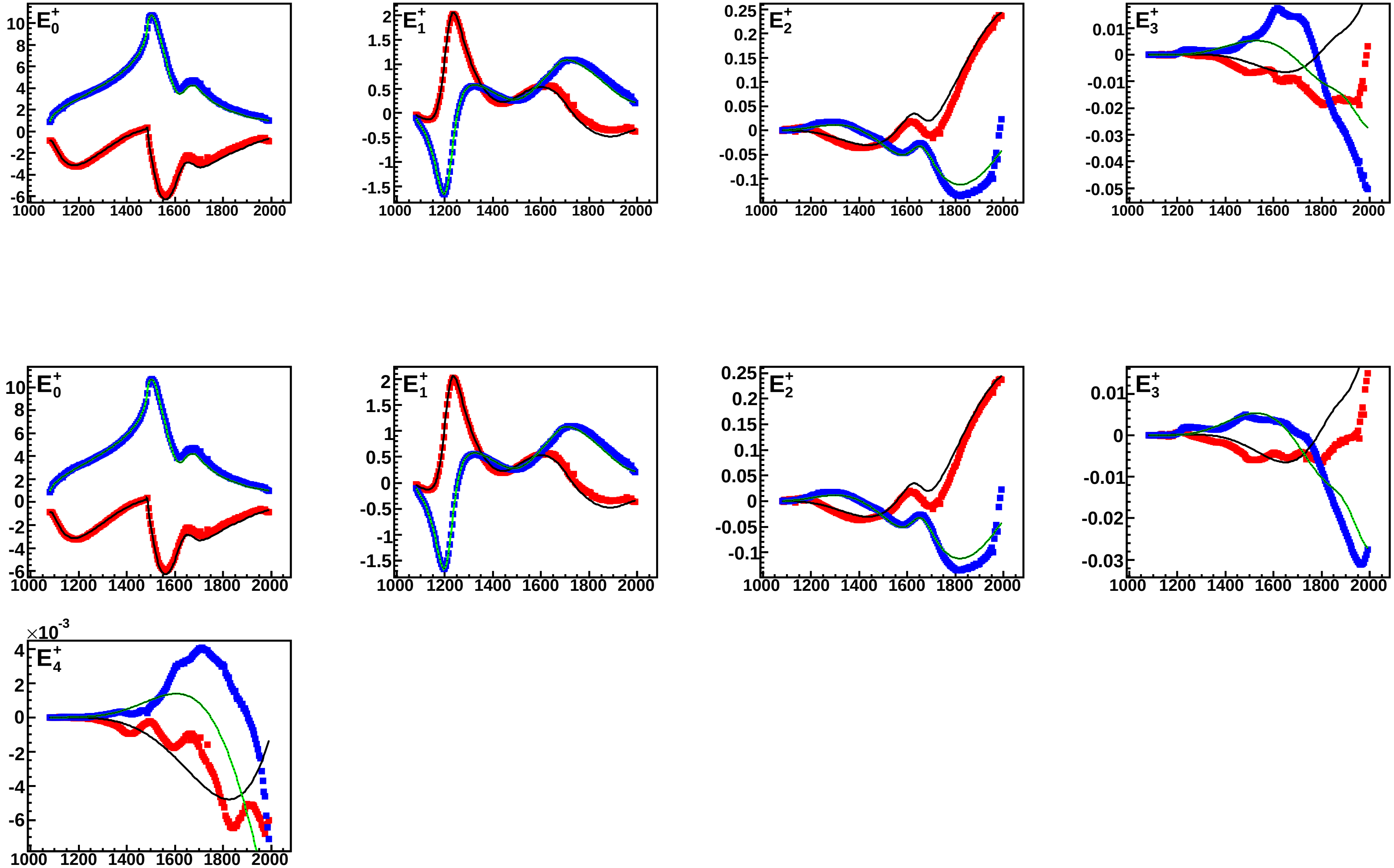
W = 2000 MeV



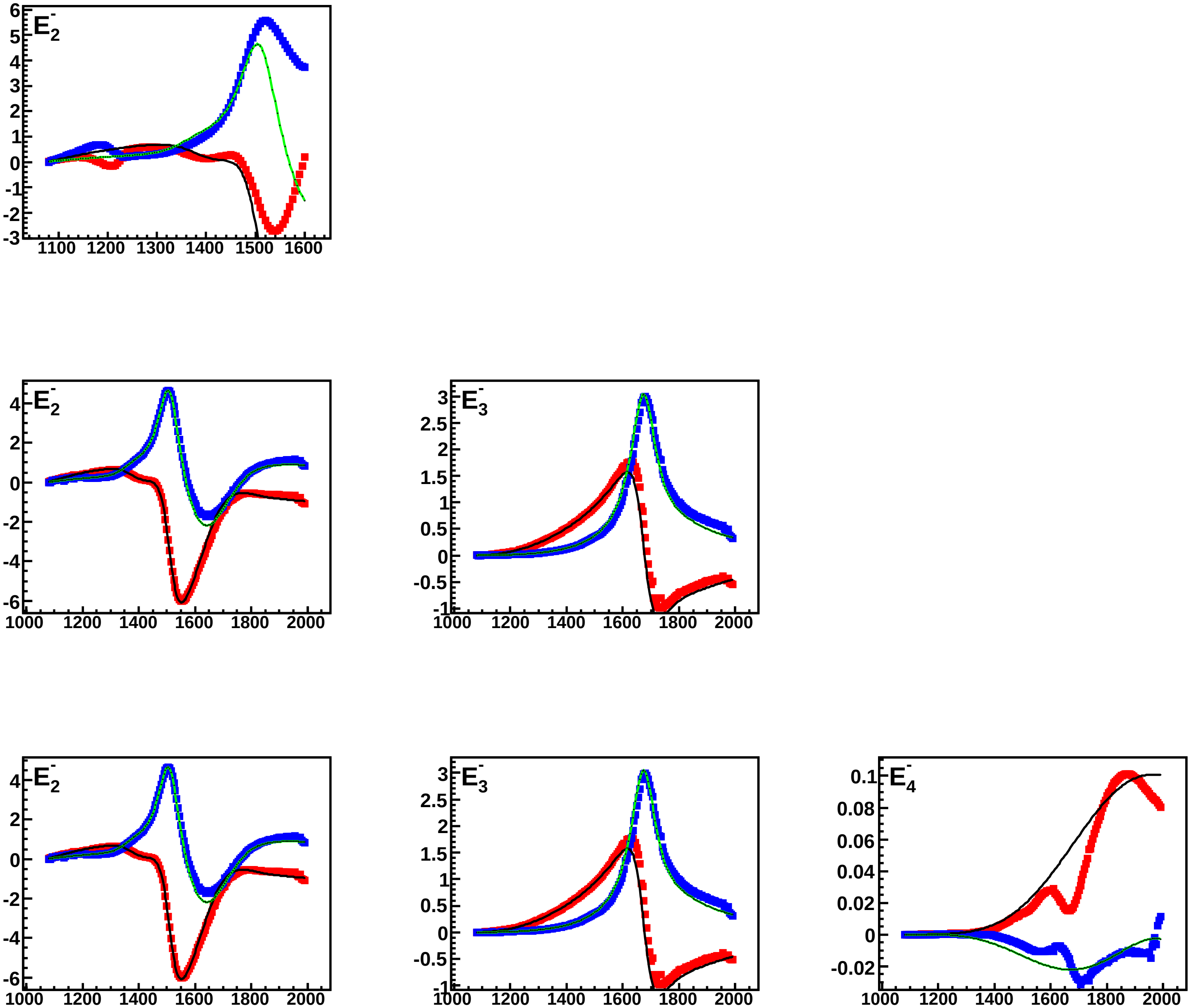
Description of the $\gamma p \rightarrow \pi^0 p$ pseudo data - multipoles up to $L = 3$ and $L = 4$



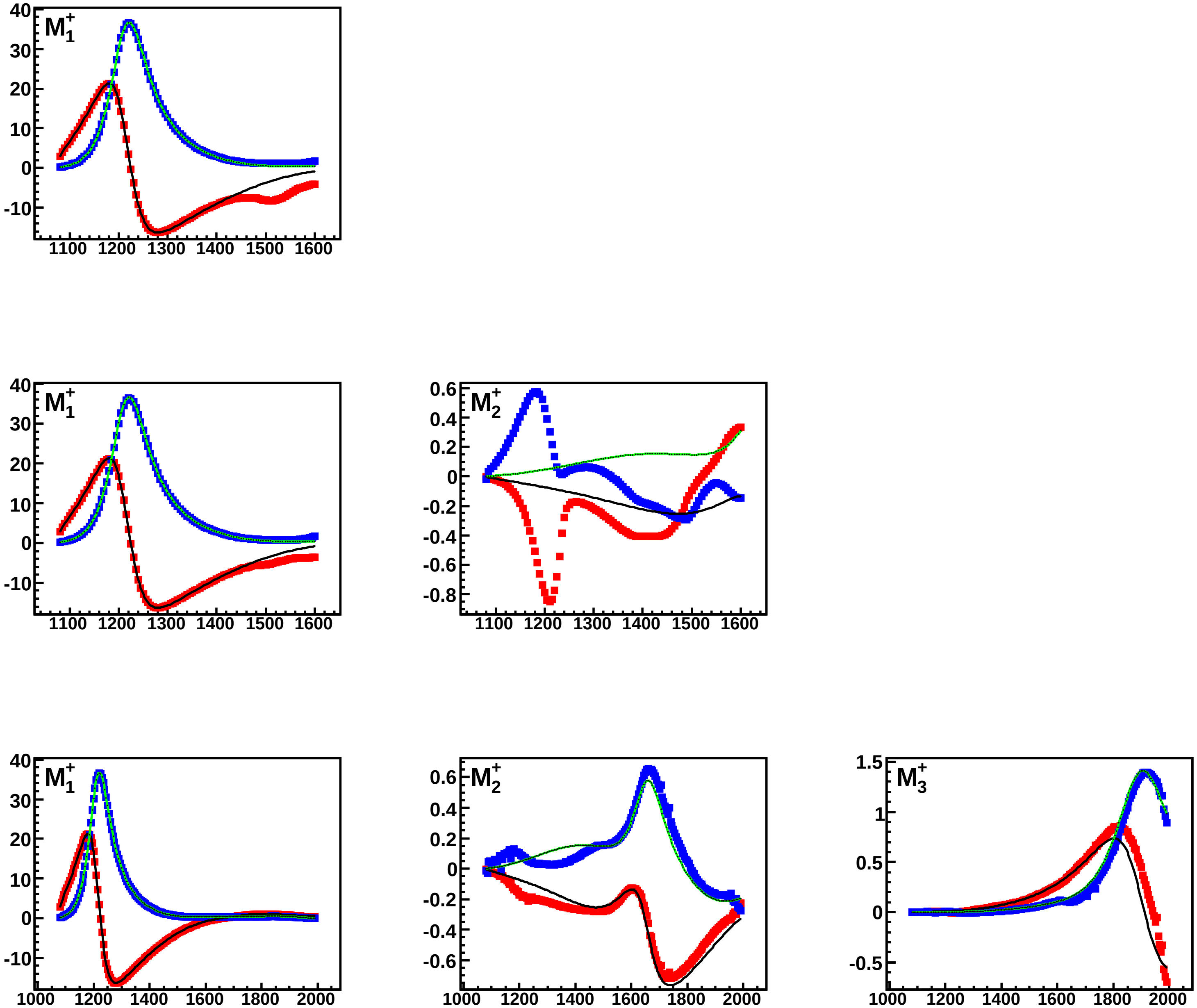
Real and imaginary parts in compare with BoGa PWA multipoles



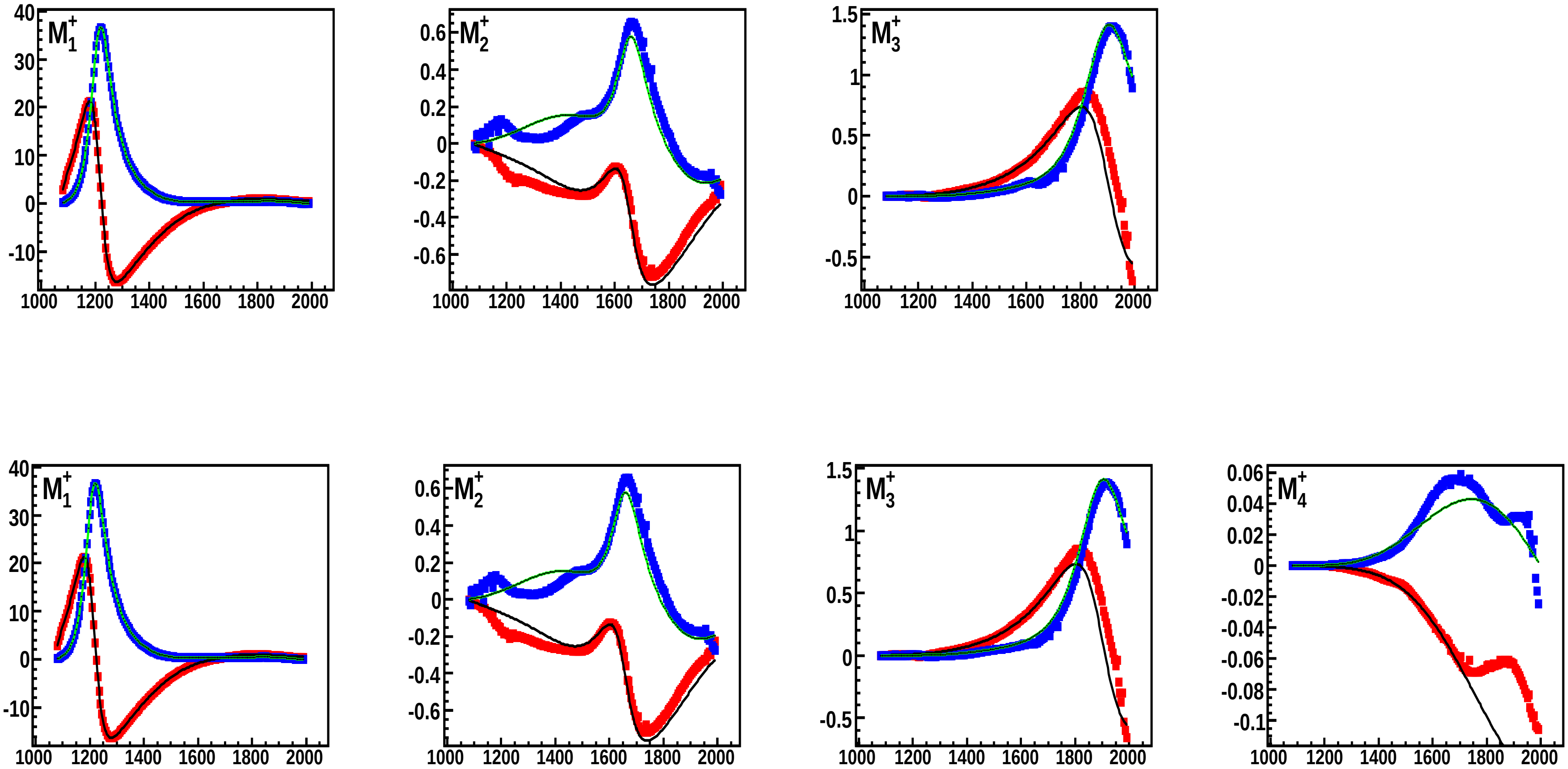
Real and imaginary parts compared with BoGa PWA multipoles



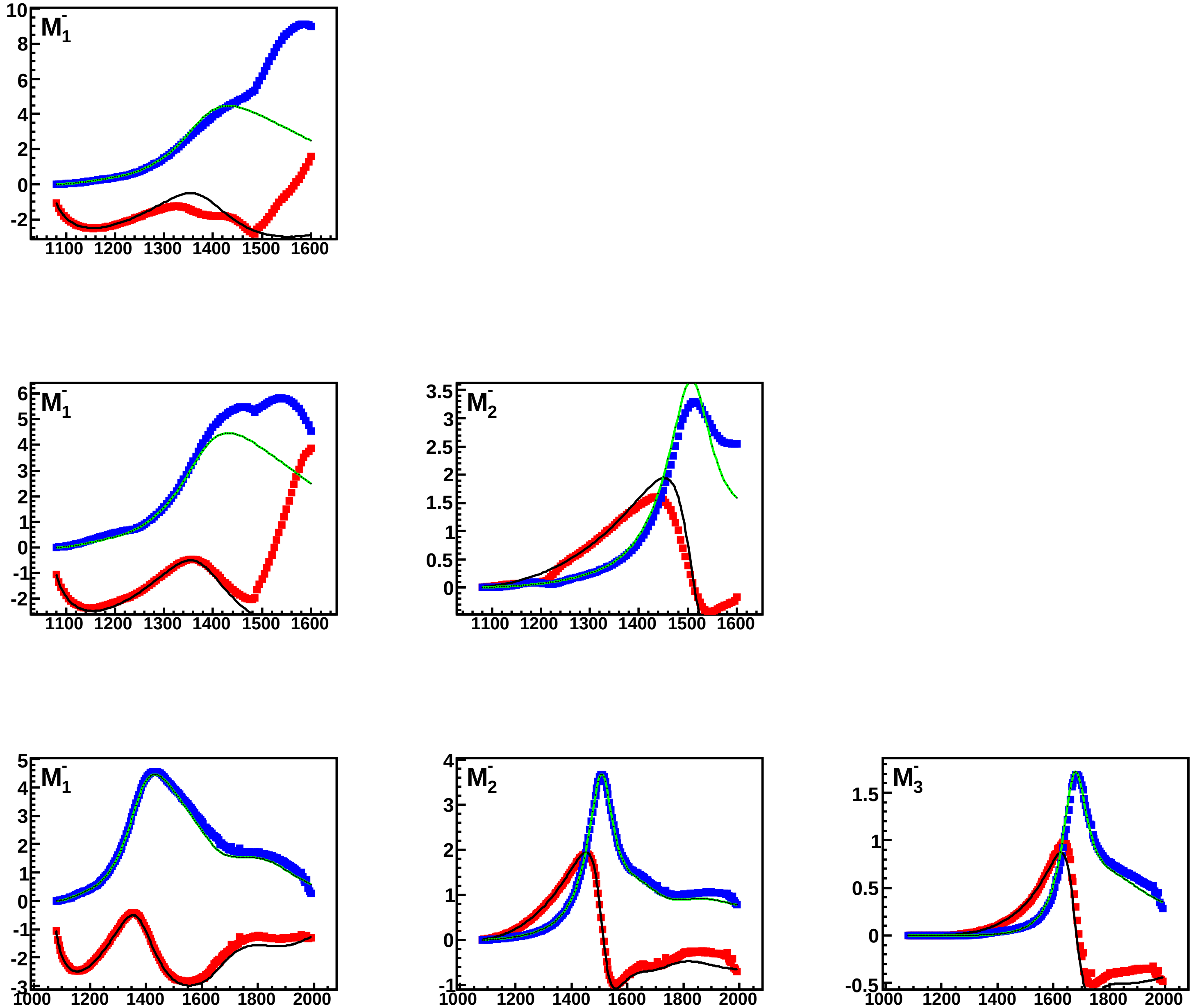
Real and imaginary parts compared with BoGa PWA multipoles



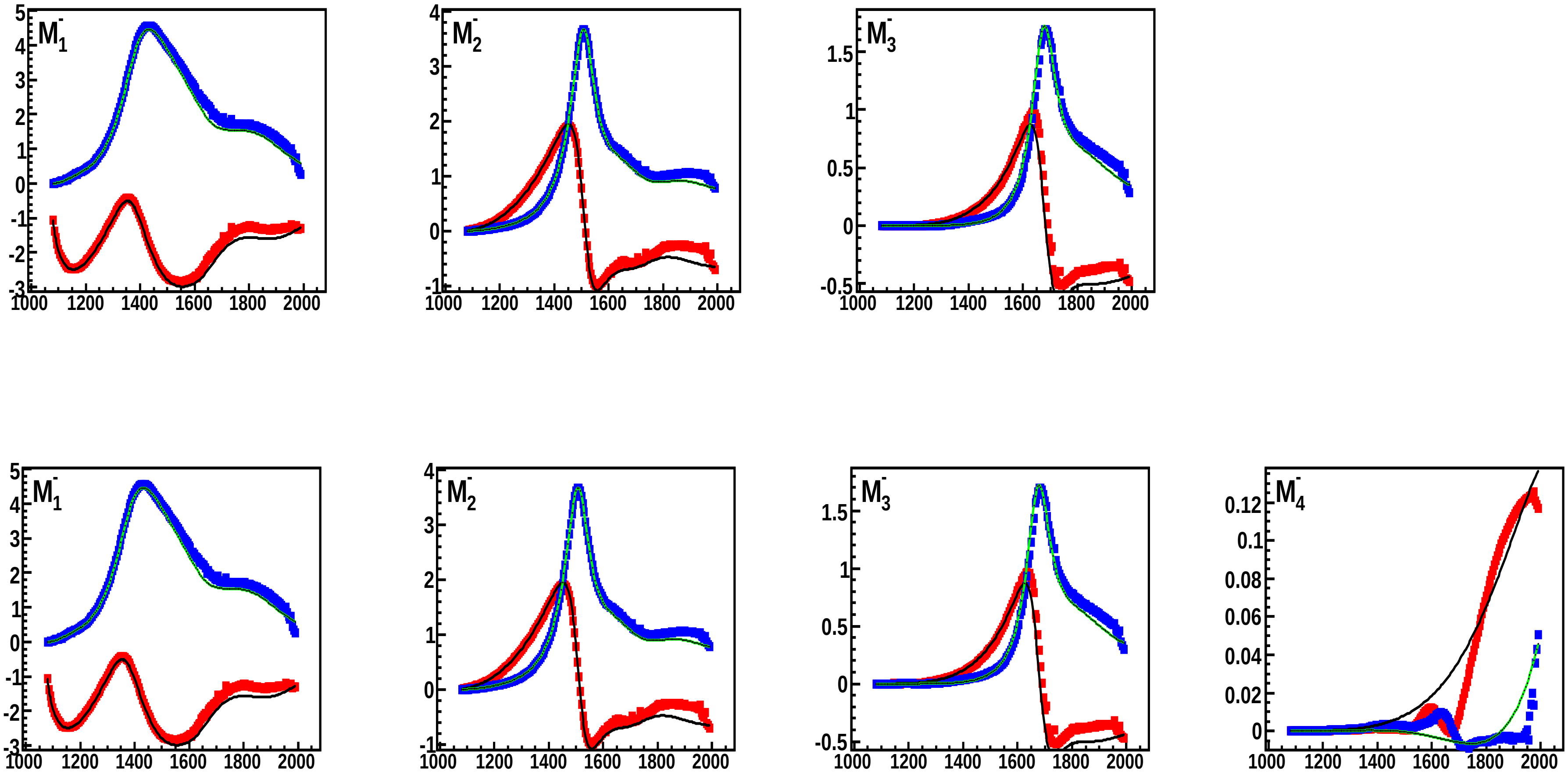
Real and imaginary parts compared with BoGa PWA multipoles



Real and imaginary parts compared with BoGa PWA multipoles



Fit: real and imaginary parts compared with BoGa PWA multipoles



Real and imaginary parts compared with BoGa PWA multipoles

Conclusion

1. **To fit pseudo $\gamma p \rightarrow \pi^0 p$ data up to $W \sim 1300$ MeV one needs up to $L = 2$ multipoles**
   At higher energy up to $W = 1600 - 1700$ MeV $L = 3$ multipoles are needed; at $W = 1900$ MeV $L = 4$ multipoles are needed

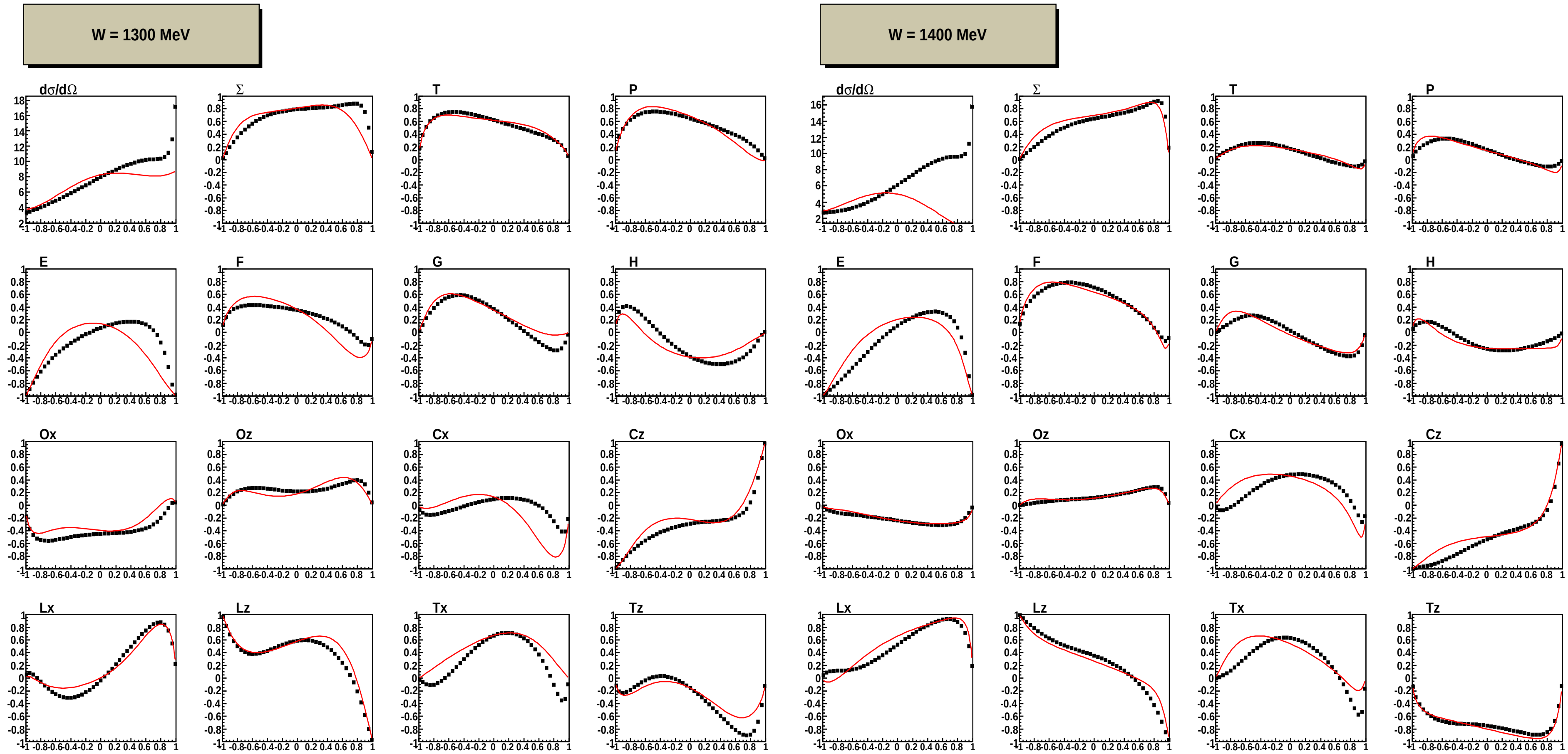
2. **Reconstructed multipoles agree well with the input multipoles from Bonn-Gatchina PWA. Higher multipoles which are omitted from the single energy fit influence only highest L multipoles taken into account.**

3. **The higher multipoles can be approximated with second order polynomials.**

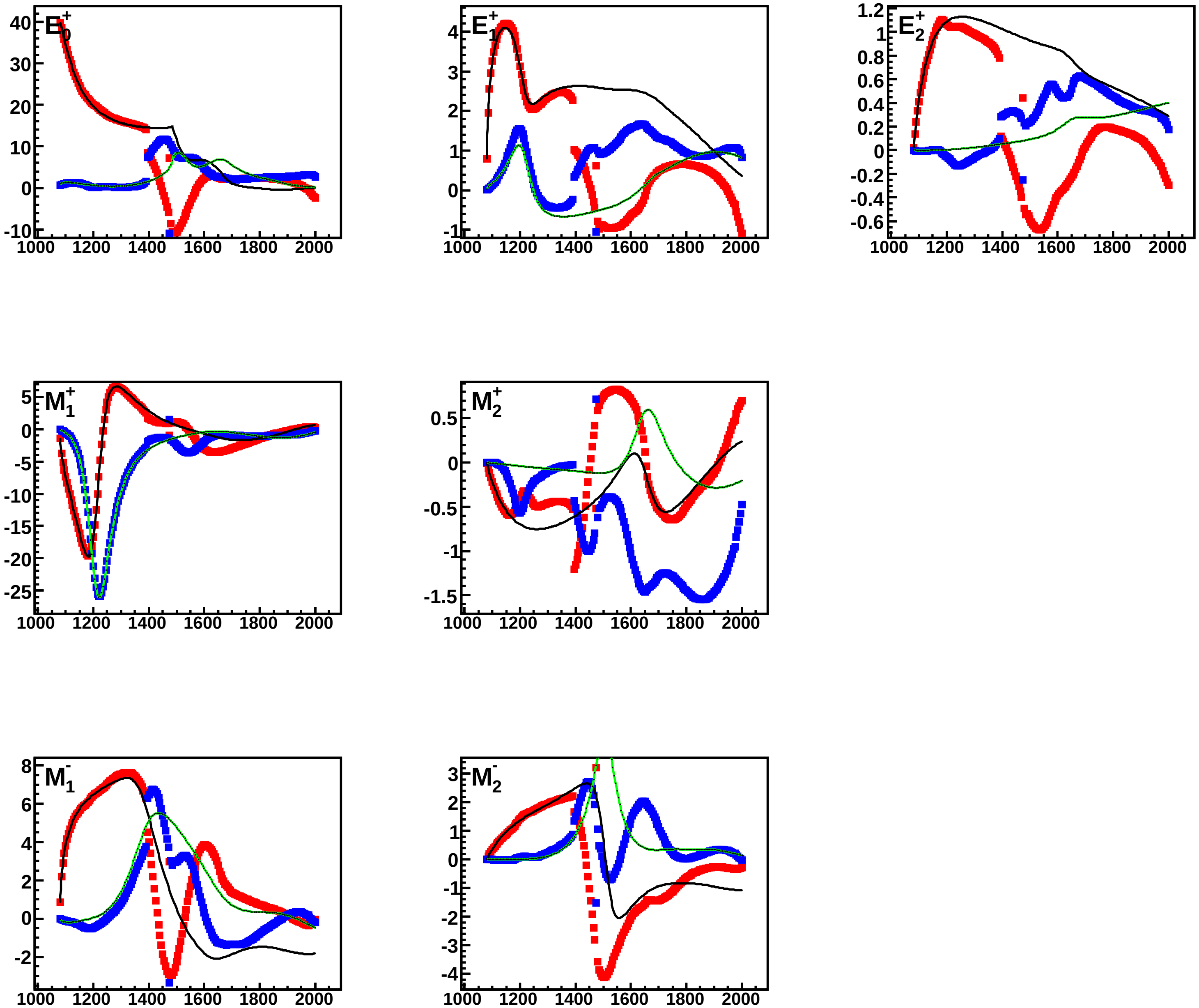
Next steps

1. **To investigate effects from acceptance (for $\gamma p \rightarrow \pi^0 p$).**

2. **To start investigation of the $\gamma p \rightarrow \pi^+ n$ reaction.**



Description of the $\gamma p \to \pi^+ n$ pseudo data - multipoles up to $L = 3$



Real and imaginary parts compared with BoGa PWA multipoles