Recent results from Bonn-Gatchina partial wave analysis

A. Sarantsev

Petersburg Nuclear Physics Institute

HISKP (Bonn), PNPI (Russia)

MENU 2010

May 31-June 4 2010; Williamsburg

Bonn-Gatchina partial wave analysis group:

A. Anisovich, E. Klempt, V. Nikonov, A. Srantsev, U. Thoma

http://pwa.hiskp.uni-bonn.de/

Petersburg Nuclear Physics Institute

Bonn-Gatchina Partial Wave Analysis

Address: Nussallee 14-16, D-53115 Bonn   Fax: (+49) 228 / 73-2505

Data Base   Meson Spectroscopy   Baryon Spectroscopy   NN-interaction   Formalism

Analysis of Other Groups	BG PWA	Useful Links
SAID	Publications	SPIRES
MAID	Talks	PDG Homepage
Giessen Uni	Contacts	Durham Data Base
		Bonn Homepage

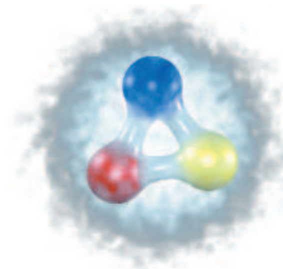
CB-ELSA Homepage

Responsible: Dr. V. Nikonov, E-mail: *nikonov@hiskp.uni-bonn.de*
Last changes: January 26th, 2010.

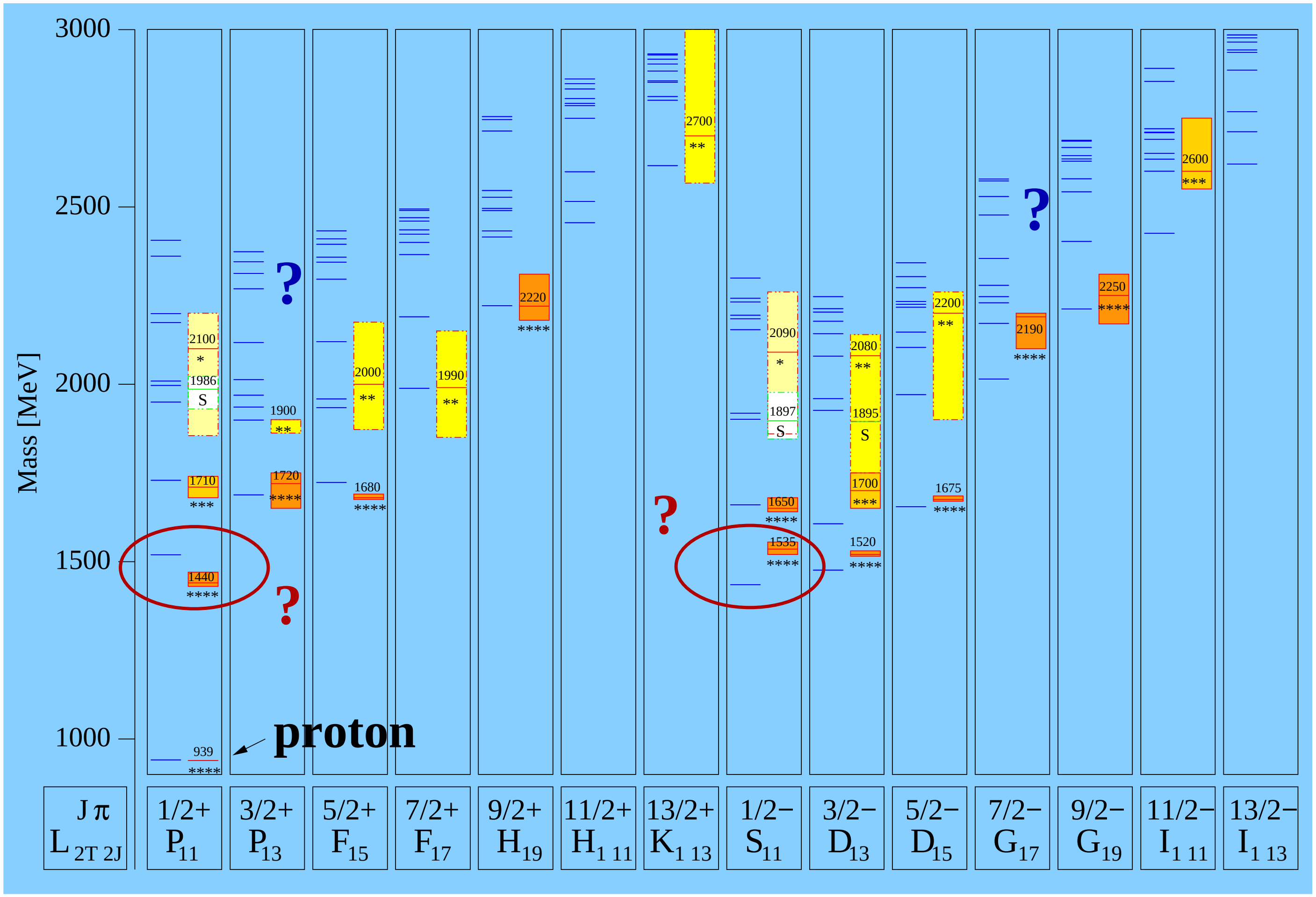
N*- resonances in the quark model

Nukleon

10^{-15} m



U. Loering, B. Metsch, H. Petry et al. (Bonn)



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Constituent quarks

Confinement-potential

Residual interaction

Problems in the baryon spectroscopy and/or quark model:

1. **Problem: Number of predicted three quark states exceeds dramatically the number of discovered baryons.**

2. **Possible solution: Most of the information comes from analyses of πN elastic reactions. Photoproduction data taken by CLAS, GRAAL, LEPS and CB-ELSA can provide an important information about missing states.**

   (a) **problem: Unambiguous analysis of photoproduction reactions can not be done without polarization information available.**

   (b) **problem: Signals in simple reactions are expected to be mostly weak. Strong signals from new resonances can be found in multi-meson final states.**

   (c) **Possible solution 1: Single polarization observables are measured now by almost all collaborations. Double polarization data are available from CLAS, GRAAL and, in nearest future, from CB-ELSA.**

   (d) **Possible solution 2: A combined analysis of large data sets.**

The latest analysis of SAID (GWU) of πN elastic data as well as $\gamma p \to \pi^0 p$ and $\gamma p \to \pi^+ n$ did not confirm the set of states observed in earlier analysis of πN elastic data. CLAS (M. Dugger et al.). Phys.Rev.C79:065206,2009.

State	PDG (Pole position)(MeV)		Bonn-Gatchina PWA (MeV)	
	Mass	**Width**	**Mass**	**Width**
$P_{11}(1710)^{***}$	1720 ± 50	230 ± 150	1710 ± 25	220 ± 20
$P_{33}(1600)^{***}$	1550 ± 100	300 ± 100	1480 ± 40	230 ± 40
$P_{33}(1920)^{***}$	1900 ± 50	200^{+100}_{-50}	1920 ± 50	330 ± 50
$D_{13}(1720)^{***}$	1680 ± 50	100 ± 50	1730 ± 30	170 ± 35
$P_{13}(1900)^{*}$	~ 1900	498 ± 78	1920 ± 30	200 ± 30
$D_{33}(1940)^{*}$	~ 1940	$200 - 500$	1990 ± 40	350 ± 50

Data Base

Pion induced reactions (χ^2 analysis).

Observable	$N_{\rm data}$	$\frac{\chi^2}{N_{\rm data}}$		Observable	$N_{\rm data}$	$\frac{\chi^2}{N_{\rm data}}$	
$\mathrm{N}^*_{1/2-}$ $\mathrm{S}_{11}(\pi\mathrm{N}\to\pi\mathrm{N})$	104	1.81	SAID	$\Delta_{1/2-}$ $\mathrm{S}_{31}(\pi\mathrm{N}\to\pi\mathrm{N})$	112	2.27	SAID
$\mathrm{N}^*_{1/2+}$ $\mathrm{P}_{11}(\pi\mathrm{N}\to\pi\mathrm{N})$	112	2.49	SAID	$\Delta_{1/2+}$ $\mathrm{P}_{31}(\pi\mathrm{N}\to\pi\mathrm{N})$	104	2.01	SAID
$\mathrm{N}^*_{3/2+}$ $\mathrm{P}_{13}(\pi\mathrm{N}\to\pi\mathrm{N})$	112	1.90	SAID	$\Delta^*_{3/2+}$ $\mathrm{P}_{33}(\pi\mathrm{N}\to\pi\mathrm{N})$	120	2.53	SAID
$\Delta^*_{3/2-}$ $\mathrm{D}_{33}(\pi\mathrm{N}\to\pi\mathrm{N})$	108	2.56	SAID	$\mathrm{N}^*_{3/2-}$ $\mathrm{D}_{13}(\pi\mathrm{N}\to\pi\mathrm{N})$	96	2.16	SAID
$\mathrm{N}^*_{5/2-}$ $\mathrm{D}_{15}(\pi\mathrm{N}\to\pi\mathrm{N})$	96	3.37	SAID	$\Delta_{5/2+}$ $\mathrm{F}_{35}(\pi\mathrm{N}\to\pi\mathrm{N})$	62	1.32	SAID
$\Delta_{7/2+}$ $\mathrm{F}_{37}(\pi\mathrm{N}\to\pi\mathrm{N})$	72	2.86	SAID				
$d\sigma/d\Omega(\pi^-p\to n\eta)$	70	1.96	Richards *et al.*	$d\sigma/d\Omega(\pi^-p\to n\eta)$	84	2.67	CBALL
$d\sigma/d\Omega(\pi^-p\to K\Lambda)$	598	1.68	RAL	$P(\pi^-p\to K\Lambda)$	355	1.96	RAL+ANL
$d\sigma/d\Omega(\pi^+p\to K^+\Sigma)$	609	1.24	RAL	$P(\pi^+p\to K^+\Sigma)$	307	1.49	RAL
$d\sigma/d\Omega(\pi^-p\to K^0\Sigma^0)$	259	0.85	RAL	$P(\pi^-p\to K^0\Sigma^0)$	95	1.25	RAL

Data Base

π and η photoproduction reactions (χ^2 analysis).

Observable	$N_{\rm data}$	$\frac{\chi^2}{N_{\rm data}}$		Observable	$N_{\rm data}$	$\frac{\chi^2}{N_{\rm data}}$	
$d\sigma/d\Omega(\gamma p\to p\pi^0)$	1106	1.34	CB-ELSA	$d\sigma/d\Omega(\gamma p\to p\pi^0)$	861	1.46	GRAAL
$d\sigma/d\Omega(\gamma p\to p\pi^0)$	592	2.11	CLAS	$d\sigma/d\Omega(\gamma p\to p\pi^0)$	1692	1.25	TAPS@MAMI
$E(\gamma p\to p\pi^0)$	140	1.23	A2-GDH	$\Sigma(\gamma p\to p\pi^0)$	1492	3.26	SAID db
$P(\gamma p\to p\pi^0)$	607	3.23	SAID db	$T(\gamma p\to p\pi^0)$	389	3.71	SAID db
$H(\gamma p\to p\pi^0)$	71	1.26	SAID db	$G(\gamma p\to p\pi^0)$	75	1.50	SAID db
$O_x(\gamma p\to p\pi^0)$	7	1.77	SAID db	$O_z(\gamma p\to p\pi^0)$	7	0.46	SAID db
$d\sigma/d\Omega(\gamma p\to n\pi^+)$	1583	1.64	SAID db	$d\sigma/d\Omega(\gamma p\to n\pi^+)$	408	0.62	A2-GDH
$\Sigma(\gamma p\to n\pi^+)$	899	3.48	SAID db	$E(\gamma p\to n\pi^+$	231	1.55	A2-GDH
$P(\gamma p\to n\pi^+)$	252	2.90	SAID db	$T(\gamma p\to n\pi^+)$	661	3.21	SAID db
$H(\gamma p\to p\pi^+)$	71	3.90	SAID db	$G(\gamma p\to p\pi^+)$	86	5.64	SAID db
$d\sigma/d\Omega(\gamma p\to p\eta)$	680	1.47	CB-ELSA	$d\sigma/d\Omega(\gamma p\to p\eta)$	100	2.16	TAPS
$\Sigma(\gamma p\to p\eta)$	51	2.26	GRAAL 98	$\Sigma(\gamma p\to p\eta)$	100	2.02	GRAAL 07
$T(\gamma p\to p\eta)$	50	1.48	Phoenics				

Data Base

Kaon photoproduction (χ^2 analysis).

Observable	$N_{\rm data}$	$\frac{\chi^2}{N_{\rm data}}$		Observable	$N_{\rm data}$	$\frac{\chi^2}{N_{\rm data}}$	
$C_x(\gamma p\to\Lambda K^+)$	160	1.23	CLAS	$C_x(\gamma p\to\Sigma^0 K^+)$	94	2.20	CLAS
$C_z(\gamma p\to\Lambda K^+)$	160	1.41	CLAS	$C_z(\gamma p\to\Sigma^0 K^+)$	94	2.00	CLAS
$d\sigma/d\Omega(\gamma p\to\Lambda K^+)$	1320	0.81	CLAS09	$d\sigma/d\Omega(\gamma p\to\Sigma^0 K^+)$	1280	2.06	CLAS
$P(\gamma p\to\Lambda K^+)$	1270	2.21	CLAS09	$P(\gamma p\to\Sigma^0 K^+)$	95	1.45	CLAS
$\Sigma(\gamma p\to\Lambda K^+)$	66	1.53	GRAAL	$\Sigma(\gamma p\to\Sigma^0 K^+)$	42	0.90	GRAAL
$\Sigma(\gamma p\to\Lambda K^+)$	45	1.65	LEP	$\Sigma(\gamma p\to\Sigma^0 K^+)$	45	1.11	LEP
$T(\gamma p\to\Lambda K^+)$	66	1.26	GRAAL 09	$d\sigma/d\Omega(\gamma p\to\Sigma^+ K^0)$	48	3.76	CLAS
$O_x(\gamma p\to\Lambda K^+)$	66	1.30	GRAAL 09	$O_z(\gamma p\to\Lambda K^+)$	66	1.54	GRAAL 09
$d\sigma/d\Omega(\gamma p\to\Sigma^+ K^0)$	72	0.74	CB-ELSA 10	$P(\gamma p\to\Sigma^+ K^0)$	24	1.06	CB-ELSA 10
$\Sigma(\gamma p\to\Sigma^+ K^0)$	15	1.13	CB-ELSA 10				

Data Base

Multi-meson final states (maximum likelihood analysis).

$d\sigma/d\Omega(\pi^- p \to n\pi^0\pi^0)$	CBALL				
$\mathrm{d}\sigma/\mathrm{d}\Omega(\gamma\mathrm{p}\to\mathrm{p}\pi^0\pi^0)$	CB-ELSA (1.4 GeV)	$\mathrm{E}(\gamma\mathrm{p}\to\mathrm{p}\pi^0\pi^0)$	16	1.91	MAMI
$\mathrm{d}\sigma/\mathrm{d}\Omega(\gamma\mathrm{p}\to\mathrm{p}\pi^0\eta)$	CB-ELSA (3.2 GeV)	$\Sigma(\gamma\mathrm{p}\to\mathrm{p}\pi^0\eta)$	180	2.37	GRAAL
$\mathrm{d}\sigma/\mathrm{d}\Omega(\gamma\mathrm{p}\to\mathrm{p}\pi^0\pi^0)$	CB-ELSA (3.2 GeV)	$\Sigma(\gamma\mathrm{p}\to\mathrm{p}\pi^0\pi^0)$	128	0.96	GRAAL
$\mathrm{d}\sigma/\mathrm{d}\Omega(\gamma\mathrm{p}\to\mathrm{p}\pi^0\eta)$	CB-ELSA (3.2 GeV)	$\Sigma(\gamma\mathrm{p}\to\mathrm{p}\pi^0\eta)$	180	2.37	GRAAL
$\mathrm{I_c}(\gamma\mathrm{p}\to\mathrm{p}\pi^0\eta)$	CB-ELSA (3.2 GeV)	$\mathrm{I_s}(\gamma\mathrm{p}\to\mathrm{p}\pi^0\eta)$	CB-ELSA (3.2 GeV)		

Energy dependent approach

In many cases an unambiguous partial wave decomposition at fixed energies is impossible. Then the energy and angular parts should be analyzed together:

$$A(s,t) \;=\; \sum_{\beta\beta' n} A_n^{\beta\beta'}(s) Q^{(\beta)+}_{\mu_1\ldots\mu_n} F^{\mu_1\ldots\mu_n}_{\nu_1\ldots\nu_n} Q^{(\beta')}_{\nu_1\ldots\nu_n}$$

1. **C. Zemach, Phys. Rev. 140, B97 (1965); 140, B109 (1965).**
2. **S.U.Chung, Phys. Rev. D 57, 431 (1998).**
3. **A.V. Anisovich** *et al.* **J. Phys. G 28 15 (2002)**

   V. V. Anisovich, M. A. Matveev, V. A. Nikonov, J. Nyiri and A. V. Sarantsev,

   Hackensack, USA: World Scientific (2008) 580 p

1. **Correlations between angular part and energy part are under control.**
2. **Unitarity and analyticity can be introduced from the beginning.**
3. **However, to fix simultaneously energy and angular dependencies of the amplitude a combined fit of many reactions is needed.**

Combined analysis of pion- and photo-production data:

For pion induced reactions:

$$A_{1i} = K_{1j}(I - i\rho K)^{-1}_{ji}$$

and

$$K_{ij} = \sum_\alpha \frac{g_i^\alpha g_j^\alpha}{M_\alpha^2 - s} + f_{ij}(s) \qquad f_{ij} = \frac{f_{ij}^{(1)} + f_{ij}^{(2)}\sqrt{s}}{s - s_0^{ij}}\,.$$

where f_{ij} is nonresonant transition part.

For the photoproduction:

$$A_k = P_j(I - i\rho K)^{-1}_{jk}$$

The vector of the initial interaction has the form:

$$P_j = \sum_\alpha \frac{\Lambda^\alpha g_j^\alpha}{M_\alpha^2 - s} + F_j(s)$$

Here F_j is nonresonant production of the final state j.

Bonn-Gatchina partial wave analysis

1. **K-matrix:** $\pi N \to \pi N$, $\pi N \to \eta N$, $\pi N \to K\Lambda$ **and** $\pi N \to K\Sigma$ **reactions.**

   Included channels: πN, ηN, $K\Lambda$, $K\Sigma$, $\pi\Delta(1232)$, $N\sigma$, $N\rho$.

   First results for the S_{11} wave fitted in the N/D approach.

2. **P-vector:** $\gamma N \to \pi N$, $\gamma N \to \eta N$, $\gamma N \to K\Lambda$ **and** $\gamma N \to K\Sigma$ **reactions.**

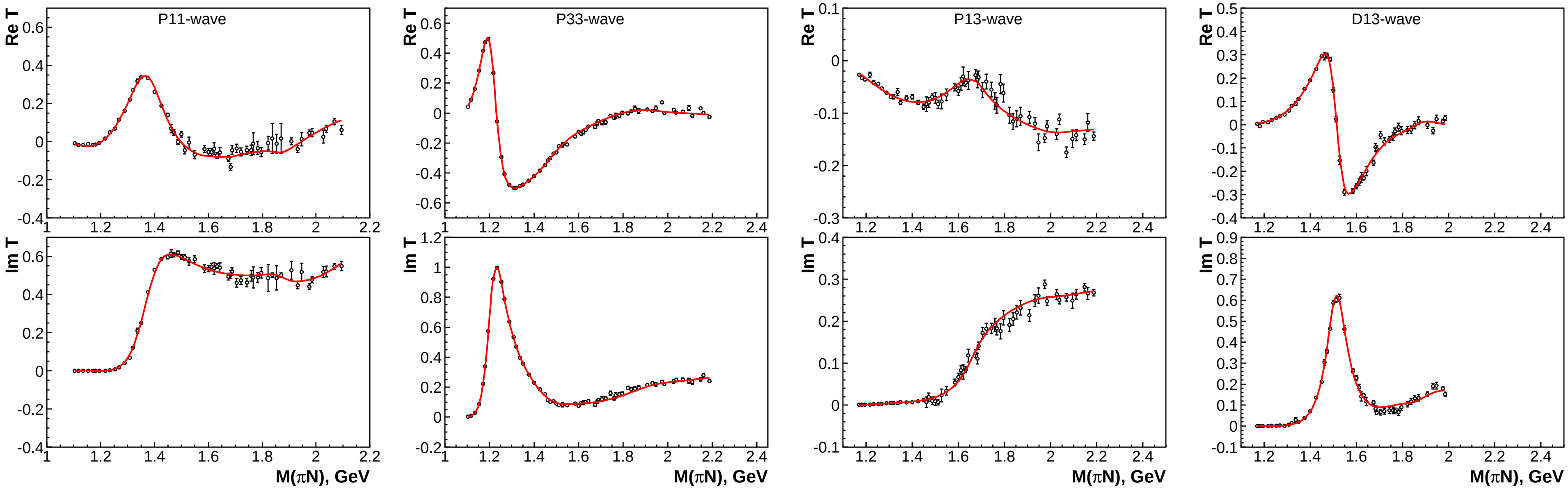
   Preliminary fit with Reggee exchanges included in the P-vectors.

3. **D-vector:** $\pi N \to \pi\pi N$

4. **PD-approach** $\gamma N \to \pi\pi N$, $\gamma N \to \pi\eta N$

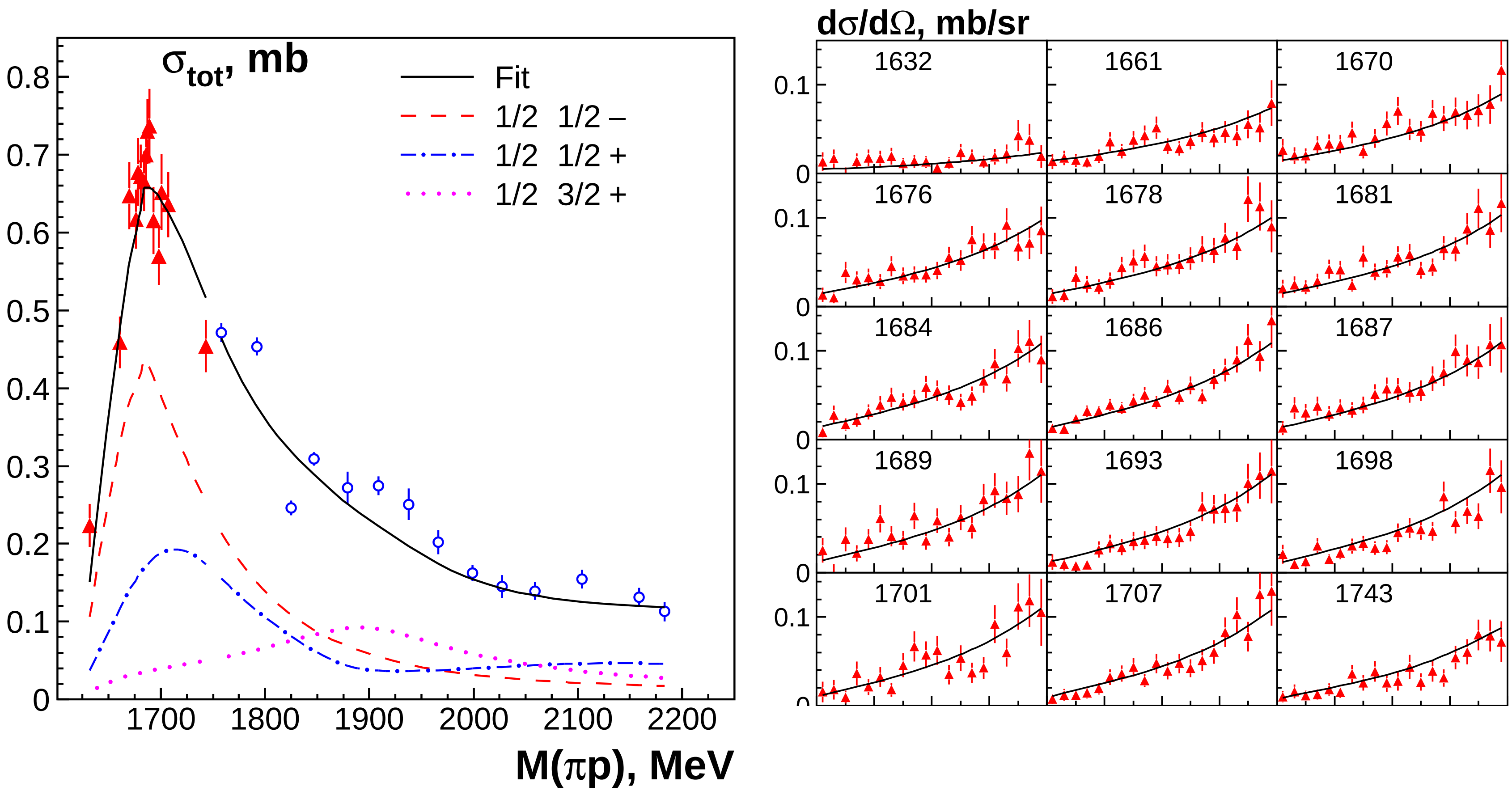
   D-vector channels: $P_{11}(1440)\pi$, $D_{13}(1520)\pi$, $F_{15}(1675)\pi$, $f_2(1275)N$, $\Delta\eta$,...

Fit of the SAID energy fixed solution for πN elastic partial waves

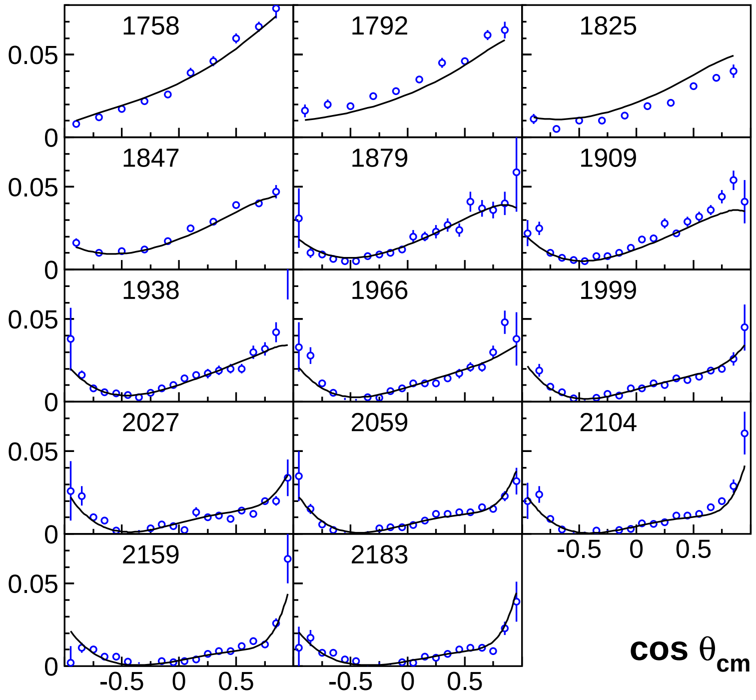


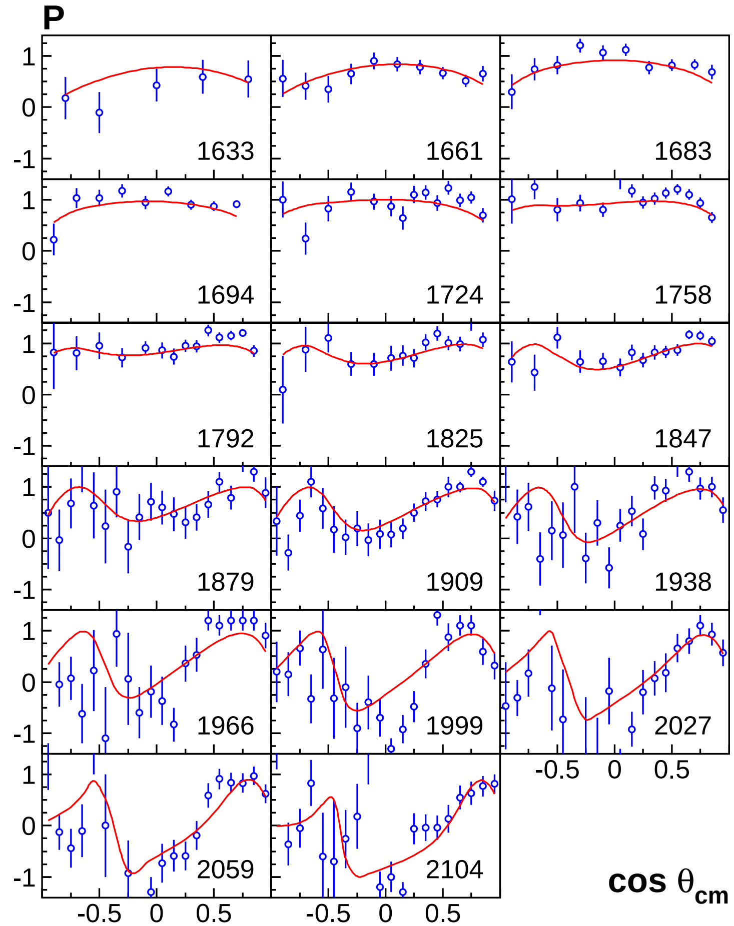
The fit of the the $\pi^- p \to K\Lambda$ reaction

The $P_{11}(1710)$ and $P_{13}(1900)$ states

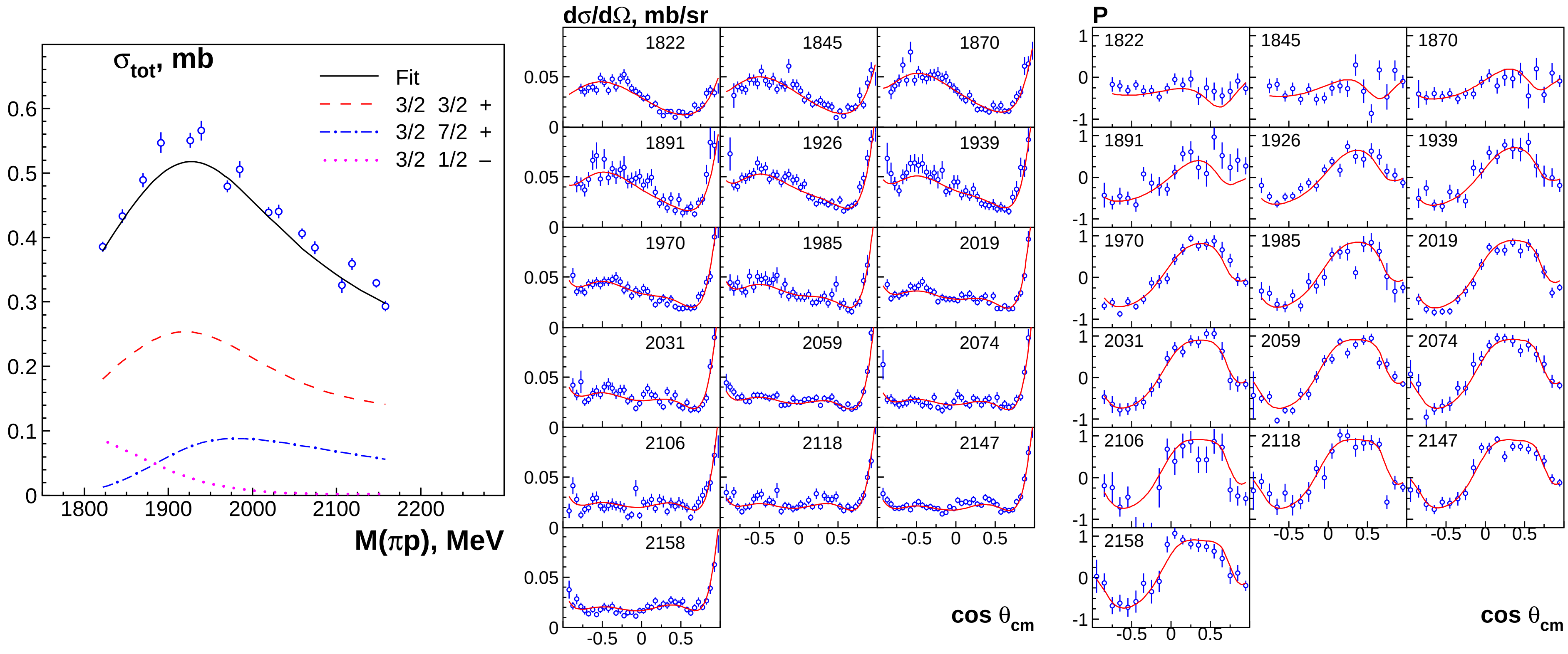


The fit of the the $\pi^- p \to K\Lambda$ reaction

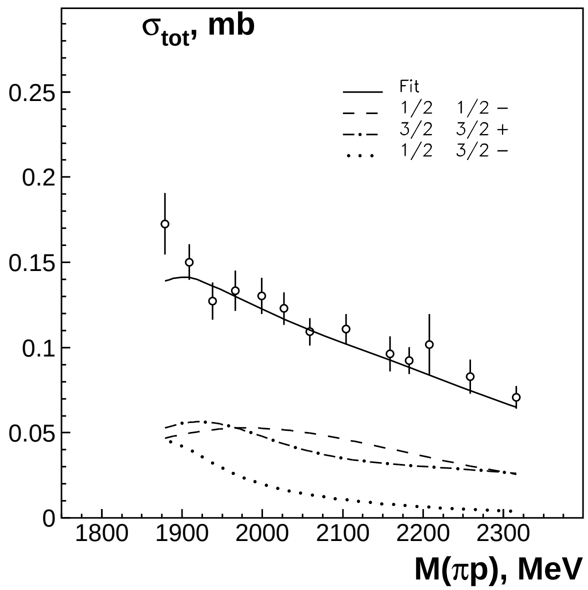


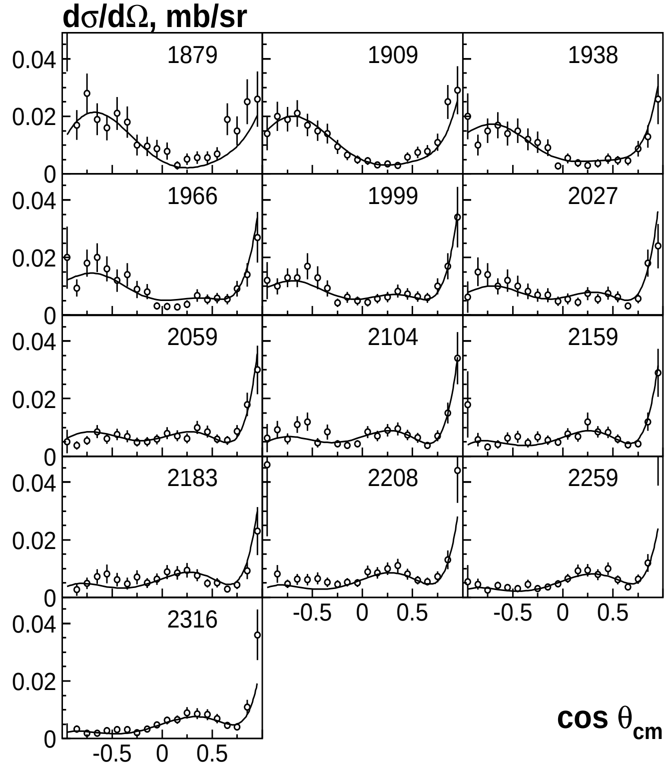


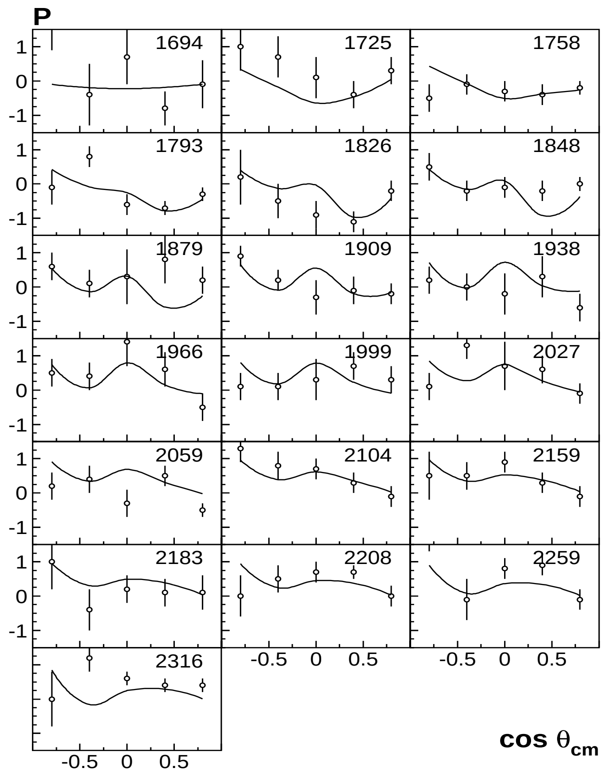
The fit of the the $\pi^+ p \to K^+\Sigma^+$ reaction



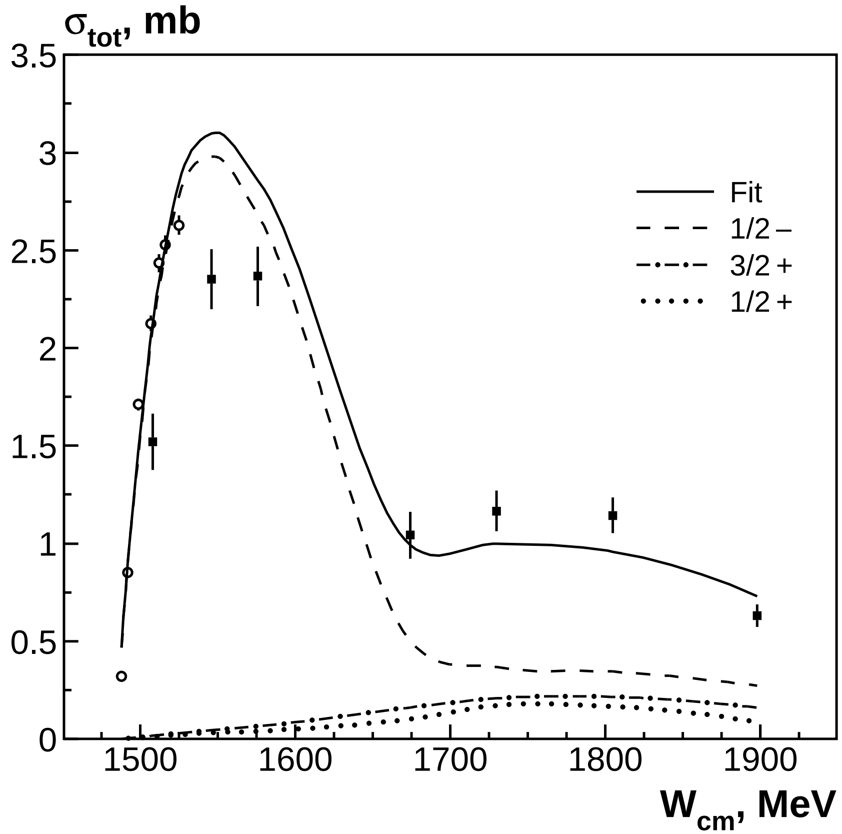
The fit of the the $\pi^- p \to K^0 \Sigma^0$ reaction

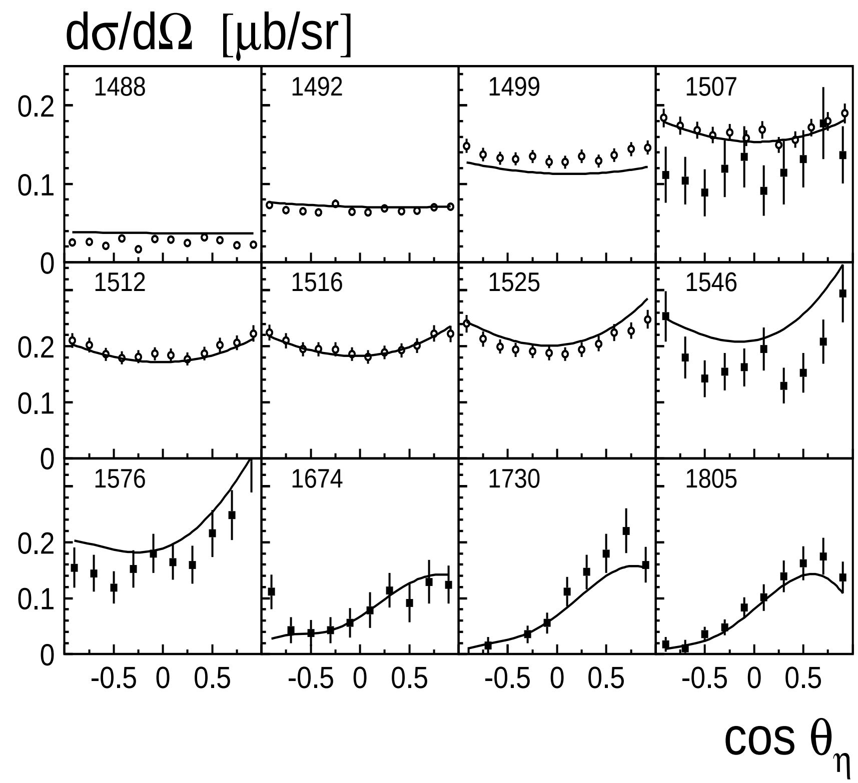




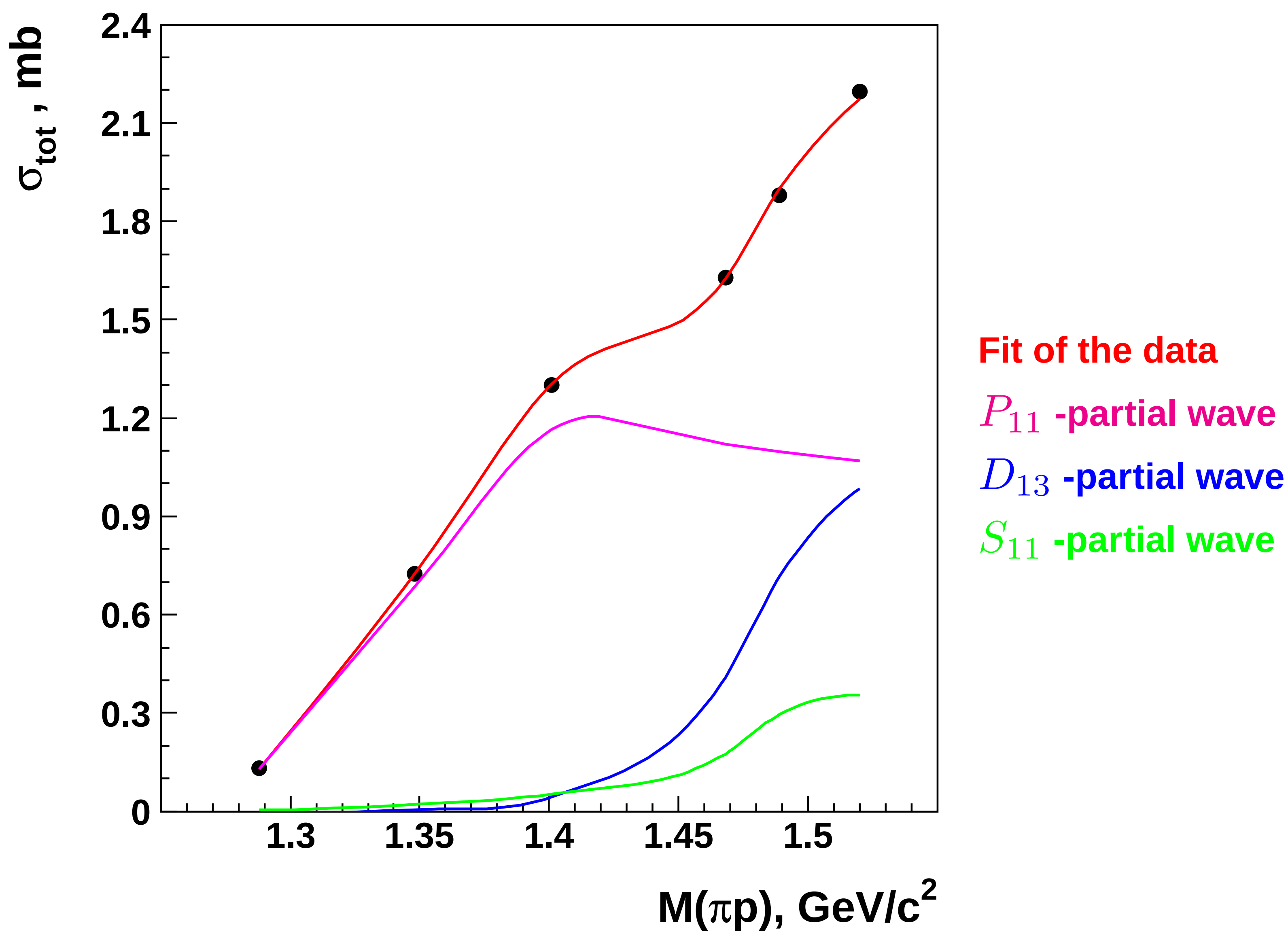


The fit of the $\pi^- p \to \eta n$ reaction



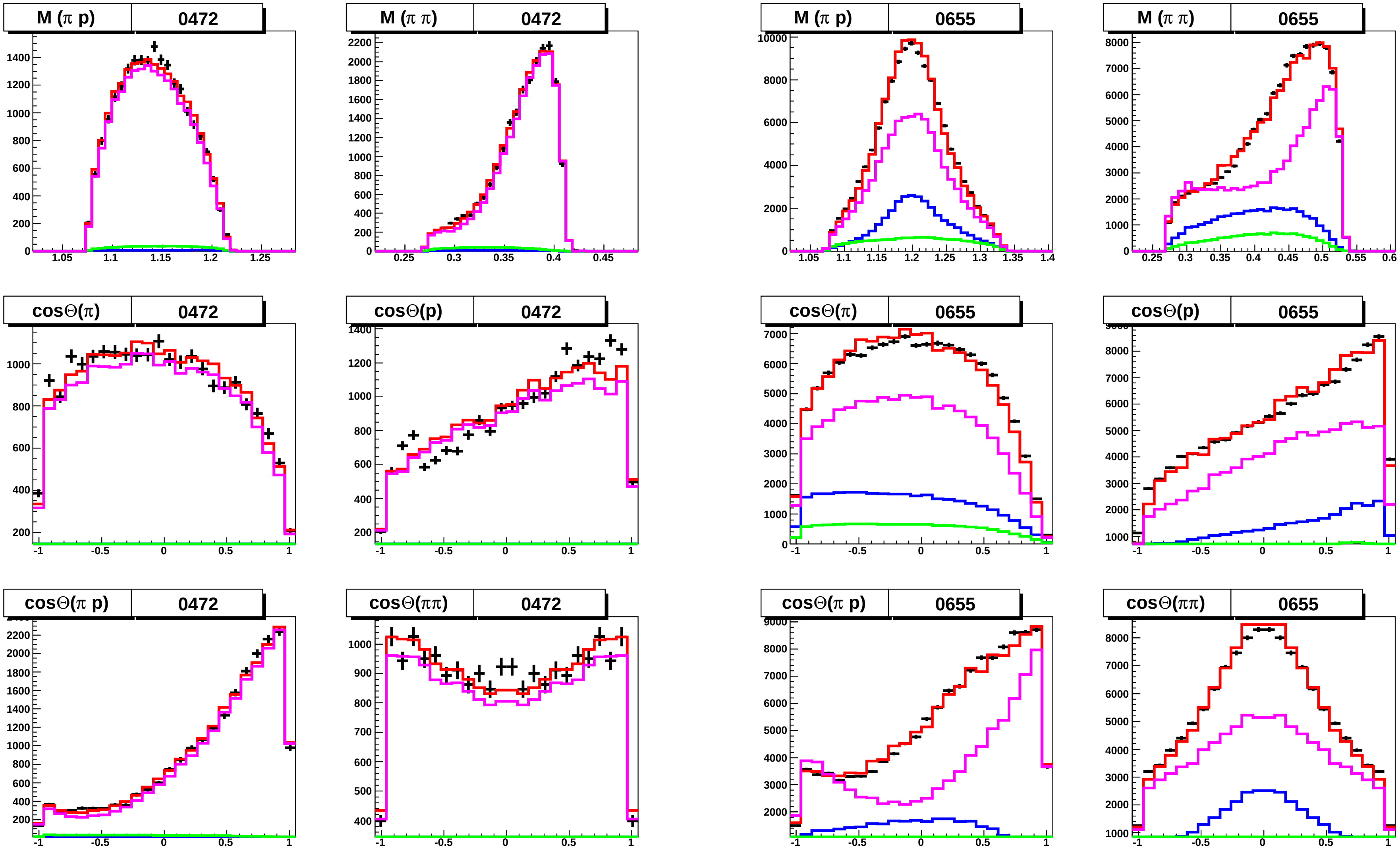


$\pi^- p \to n\pi^0\pi^0$ (Crystal Ball) total cross section



$\pi^- p \to n\pi^0\pi^0$ (Crystal Ball)

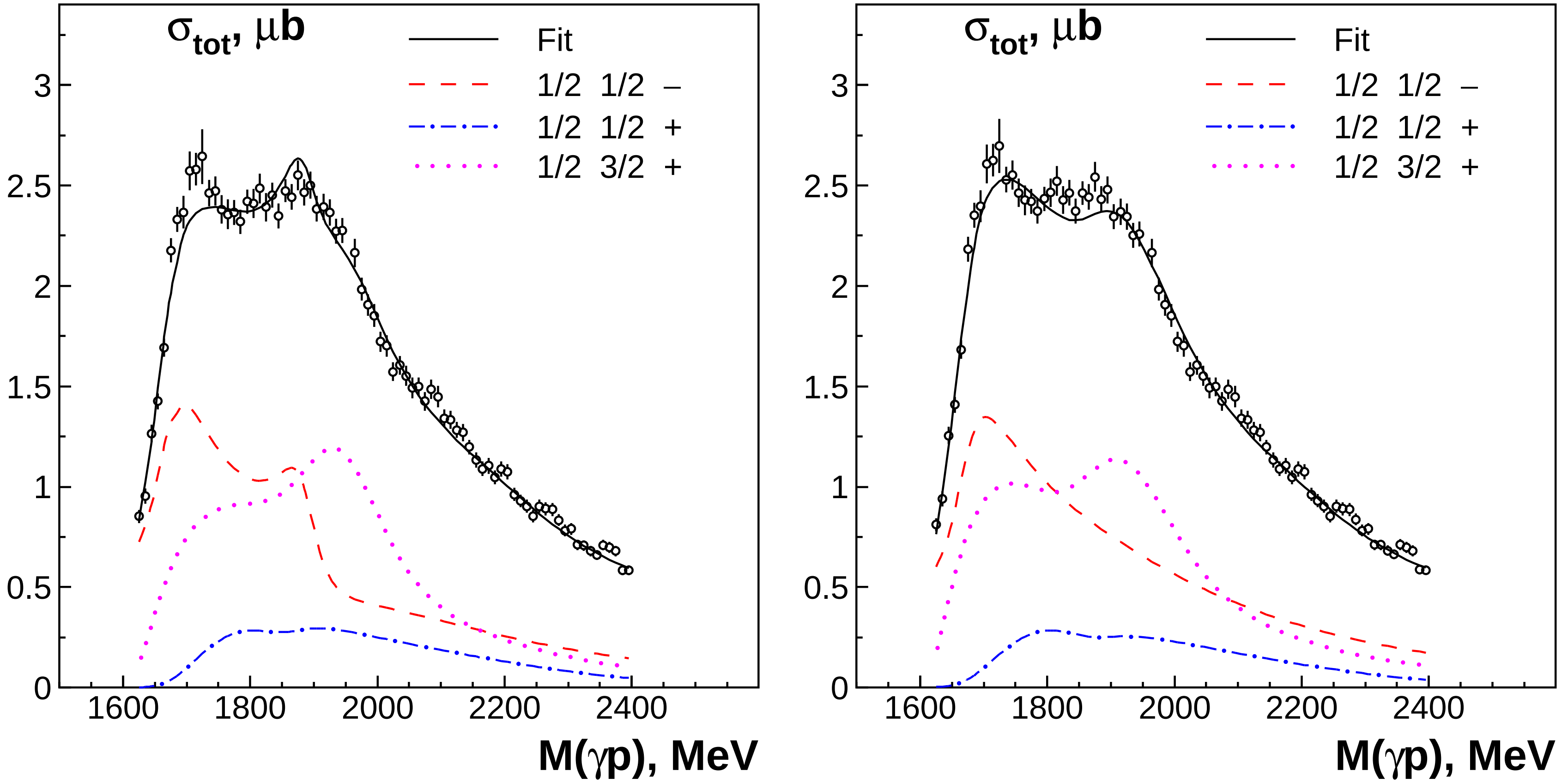
Differential cross sections for 472 and 665 MeV/c data.



Description of all fitted single meson photoproduction observables as well as multipoles can be downloaded in numerical form or as PDF figures from

PWA.HISKP.UNI-BONN.DE

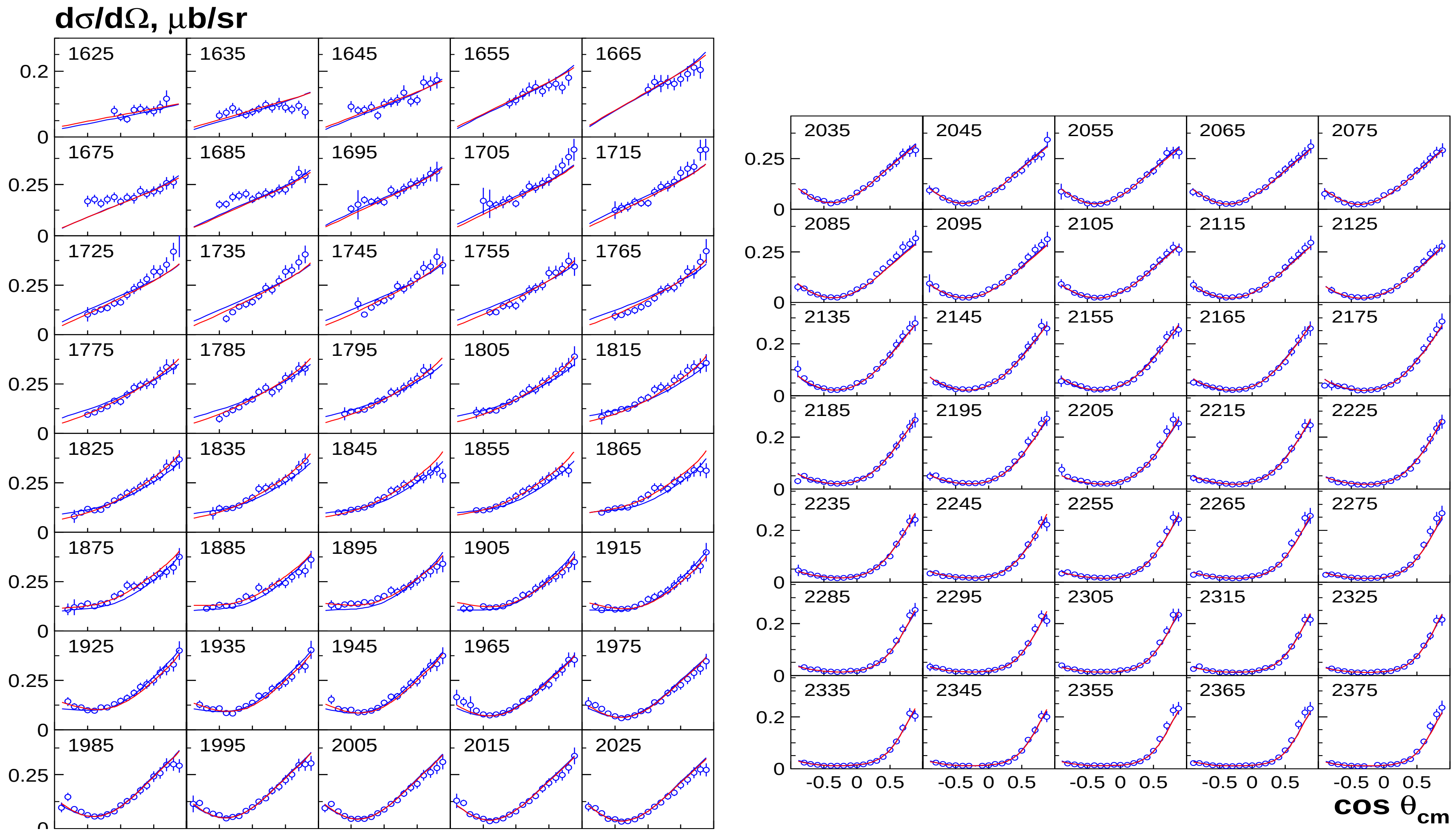
The $\gamma p \to K\Lambda$ reaction (CLAS 2009)



In the first solution the new S_{11} state with mass 1890 ± 10 MeV and width 90 ± 10 MeV is introduced in the fit.

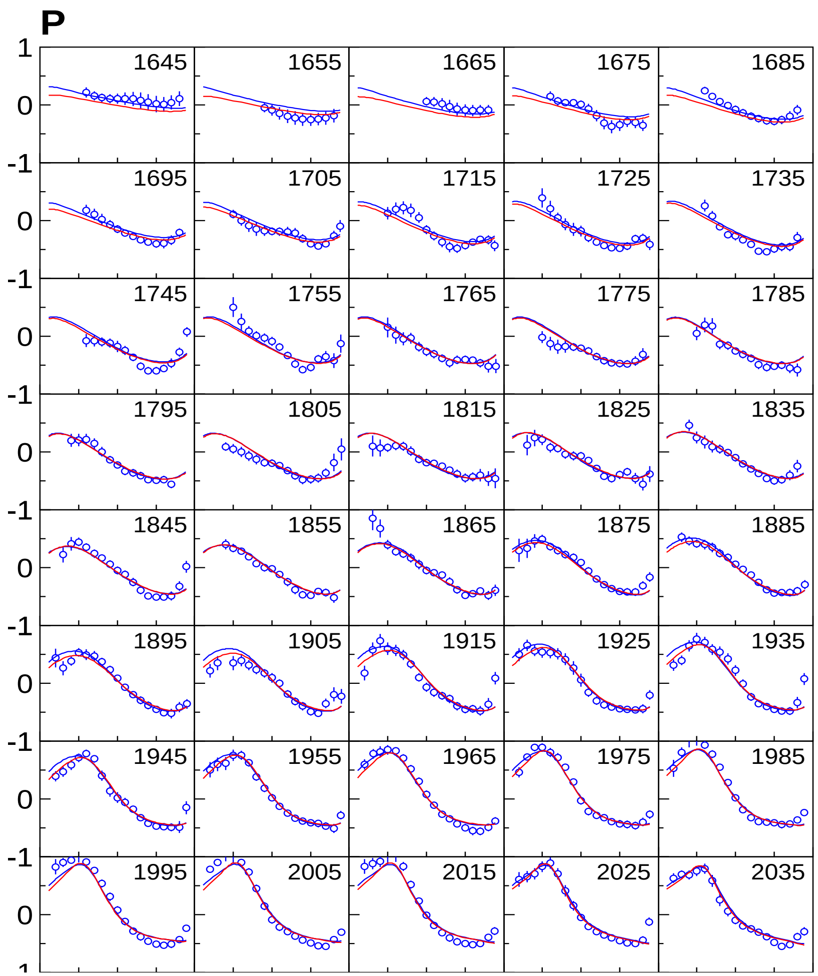
The fit of the $\gamma p \rightarrow K\Lambda$ differential cross section

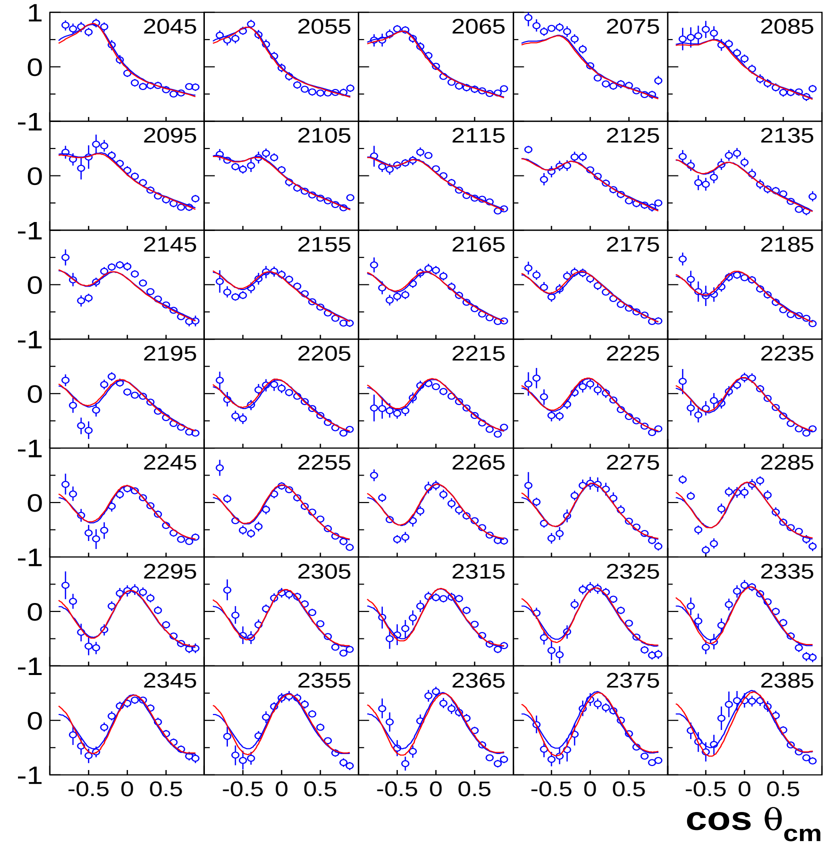
(CLAS 2009)



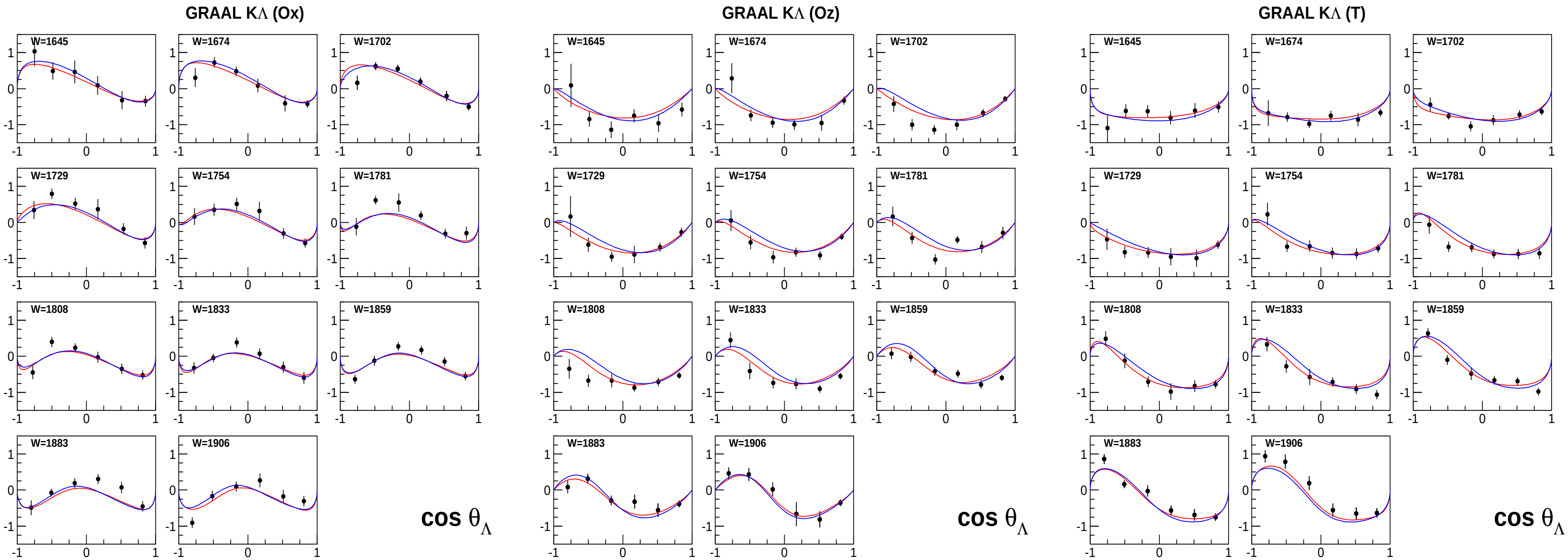
The fit of the $\gamma p \to K\Lambda$ recoil asymmetry

(CLAS 2009)

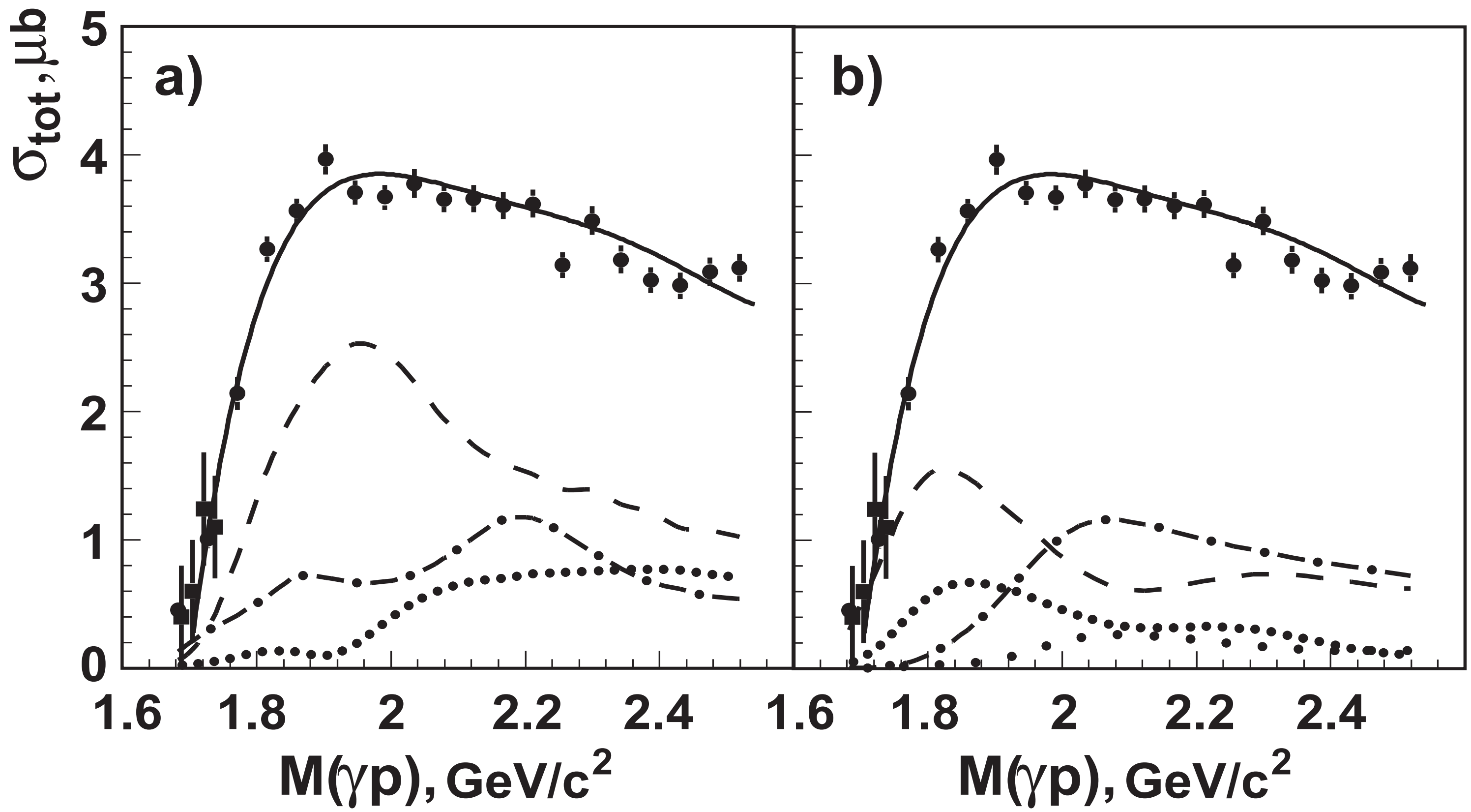




The O_x, O_z and T observables from the $\gamma p \to K\Lambda$ reaction (GRAAL)



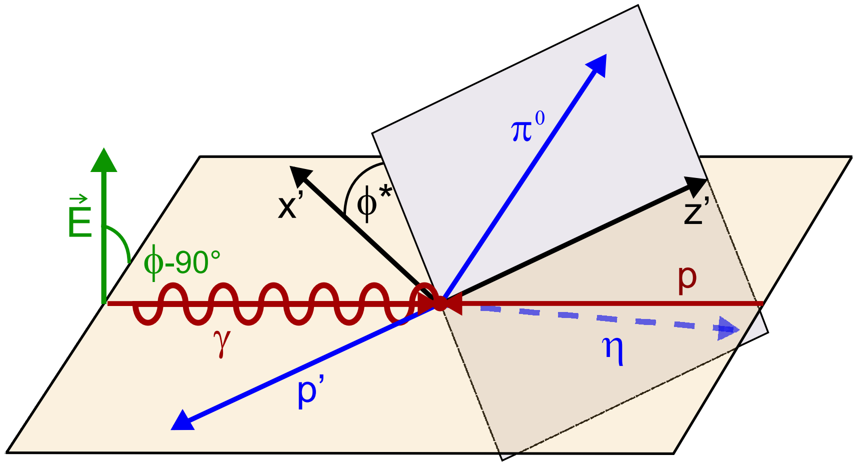
$\gamma p \to p\pi^0\eta$ (CB-ELSA)



Left panel : contributions from $\Delta(1232)\eta$ (dashed), $S_{11}(1535)\pi$ (dashed-dotted) and $N\,a_0(980)$ final states.

Right panel: D_{33} partial wave (dashed), P_{33} partial wave (dashed-dotted), $D_{33} \to \Delta(1232)\eta$ (dotted) and $D_{33} \to N\,a_0(980)$ (wide dotted).

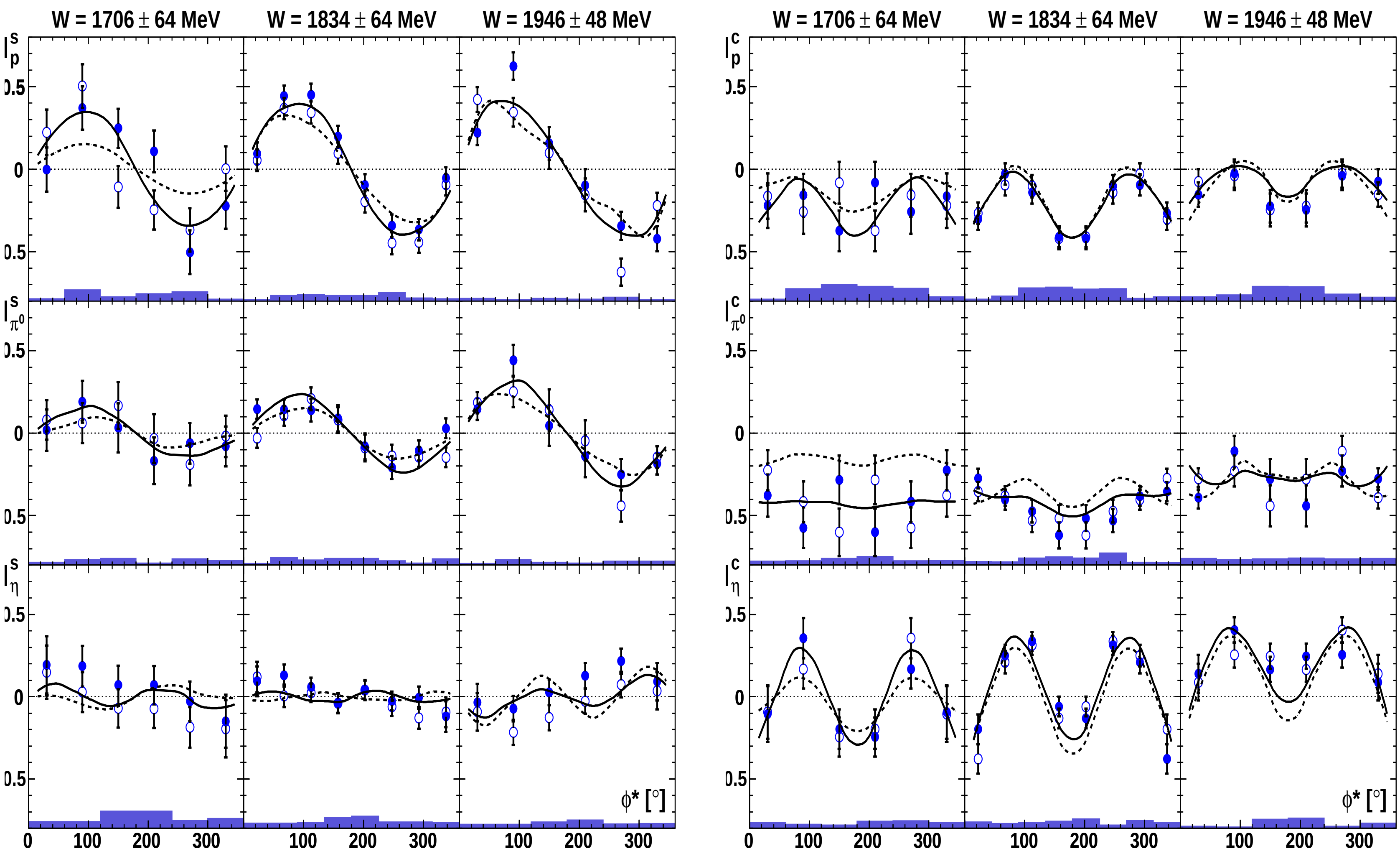
$\gamma p \to p\pi^0\eta$ (CB-ELSA)with linear polarized photon



$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_0 \{1 + \delta_l [I^s \sin(2\phi) + I^c \cos(2\phi)]\}\,, \tag{1}$$

$$\Sigma = \int_0^{2\pi} I^c d\phi^*$$

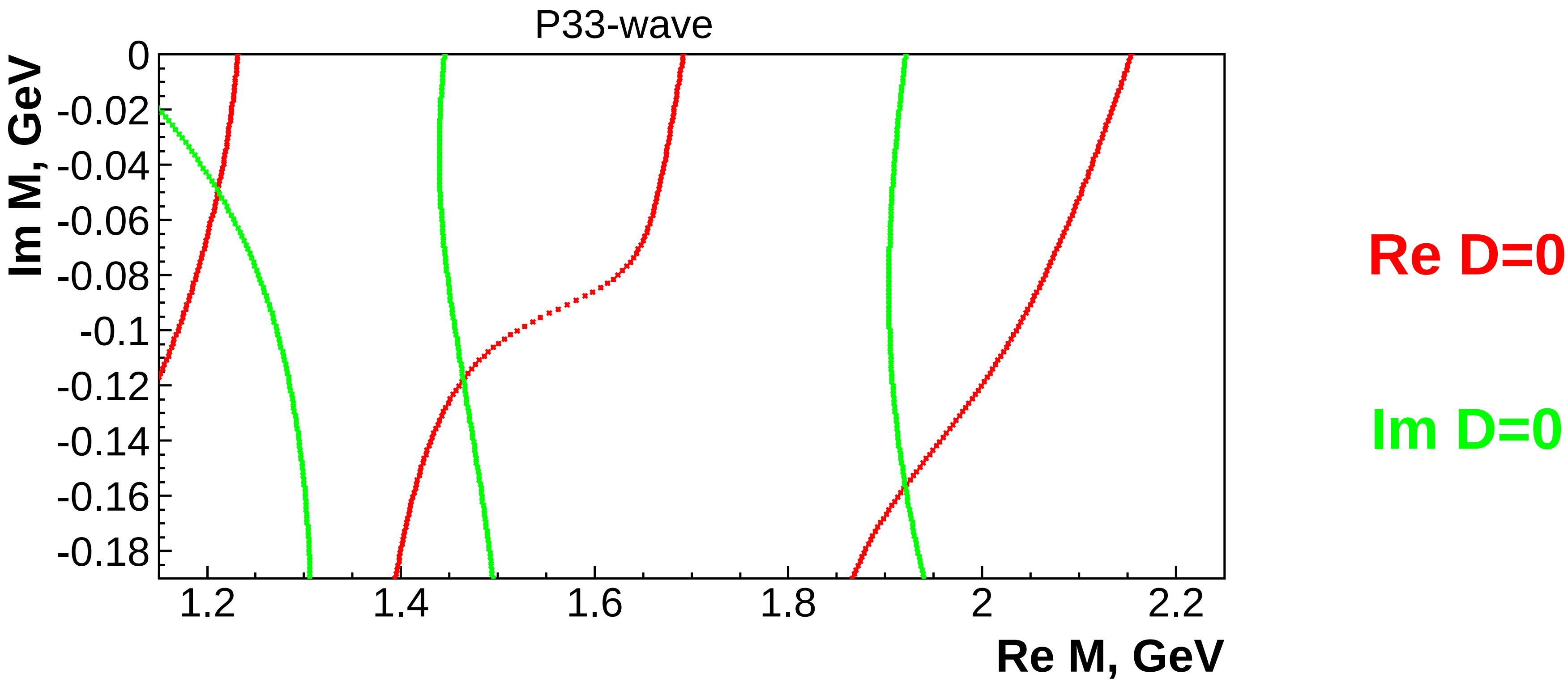
I^c and I^s for $\gamma p \to p\pi^0\eta$ (CB-ELSA)



Search for the pole position in the complex plane

P_{33} wave (4 pole 6 channel K-matrix)

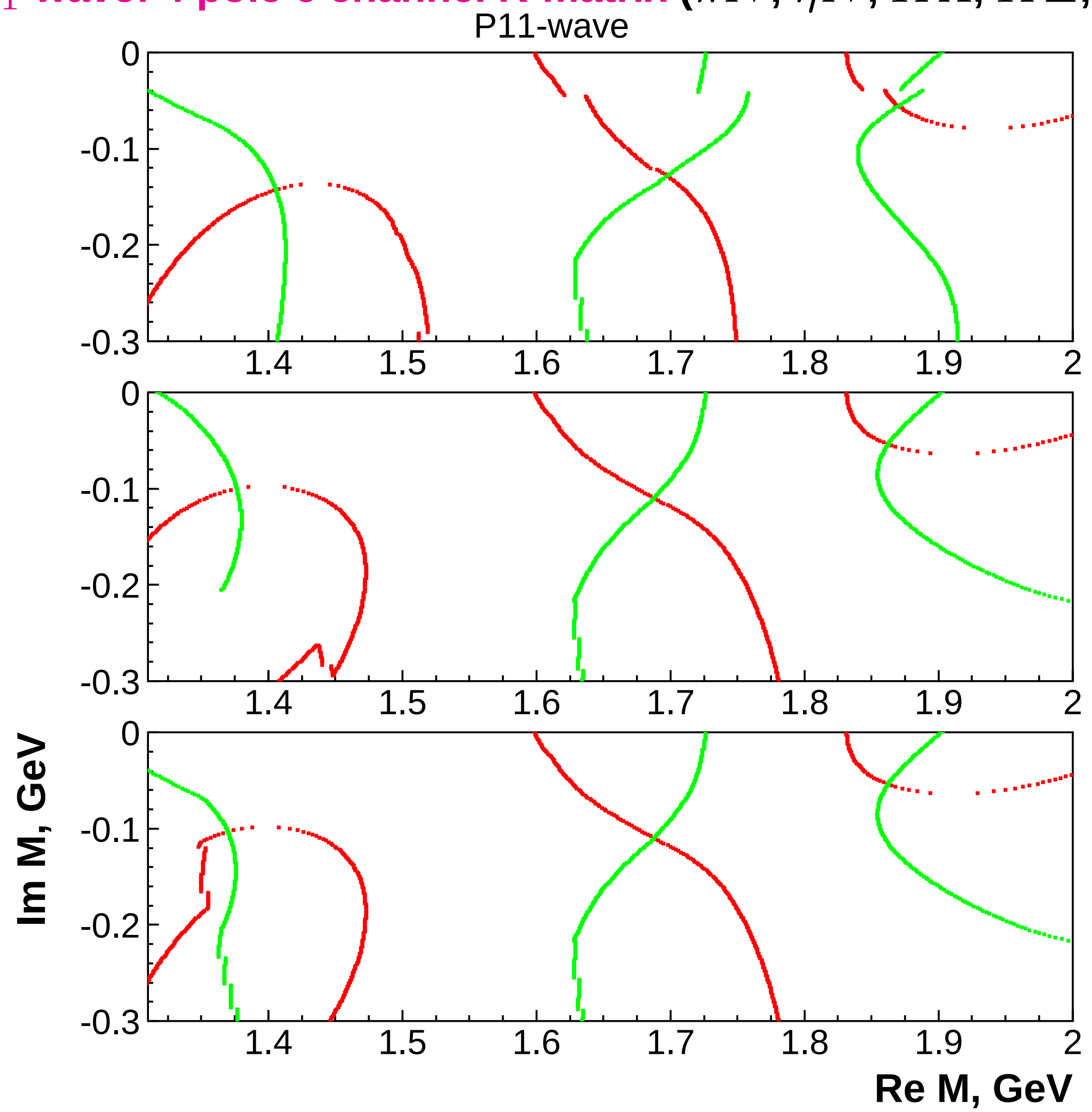
$$D = det(I - i\rho K)\prod_i (M_i^2 - s) \qquad 1-\text{pole}: D = M^2 - s - i\sum_j \rho_j(s) g_j^2$$



T-matrix poles: $M = 1210 - i50$ **MeV;**

$M = 1485 - i120$ **MeV** $\qquad M = 1920 - i160$ **MeV**

P_{11} wave: 4 pole 6 channel K-matrix (πN, ηN, $K\Lambda$, $K\Sigma$, $\pi\Delta(1232)$, $N\sigma$. Solution 1.)



Re D=0

Im D=0

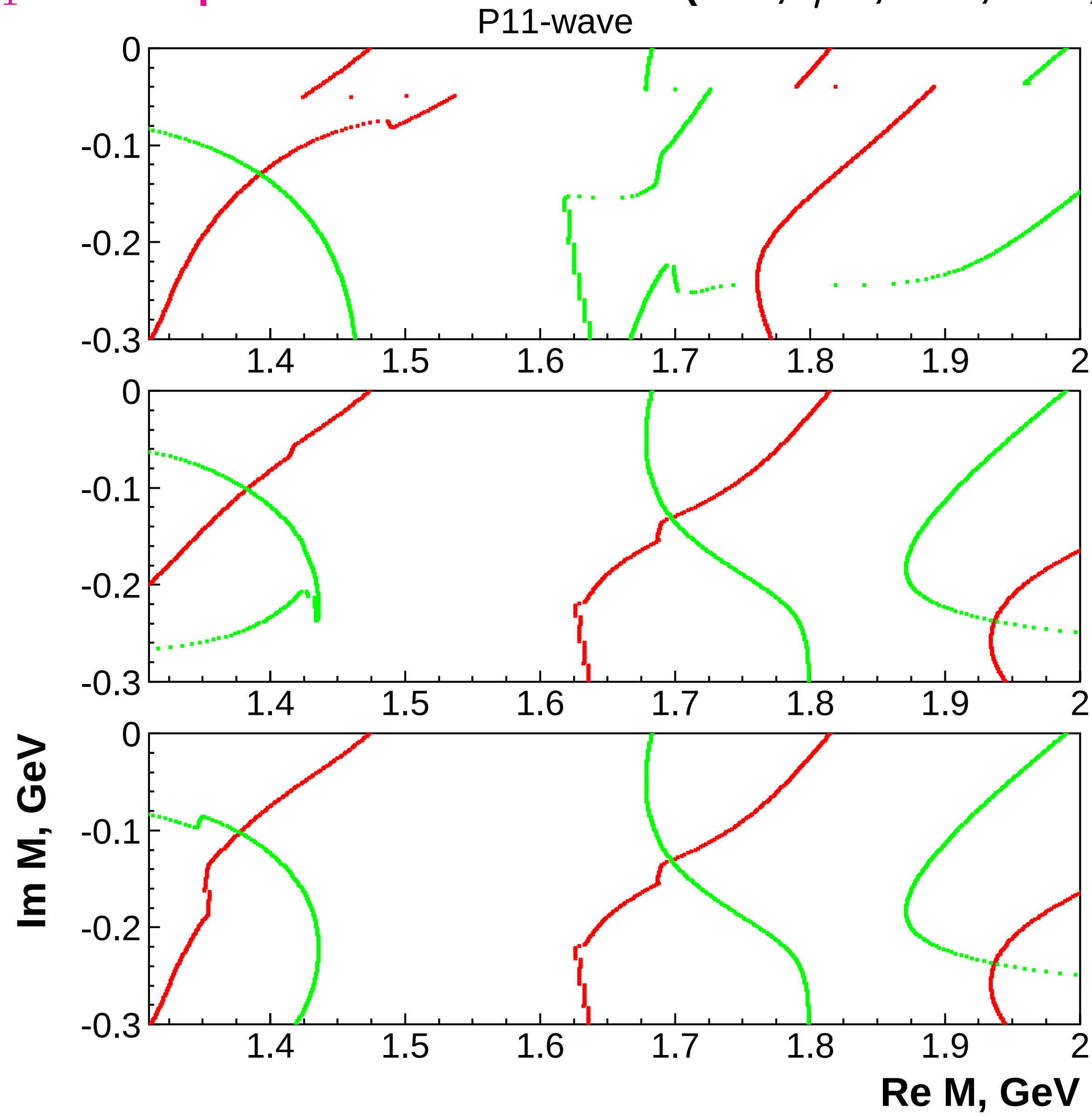
T-matrix poles:

$M = 1370 - i100$ **MeV;**

$M = 1695 - i105$ **MeV**

$M = 1860 - i60$ **MeV**

P_{11} wave: 4 pole 6 channel K-matrix (πN, ηN, $K\Lambda$, $K\Sigma$, $\pi\Delta(1232)$, $N\sigma$. Solution 2.)



Re D=0

Im D=0

T-matrix poles:

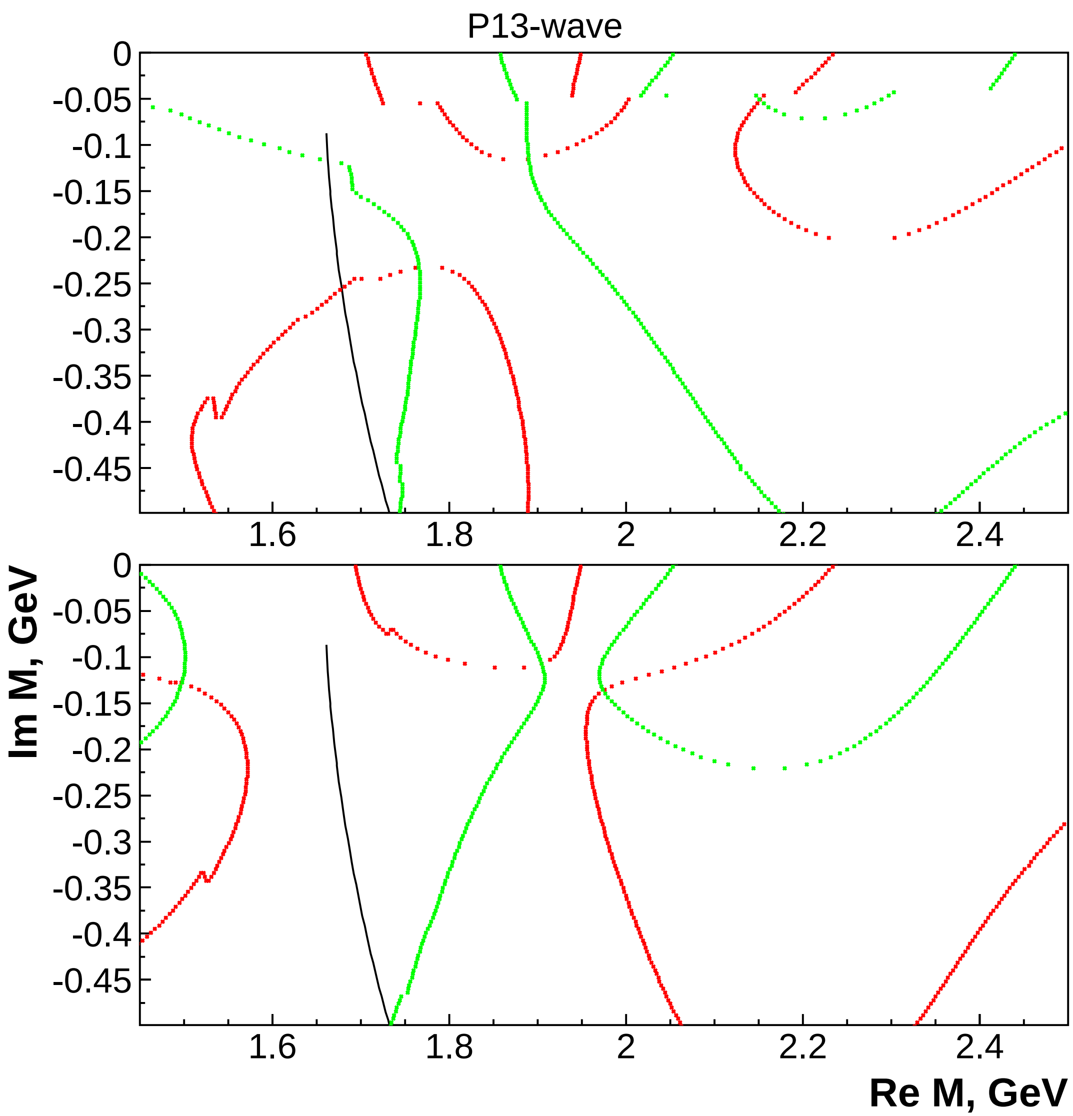
$M = 1370 - i100$ **MeV;**

$M = 1695 - i115$ **MeV**

$M = 1940 - i220$ **MeV**

P_{13}: 3-pole 8-channel K-matrix

(πN**,** ηN**,** $K\Lambda$**,** $K\Sigma$**,** $\pi\Delta(1232)$**(P,F),**$N\sigma$**,** $D_{13}(1520)\pi$**)**



Re D=0 **Im D=0**

I sheet: closest to the physical region below $D_{13}(1520)\pi$ **threshold.**
$M = 1730 - i230$ **MeV;**

II sheet: closest to the physical region above $D_{13}(1520)\pi$ **threshold.**
$M = 1500 - i125$ **MeV**
$M = 1900 - i100$ **MeV**
$M = 1980 - i140$ **MeV**

Tabelle 1: Pole position (in MeV), πN, ηN, $K\Lambda$ and $K\Sigma$ couplings (in GeV) and photo-couplings (in GeV$^{-1/2}10^3$).

State	$P_{11}(1440)$	$P_{11}(1710)$
Re(pole)	1375 ± 6 **(**1365 ± 15**)**	1690^{+25}_{-10} **(**1720 ± 50**)**
-2Im(pole)	200 ± 10 **(** 190 ± 30**)**	230^{+30}_{-20} **(** 230 ± 150**)**
$g(\pi N)$	0.49 ± 0.03 **/**-40 ± 6^o	0.16 ± 0.06 **/**-$(5^{+20}_{-50})^o$
$g(\eta N)$	-0.12 ± 0.05 **/** 20 ± 10^o	-0.16 ± 0.05 **/**-20 ± 25^o
$g(K\Lambda)$		0.70 ± 0.20 **/**-8 ± 10^o
$g(K\Sigma)$		0.10 ± 0.05 **/**$(60^{+60}_{-30})^o$
$A^{1/2}(\gamma p)$	-44 ± 10 **/**-$37^o\pm10^o$	-65 ± 25 **/** $-65^o\pm20^o$
State	$P_{11}(1840)$	$P_{13}(1720)$
Re(pole)	1860 ± 10 **( )**	1720 ± 50 **(**1675 ± 15**)**
-2Im(pole)	110^{+30}_{-10} **( )**	420 ± 80 **(**190 ± 85**)**
$g(\pi N)$	0.12 ± 0.04 **/** $(15^{+15}_{-25})^o$	0.78 ± 0.12 **/**-35 ± 10^o
$g(\eta N)$	-0.46 ± 0.10 **/** 25 ± 12^o	0.75 ± 0.15 **/**-15 ± 10^o
$g(K\Lambda)$	-$(0.07^{+0.10}_{-0.05})$ **/** $0^{+12}_{-22}\ ^o$	0.60 ± 0.35 **/**-15 ± 20^o
$g(K\Sigma)$	0.30 ± 0.10 **/** $40^{+60}_{-30}\ ^o$	1.15 ± 0.60 **/**-10 ± 10^o
$A^{1/2}(\gamma p)$	-14 ± 6 **/**$50^o\pm50^0$	160 ± 30 **/**-$25^o\pm35^o$
$A^{3/2}(\gamma p)$		150 ± 60 **/**$50^o\pm40^o$

Tabelle 2: Pole position (in MeV), πN, ηN, $K\Lambda$ and $K\Sigma$ couplings (in GeV) and photo-couplings (in GeV$^{-1/2}10^3$).

State	$P_{13}(1960)$	$P_{13}(1900)$
Re(pole)	1970 ± 12 **(~1900)**	1890 ± 50 **( )**
-2Im(pole)	300 ± 60 **( )**	270^{+200}_{-100} **( )**
$g(\pi N)$	0.13 ± 0.20 / 20 ± 50^o	0.15 ± 0.10 / $(20^{+50}_{-100})^o$
$g(\eta N)$	-0.70 ± 0.20 / 5 ± 15^o	-$(0.40^{+0.40}_{-0.30}$ /-$(5^{+70}_{-50})^o$
$g(K\Lambda)$	-$(1.10^{+0.50}_{-0.30})$ / 0 ± 15^o	-0.70 ± 0.35 / $(5^{+70}_{-35})^o$
$g(K\Sigma)$	-0.40 ± 0.15 / $(35^{+15}_{30})^o$	$0.40^{+0.50}_{-0.25}$ /-$(5^{+40}_{-100})^o$
$A^{1/2}(\gamma p)$	9 ± 7 /-2 ± 10^o	63 ± 20 / $65^o\pm20^o$
$A^{3/2}(\gamma p)$	50 ± 40 / $55^o\pm40^o$	63 ± 15 / $80^o\pm30^o$
State	$P_{33}(1600)$	$P_{33}(1920)$
Re(pole)	1480 ± 40 **(**1550 ± 100**)**	1925 ± 40 **(**1900 ± 50**)**
-2Im(pole)	230 ± 40 **(**300 ± 100**)**	320 ± 50 **(**300 ± 100**)**
$g(\pi N)$	0.40 ± 0.10 / 85 ± 15^o	0.45 ± 0.15 /-30 ± 25^o
$g(K\Sigma)$	-0.15 ± 0.08 /-15 ± 15^o	-0.20 ± 0.10 / 20 ± 15^o
$A^{1/2}(\gamma p)$	20 ± 12 / $55^o\pm20^o$	100 ± 20 /-$55^o\pm15^o$
$A^{3/2}(\gamma p)$	14 ± 10 /-$5^o\pm20^o$	-73 ± 12 / $35^o\pm15^o$

Summary

1. **Analysis of the $\pi^- p \to K^0 \Lambda$ reaction confirms firmly the $P_{11}(1710)$ state. It also confirms existence of the $P_{11}(1860)$ state however there are two solutions which give very different widthes for this state.**

2. **The data on $\pi^+ p \to K^+ \Sigma^+$ confirm the $P_{33}(1600)$ and $P_{33}(1920)$ resonances.**

3. **The fit of new polarization observable I_c on $\gamma p \to \eta \pi^0 n$ confirms the solution published in Eur.Phys.J.A38:173-186,2008: $D_{33}(1980)$.**

4. **The combined analysis of pion- and photoproduction reactions confirms the $P_{13}(1900)$ state. Moreover, there is a strong indication for a double pole structure in this region.**

5. **The new data on the $\gamma p \to K\Lambda$ reaction (CLAS: $d\Sigma/d\Omega$, P; GRAAL: O_x, O_z, T shows an indication for the third S_{11} state with mass 1890 ± 10 MeV and width 90 ± 10 MeV.**

Problem: how to compare our results with other analyses?

For example with Breit-Wigner parameters given in PDG.

We construct the following amplitude:

$$A_{ij}^{BW} = \frac{g_i^{BW} g_j^{BW}}{M_{BW}^2 - s - i\beta \sum_i g_i^2 \rho_i}$$

where M_{BW} and β are fitted to reconstruct pole position and g_i^{BW} to reconstruct residues in the pole.

As a cross-check: the procedure works very well for the relativistic Breit-Wigner amplitude

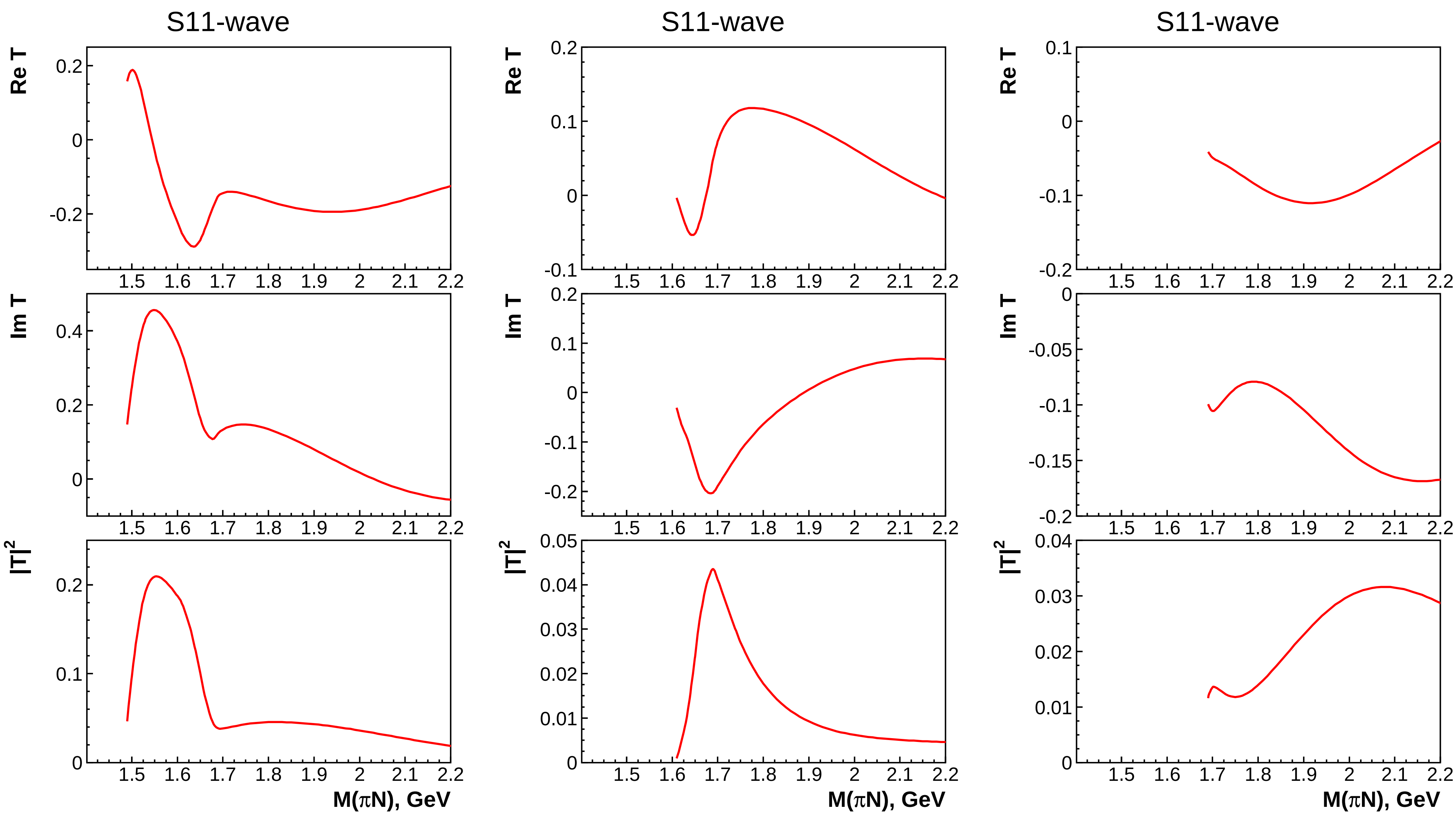
$$(g_i^{BW})^2 \sim \beta g_i^2$$

and width can be estimated as:

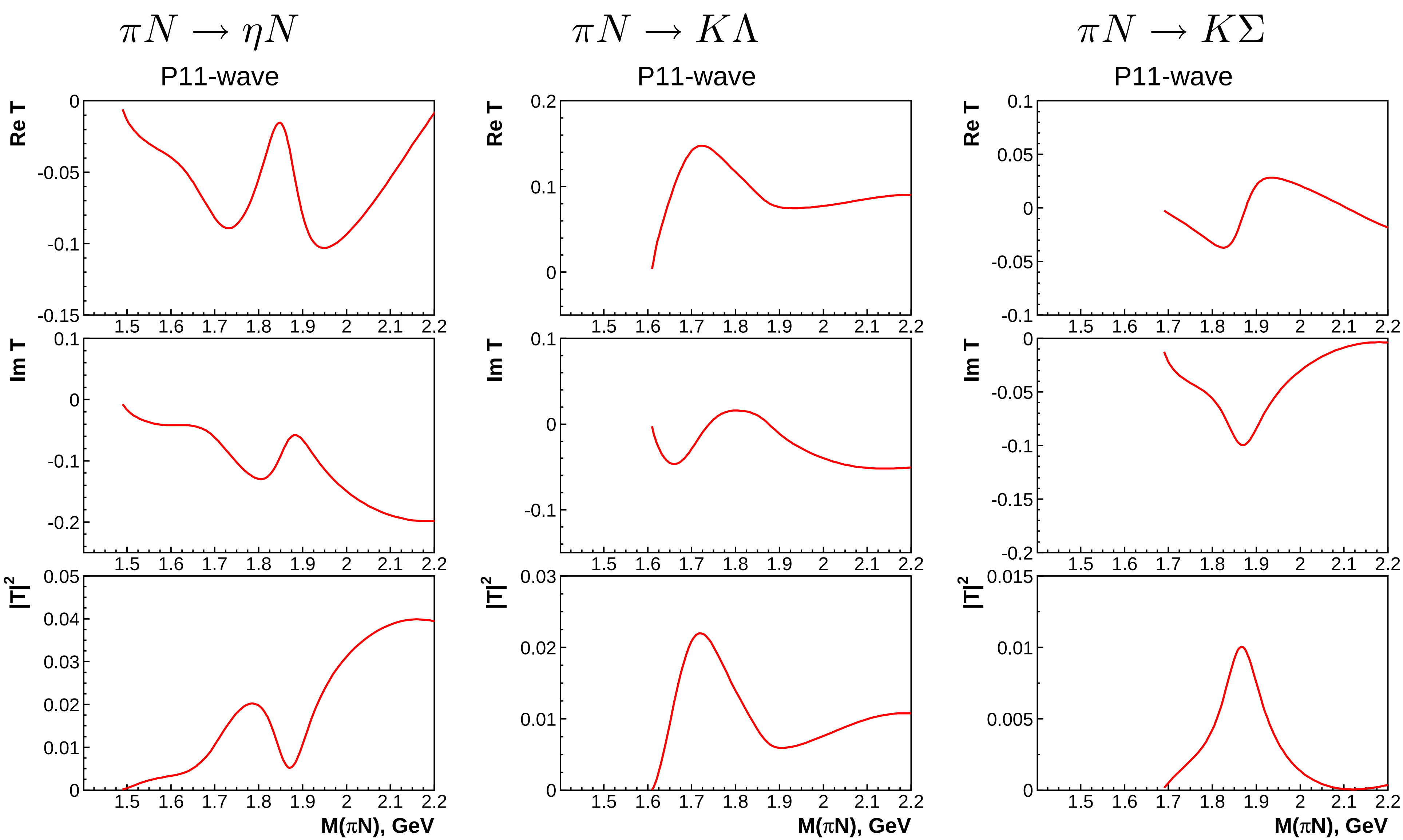
$$M_{BW}\Gamma_{tot}^{BW} = Im(i\beta \sum_i g_i^2 \rho_i)$$

S11-wave transition amplitudes

$\pi N \to \eta N$ $\pi N \to K\Lambda$ $\pi N \to K\Sigma$

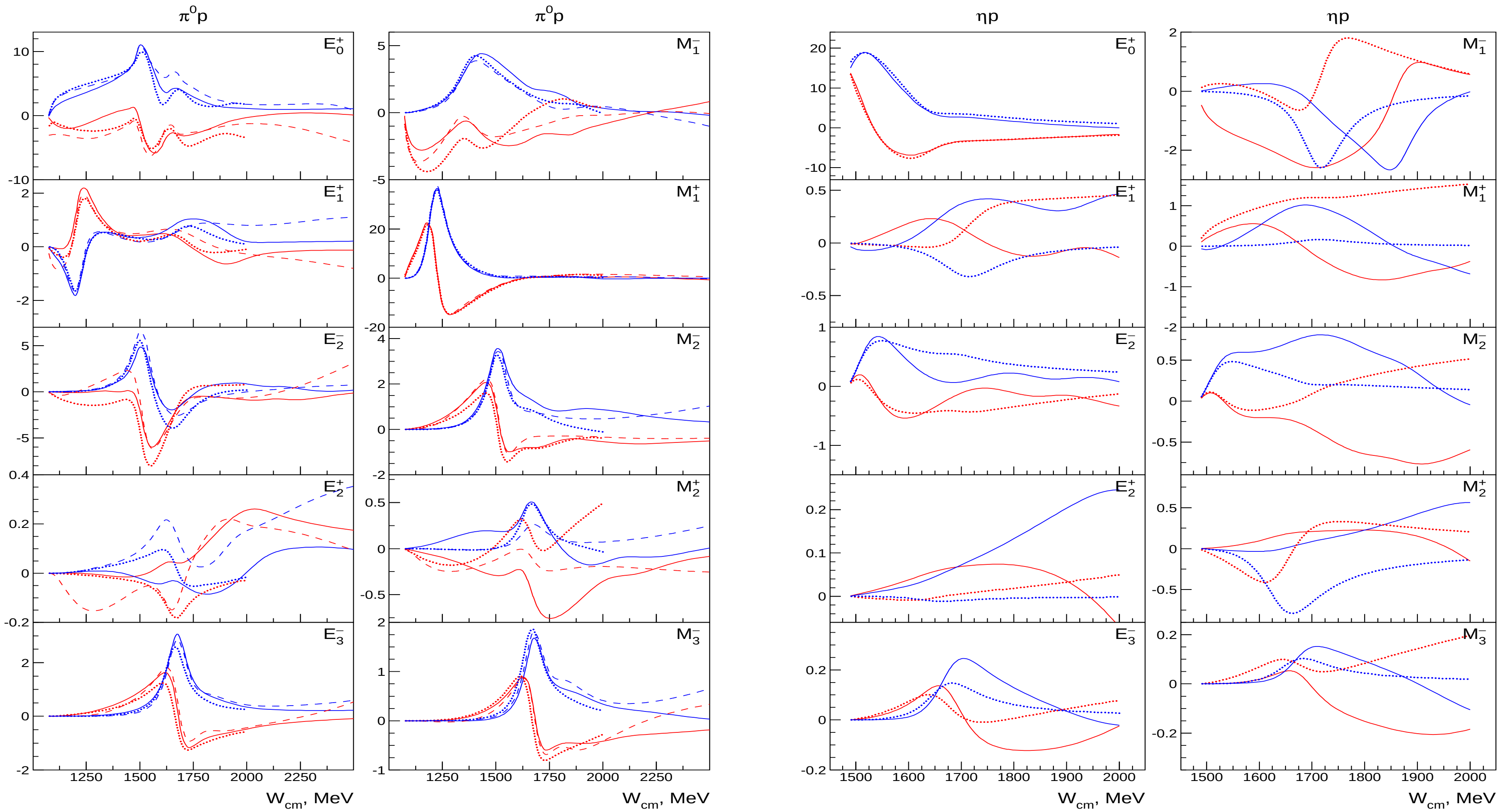


P11-wave transition amplitudes

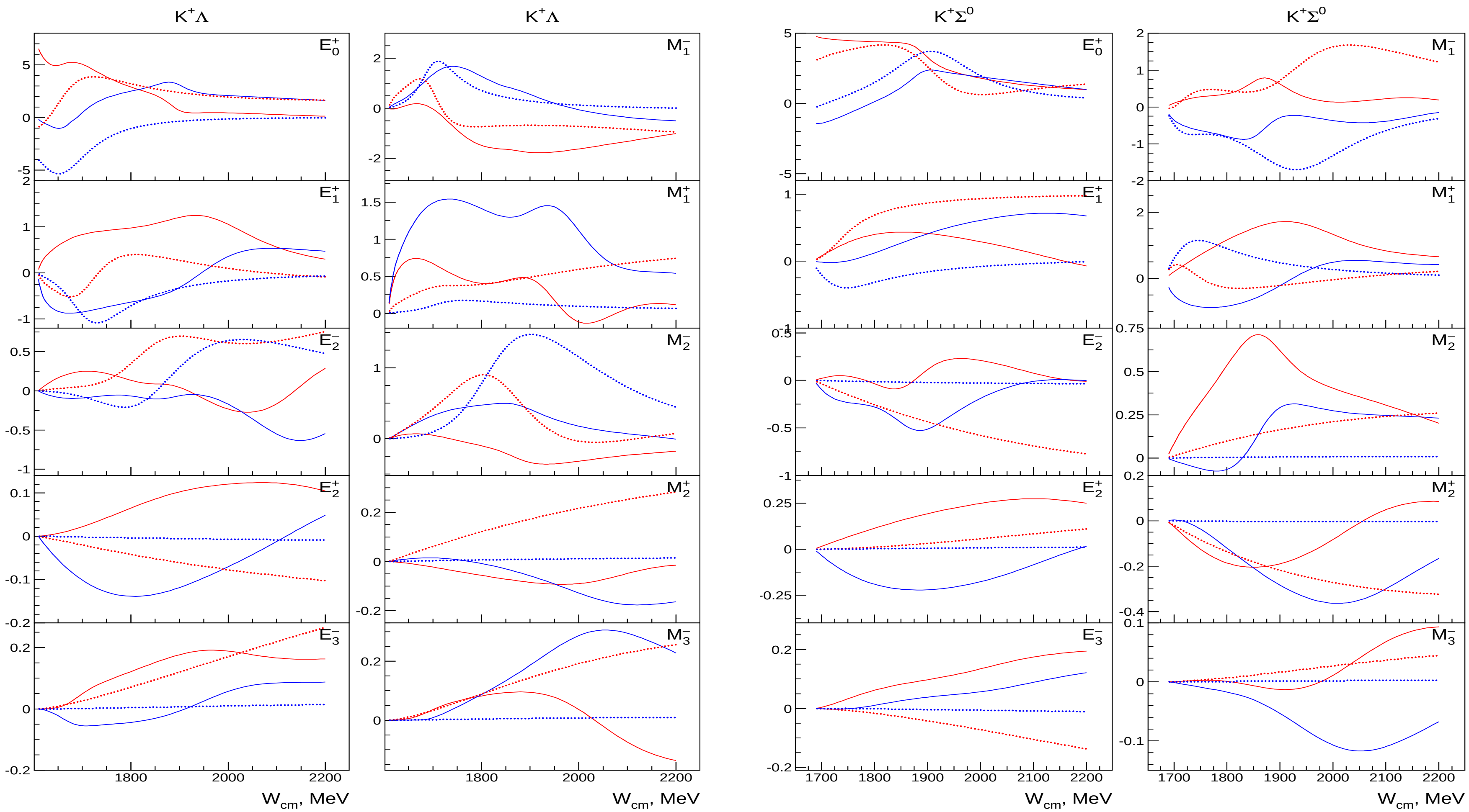


Multipoles for the single π^0 and η production. Red - real part, Blue - imaginary part.

Solid curves is our solution, dashed curves - SAID solution, dotted - MAID 2009.

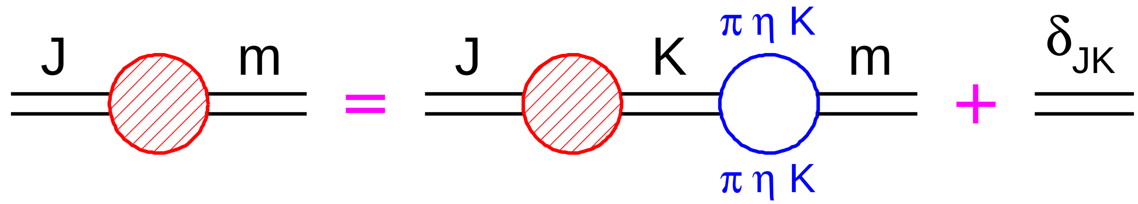


Multipoles for the $K\Lambda$ and $K\Sigma$ final states. Red - real part, Blue - imaginary part. Solid curves is our solution, dashed curves - - MAID 2009.



N/D based analysis of the data

In the case of resonance contributions only we have factorization and Bethe-Salpeter equation can be easily solved:



$$A_{jm} = A_{jk} \sum_{\alpha} B^{km}_{\alpha}(s) \frac{1}{M_m - s} + \frac{\delta_{jm}}{M_j^2 - s} \qquad B^{km}_{\alpha}(s) = \int\limits_{4m_j^2}^{\infty} \frac{ds'}{\pi} \frac{g^{(k)}_{\alpha}(s')\rho(s')g^{(m)}_{\alpha}(s')}{s' - s - i0}$$

$$\hat{A} = \hat{\kappa}(I - \hat{B}\hat{\kappa})^{-1} \qquad \kappa_{ij} = \frac{\delta_{ij}}{M_i^2 - s} \qquad B^{ij} = \sum_{\alpha} B^{km}_{\alpha}(s)$$

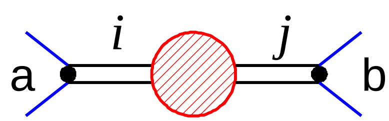
For non-resonant contributions: there is no factorization and the amplitude can have a complicated energy dependence. However in majority of K-matrix analysis the nonresonant contributions are constant or have a simple energy dependence. Non-factorization can be taken into account by introduction of two transitions with fixed left and right vertices.

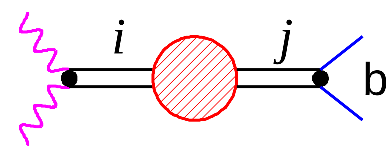
Parameterization of P_{13} wave: 3 resonances 8 channels, 4 non-resonant contributions $\pi N \to \pi N$, $\pi N \to \eta N$, $\pi N \to K\Sigma$, $\pi N \to \Delta\pi$. It corresponds to 8×8 channel K-matrix and 5×5 N/D-matrix.

In many cases (fixed form-factor or subtraction procedure) the real part can be calculated in advance (for S-wave):

$$B(s) = ReB(M^2) + \frac{g^2}{\pi}[\rho(s)\ ln\frac{1-\rho(s)}{1+\rho(s)} - \rho(M^2)\ ln\frac{1-\rho(M^2)}{1+\rho(M^2)}] + i\rho(s)g^2$$

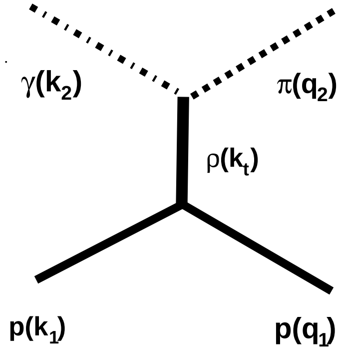
The P-vector approach is strait forward:

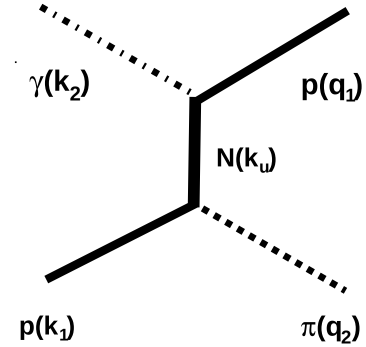
$A_{ab} = \sum_{ij}$ 

$P_b = \sum_{ij}$ 

1. **This approach satisfies analyticity and two body unitarity conditions. It takes left-hand side singularities into account.**
2. **The approach is suitable for the analysis of high statistic data in combined analysis of many reactions.**
3. **However: a treatment of the real part for interfering resonances is model dependent.**

Reggezied exchanges:





The amplitude for t-channel exchange:

$$A = g_1(t)g_2(t)R(\xi,\nu,t) = g_1(t)g_2(t)\frac{1+\xi exp(-i\pi\alpha(t))}{\sin(\pi\alpha(t))} {\frac{\nu}{\nu_0}}^{\alpha(t)} \qquad \nu = \frac{1}{2}(s-u).$$

Here $\alpha(t)$ is Reggion trajectory, and ξ is its signature:

$$R(+,\nu,t) = \frac{e^{-i\frac{\pi}{2}\alpha(t)}}{\sin(\frac{\pi}{2}\alpha(t))\Gamma\frac{\alpha(t)}{2}} {\frac{\nu}{\nu_0}}^{\alpha(t)},$$

$$R(-,\nu,t) = \frac{ie^{-i\frac{\pi}{2}\alpha(t)}}{\cos(\frac{\pi}{2}\alpha(t))\Gamma\frac{\alpha(t)}{2}+\frac{1}{2}} {\frac{\nu}{\nu_0}}^{\alpha(t)}.$$